

A Bayesian Network for the Analysis of Traffic Accidents in Peru

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Abstract: Traffic accidents are a problem that affects the State and society, because they cause material damage, injuries and even the death of a person. This has led countries such as China, Switzerland and Australia to carry out studies using Bayesian networks to determine the main causes and, based on them, propose measures to reduce the number of traffic accidents. Following this trend, we, without having any expert knowledge on the subject, decided to analyze the data of traffic accidents on the Pan-American Highway in Lima, Peru. This analysis was done by means of directed graph learning with the Hill Climbing Search, Chow-Liu, K2, BIC and BDEU. In addition, we used a Bayesian estimator to calculate the conditional probability distribution for our dataset. This dataset contains observations from the years 2017 to 2019 and approximately 16 km of this highway. Our results show that it is possible to identify the possible causes of excess accidents in specific areas of the Pan-American Highway in certain shifts i.e., 32% of fatal accidents occur between 12 am and 7 pm in the Rimac district and of these 20% are due to pedestrians on the highway.

1 INTRODUCTION

Traffic accidents are a problem that affects people and societies. This is because it does not allow the Peruvian State to guarantee the life and physical integrity of its people as indicated in the Peruvian constitution. Furthermore, it causes material damages, injuries and can even prematurely end a person's life. The National Institute of Statistics and Informatics (INEI) indicates that the number of fatal vehicle accidents throughout Peru reported from 2012 to 2019, is on average 3,000. Additionally, according to the Ministry of Health (MINSA), the World Health Organization (WHO), places Peru as the third country with the highest mortality from vehicular accidents¹.

Due to all the above, a method is required to analyze this problem based on the dependence of probabilities between variables that these accidents occur in detail, considering variables and their specific values, in order to allow us make inferences and based on the results of these, the citizens and the corresponding authorities adopt the corresponding measures to prevent these unfortunate events. Currently there are machine learning algorithms that allow predicting events

based on a set of data, which could contribute to this problem, however these only provide a numerical or nominal value as a result, other methods are required to understand how the algorithm arrived to that conclusion and do not provide sufficient information to adopt the corresponding measures to prevent these accidents (for example, knowing at what time the most accidents are likely to occur in order to deploy more traffic police officers).

A Bayesian Network (BN) is a probabilistic graphical model (PGM) that is represented by a directed acyclic graph (DAG) that depicts a set of variables and their conditional relationships. Bayesian networks are perfect for forecasting the likelihood that any one of numerous possible known causes contributed to an event that occurred. A BN, for example, could be used to illustrate the probability correlations between weather and climate conditions. The network may be used to calculate the chances of certain weather being present based on climate conditions. In Bayesian networks, efficient algorithms can do inference and learning. The repercussions of highway accidents can be characterized and assessed in terms of accident severity, which can range from catastrophic events with fatalities to minor fender bender damage. In most nations, official accident data exist that categorize accidents according to severity using the following simple scale: fatal accident, accident

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¹“8'929 accidentes de tránsito registra nuestro país a causa de la ebriedad del conductor” - MINSA - 2012

resulting in serious injury, accident resulting in minor injury, and accident resulting in merely property damage. The severity level is first defined when the incidents are documented, and these basic classifications are not consistent between countries.

Our contributions are as follows:

- We develop a Bayesian Network obtain with the Hill Climbing Search with Bdeu score and Chow-Liu Algorithm with K2 score.
- We carry out experiments to demonstrate that it is possible to identify that very serious accidents are generated with greater probability in the sections of the Pan-American Highway at a belonging to different districts, in specific shifts, due to different reasons and in different types.

Our work is structured as: in Section 2 presents the related works. Then, Section 3 presents the background of our approach and the structure of our model. Afterwards, Section 4 presents the experimentation and finally in Section 5, our conclusions.

2 RELATED WORKS

In (Deublein et al., 2015), the authors seek to predict the rates of minor, serious and fatal accidents that will occur on the road segments in Switzerland. They perform a multivariate regression analysis on the original data to obtain a base model. To this model they applied an algorithm called "expectation - maximization" to train the base model. And with this later model they calculate the conditional probabilities of the number of accidents per road segment. The purpose of this Bayesian model is to identify the road segments with the highest probability of accident occurrence for risk reduction.

In (Hongguo et al., 2010), the authors explain that the vehicular traffic system is complex, since it has as actors: people, vehicles, roads and the environment. And a traffic accident is caused by alteration of one of these components. The purpose of this paper is to find, through a Bayesian network, the causality in vehicular traffic accidents. The authors indicate that in previous studies only a unitary analysis is made and this can only reveal the inherent laws of traffic accident in a certain aspect, but does not contemplate that the causality of traffic accidents is multidimensional and there are correlations and logical relationships between causality factors. To learn the Bayesian graph, they used the K2 algorithm and the tool that Matalab offers for working with BN.

In (Zou and Yue, 2017), the authors give a general consideration of the factors affecting road safety

assessments, Bayesian network theory based on probability risk analysis is applied in the causality analysis of road accidents. Taking as a case the Adelaide Central Business District (CBD) in South Australia, the structure of the Bayesian network was established by integrating the K2 algorithm with the knowledge of experts, and the Expectation-Maximization algorithm was adopted that could process the missing data to perform parameter learning, thus establishing the Bayesian network model for the analysis of the causality of traffic accidents. The results showed that the Bayesian network model could effectively explore the complex logical relationship in traffic accidents and express the uncertain relationship between the related variables. Not only was the model able to quantitatively predict the probability of an accident under certain traffic conditions, it can also find the key reasons and worst-case combination that leads to the occurrence of an accident. Their results can provide theoretical support for urban road management authorities to thoroughly analyze the drivers of road accidents and then lay the basis for improving the safety performance of the urban road traffic system.

In (Makaba et al., 2021), the authors investigate the cost-implications of road traffic collision factors for the economy, and transport policies. They develop a Bayesian network framework using real-life road traffic collision data and expert knowledge to assess the cost of road traffic collisions.

In the first work, an analysis focused on a specific zone and evaluate where the most traffic accidents occur and provide improvement actions. The second work motivate us to map the different actors we had in the dataset to find causal relationships. In addition, it motivate us to explore and test with more Bayesian Network generation algorithms, not only K2. The third work help us to emphasize finding the probabilistic dependency relationships from the variables to improve urban road safety management.

3 MATERIAL

In this section we present and explain the main concepts that are used as the foundation of our work.

3.1 Preliminary Concepts

A Bayesian network is a probabilistic graphical model that consists of a set of random variables and their conditional dependencies represented by a directed acyclic graph (cause-effect relationships). If a variable has a parent node then it will have a table of conditional probability. These graphs have as main appli-

cations: classification, diagnosis, among others. Another advantage is that we can generate the Bayesian graphs without the complete dependence on field experts. This thanks to algorithms such as exhaustive search, the K2 algorithm and Hill-climbing. The basis of the Bayesian network is Bayes’ formula (see Equation (1)) for conditional probability:

$$P(H | D) = \frac{P(H) \times P(D | H)}{P(D)} \tag{1}$$

3.2 Methods

Now, we present some methods used to generate Bayesian Networks from the data.

3.2.1 Hill Climbing Search

It is a mathematical optimization algorithm, whose purpose is to find the best solution to a problem that has a large number of possible solutions in a short time, which is probably not the global optimal. Fig 1 depicts an elevation related to the goal function in a one-dimensional state-space landscape. The objective is to locate the global maximum. As indicated by the arrow, hill-climbing search adjusts the existing condition in an attempt to enhance it. For our case, we are seeking to maximize the score obtained for the graphs generated during graph learning.

3.2.2 Chow-Liu Algorithm

It is a method that learns a Bayesian network with a tree structure that maximizes the probability of the training data. This algorithm uses the mutual information, compute weight, between the events of the variables by the Equation (2).

$$I(X, Y) = \sum_{x \in \text{values}(X)} \sum_{y \in \text{values}(Y)} P(x, y) \log_2 \frac{P(x, y)}{P(x)P(y)} \tag{2}$$

Then it finds the maximum weight spanning tree, which connects all vertices of a graph. And finally,

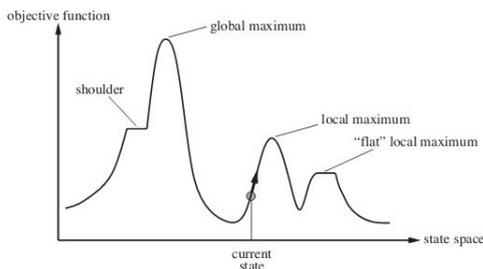


Figure 1: An overview of a one dimension function (Russell and Norvig, 2020).

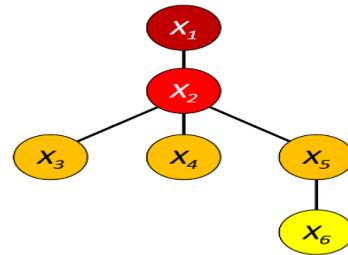


Figure 2: A first-order dependency tree representing the product on the left (Chow and Liu, 1968).

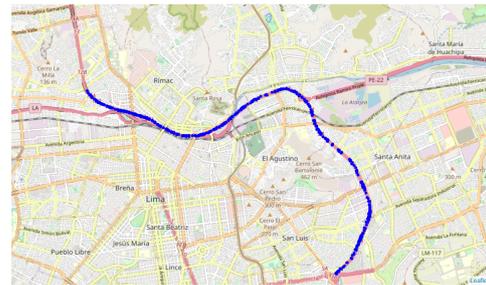


Figure 3: Accidents that occur on the Pan-American Highway at Lima.

it assigns the directions. Fig 2 depicts a first-order dependency tree representing the product on the left.

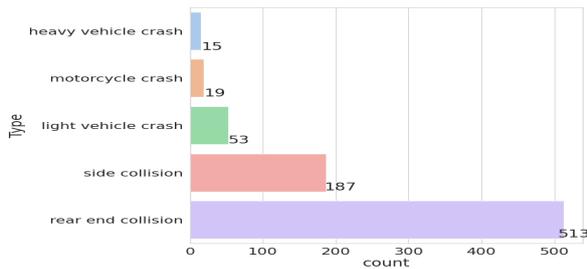
3.3 Model

The data was obtained from the governmental open data portals (see Section 4) which consists of traffic accidents records that occurred at the Pan-American Highway in Lima (i.e., material damage, injured victims and fatalities). Fig. 3 depicts these accidents with blue dots. Table 1 shows the traffic accident variables.

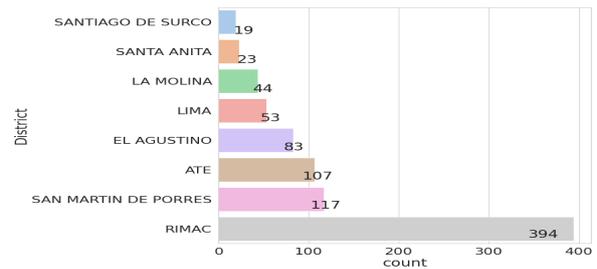
Table 1: Variables related to car accidents in Peru.

Variable	Description
Day	Day of accident occurrence
Shift	Shift corresponding to the time
Sense	Direction in which a vehicle is traveling
Vehicles_Involved	Vehicles involved in the accident
Reason	Cause of the accident generation
District	District of the Lima region where it occurred
Type	Type of accident that occurred
Severity	Importance of an accident according to its effects

- **Material Accidents:** Fig. 4a depicts the count of material accidents by their types. For instance, the most common type of material accidents are rear collision. Fig. 4b depicts the count of material accidents by district.
- **Accidents with Injuries:** Fig. 5a depicts the count of accidents with injuries by their types. For instance, the most common type of material acci-

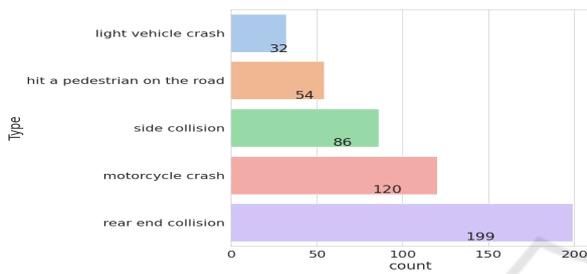


(a) Accident types.

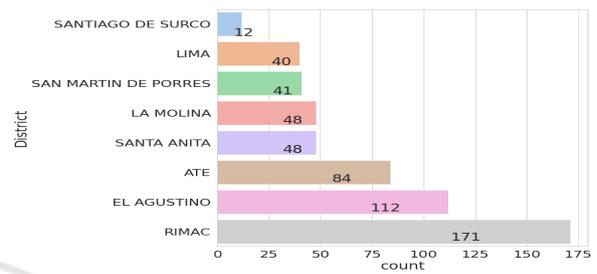


(b) Count of Accidents for Districts.

Figure 4: Material Accident Data related to car accidents in Peru.

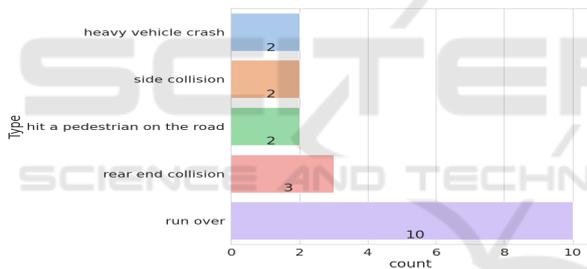


(a) Accident types.

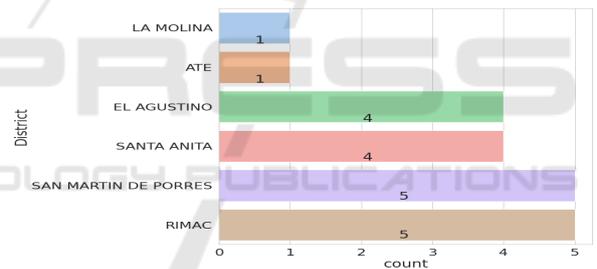


(b) Count of Accidents for Districts.

Figure 5: Accidents with Injuries Data related to car accidents in Peru.



(a) Accident types.



(b) Count of Accidents for Districts.

Figure 6: Deathly Accident Data related to car accidents in Peru.

dents are rear collisions or motorcycle collisions. Fig. 5b depicts the count of accidents with injuries by district.

- **Deathly Accidents:** Fig. 6a depicts the count of deathly accidents by their types. For instance, the most common type of deathly accidents are runs over or rear collisions. Fig. 6b depicts the count of deathly accidents by district.

3.3.1 Scores

Bayesian Information Criterion (BIC) (Schwarz, 1978) is a criterion for model selection among a finite set of models; models with lower BIC are generally preferred. It is based, in part, on the likelihood function. When fitting models, it is possible to increase the likelihood by adding parameters, but it may result

in overfitting. BIC is formally defined as:

$$BIC = k \ln(n) - 2 \ln(\hat{L}). \quad (3)$$

where :

- \hat{L} = the maximized value of the likelihood function of the model M
- x = the observed data
- n = the sample size
- k = the number of parameters estimated by the model

K2 (Cooper and Herskovits, 1992) finds the structure that maximizes each factor of a BN. It is derived by assuming uniform prior distributions on the values of an attribute for each possible instantiation of its parent attributes. This assumption introduces a tendency

to select simpler network structures. The K2 metric is formally defined as:

$$K2(B, T) = \log(P(B)) + \sum_{i=1}^n \sum_{j=1}^{q_i} \left(\log \left(\frac{(r_i-1)!}{(N_{ij}+r_i-1)!} \right) + \sum_{k=1}^{r_i} \log(N_{ijk}!) \right)$$

where

- B : a network
- $P(B)$: represents the prior probability of the network B
- T : data
- r_i : number of states of the finite random variable X_i
- N_{ijk} : number of instances in the data T where the variable X_i takes its k -th value x_{ik} and the variables in Π_{X_i} take their j -th configuration w_{ij}

BDeu (Heckerman et al., 1995) is a special case of BDe, where BDe means Bayesian Dirichlet scoring, and, thanks to likelihood equivalence, yields the same score for any two Markov equivalent structures given D and a prior network from which the priors are derived. For BDeu, they use uniform priors.

$$BDeu(B, T) = \log(P(B)) + \sum_{i=1}^n \sum_{j=1}^{q_i} \left(\log \left(\frac{\Gamma(\frac{N'}{q_i})}{\Gamma(N_{ij} + \frac{N'}{q_i})} \right) + \sum_{k=1}^{r_i} \log \left(\frac{\Gamma(N_{ijk} + \frac{N'}{r_i q_i})}{\Gamma(\frac{N'}{r_i q_i})} \right) \right)$$

where

- B : a network
- $P(B)$: represents the prior probability of the network B
- Γ : the Gamma function
- T : data
- r_i : number of states of the finite random variable X_i
- N_{ijk} : number of instances in the data T where the variable X_i takes its k -th value x_{ik} and the variables in Π_{X_i} take their j -th configuration w_{ij}

According to (Liu et al., 2012), BIC score can still work well for large sample sizes, however it can perform arbitrarily worse than other functions for small data sets. Additionally, according to (Riggelsen, 2008) the methodology for calculating the K2 score is analogous to that of BDeu, however it differs in that K2 makes use of a priori parameters equal to one.

4 EXPERIMENTATION

In this section we present our experimental study to show the results of our study.

4.1 Experimental Protocol

For the development of our approach, we used the following resources:

1. **Software:** We develop our proposal with Python 3.7 with Google Colab Pro
2. **Hardware:** We use Google Colab Pro service with SSD 125GB for storage, 24GB of RAM, a GPU Nvidia® Tesla V100-SXM2 16 GB and a CPU Intel® Xeon® CPU @ 2.20GHz

3. **Dataset:** We used the public dataset from Municipality of Lima: <https://aplicativos.munlima.gob.pe/extranet/datos-abiertos/>.

Our code is publicly available at: https://colab.research.google.com/drive/1rpSPGnCCQsWmUlyGZCnonOI20_7mPAiny?usp=sharing

4.2 Results

The algorithms (see Section 3.2) and the scores (see Section 3.3.1) are evaluated in order to determine the best possible model for a BN.

4.2.1 BN Model

Hill Climbing Search: When applying the Hill Climbing Search method (see Section 3.2.1), the following Bayesian networks were obtained with their respective scores. Fig. 7a depicts the best BN obtained with hill climbing algorithm for BIC score, as we can see, it only considers three variables letting aside the other ones. Hence it is not useful in practice, since the queries cannot be made over the variables that are not taken into account.

Fig. 7b (resp. Fig. 7c) depicts the best BN obtained with hill climbing algorithm for K2 score (resp. BDeu score), as we can see, it considers all five variables. Furthermore, Fig. 7b and Fig. 7c show that with K2 score and BDeu scores obtain the same BN.

Chow-Liu Algorithm. When applying the Chow-Liu algorithm (see Section 3.2.2), the following Bayesian networks were obtained with their respective scores. Fig. 8a, Fig. 8b and Fig. 8c show that all three scores (i.e., BIC score, K2 score and BDeu score) obtain the same BN as a tree centered on “Vehicles.Involved” variable.

4.2.2 Queries

Thanks to the Variable Elimination method (Zhang and Poole, 1994), we can use the obtained BN models to make inference queries.

Hill Climbing Search: We use the BN obtained by hill climbing method with BDeu score (see Fig. 7c). For instance, Table 2 shows the distribution for districts knowing that the reason of an accident is an animal on the road (i.e., $Q_0 : P(District | Reason = animal\ on\ the\ road)$).

Now, we are going to make some queries with the BN model according to the data:

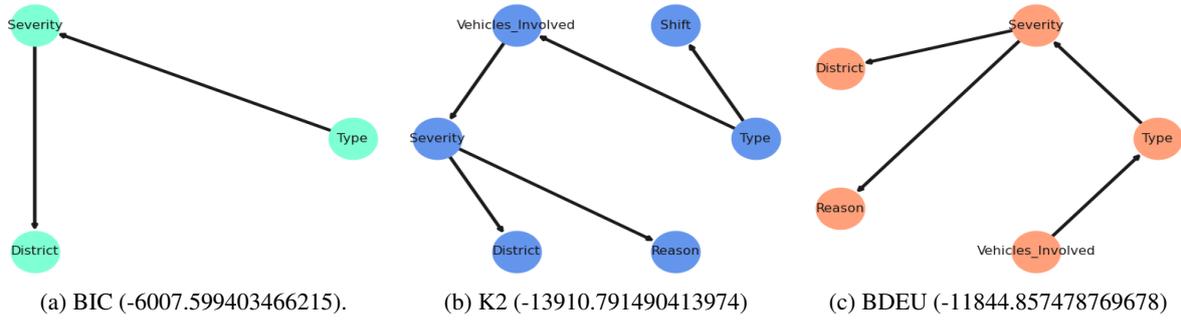


Figure 7: BN obtained with Hill Climbing for various scores.

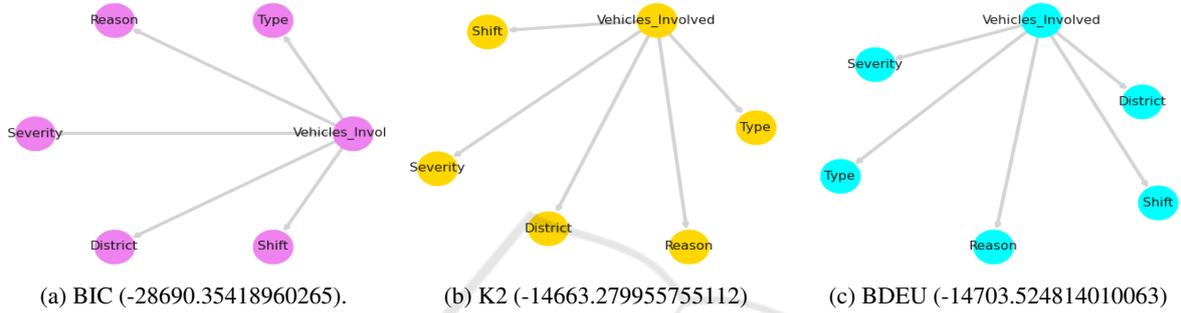


Figure 8: BN obtained with Chow Liu Algorithm for various scores.

Table 2: Percentage values for query Q_0 .

District	Percentage (%)
Rimac	32.2897
Ate	20.4259
El Agustino	14.9067
San Martín de Porres	12.7724
Santa Anita	7.0057
Lima	6.0446
La Molina	4.4526
Santiago de Surco	2.1024

$Q_1 : P(District \mid Severity = deathly)$, this query helps to obtain the distribution of Districts knowing that a deathly accident has happened. Table 3 shows the distribution for this query, where we can pick the first four districts according to their percentage (i.e., Rimac, El Agustino, ate and San Martin de Porres).

Table 3: Percentage values for query Q_1 .

District	Percentage (%)
Rimac	31.2005
El Agustino	25.3065
ate	13.0754
San Martín de Porres	10.4243
Santa Anita	8.2161
Lima	5.6259
La Molina	4.8440
Santiago de Surco	1.3074

$Q_2 : P(Reason, Type \mid Severity = deathly, District = Rimac)$, this query helps to obtain the distribution of Reasons and Types of accidents knowing that a deathly accident at Rimac District has happened. Table 4 shows the distribution for this query, the main reasons and types are pedestrian on the road, reckless driver with rear collision and side collision.

Table 4: Percentage values for query Q_2 .

Reason	Type	Percentage (%)
pedestrian on the road	hit a pedestrian on the road	19.8853
reckless driver	rear collision	18.5141
reckless driver	side collision	8.3092
reckless driver	hit a pedestrian on the road	7.7709
pedestrian on the road	run over	5.6028

$Q_3 : P(Reason, Type \mid Severity = deathly, District = El Agustino)$, this query helps to obtain the distribution of Reasons and Types of accidents knowing that a deathly accident at El Agustino District has happened. Table 5 shows the distribution for this query, the main reasons and types are pedestrian on the road, homeless on the road that are runned over and reckless drivers.

Table 5: Percentage values for query Q_3 .

Reason	Type	Percentage (%)
pedestrian on the road	hit a pedestrian on the road	24.8418
homeless on the road	run over	15.2892
reckless driver	hit a pedestrian on the road	10.1552
reckless driver	rear collision	6.1704
reckless driver	side collision	5.1654

$Q_4 : P \left(Reason, Type \mid \begin{matrix} Severity = deathly, \\ District = ate \end{matrix} \right)$, this query helps to obtain the distribution of Reasons and Types of accidents knowing that a deathly accident at ate District has happened. Table 6 shows the distribution for this query, the main reasons and types are pedestrian on the road, homeless on the road that are runned over and reckless drivers.

Table 6: Percentage values for query Q_4 .

Reason	Type	Percentage (%)
pedestrian on the road	hit a pedestrian on the road	24.8418
homeless on the road	run over	15.2892
reckless driver	hit a pedestrian on the road	10.1552
reckless driver	rear collision	6.1704
reckless driver	side collision	5.1654

$Q_5 : P \left(Reason, Type \mid \begin{matrix} Severity = deathly, \\ District = San Martin de Porres \end{matrix} \right)$, this query helps to obtain the distribution of Reasons and Types of accidents knowing that a deathly accident at ate District has happened. Table 7 shows the distribution for this query, the main reasons and types are pedestrian on the road, reckless drivers and collisions.

Table 7: Percentage values for query Q_5 .

Reason	Type	Percentage (%)
pedestrian on the road	hit a pedestrian on the road	28.6446
reckless driver	hit a pedestrian on the road	10.4051
reckless driver	rear collision	7.5471
pedestrian on the road	run over	4.9270
reckless driver	side collision	4.1947

Chow-Liu Algorithm: We use the BN obtained by Chow-Liu Algorithm with K2 score (see Fig. 8b). For instance, Table 8 shows the distribution for districts and severity knowing that the reason of an accident is a drunk driver.

Now, we are going to make some queries with the

Table 8: Percentage values for query $P(District \mid Reason = animal\ on\ the\ road)$.

Severity	District	Percentage (%)
mild	Rimac	20.4145
serious	Rimac	15.2064
mild	Ate	9.1750
serious	Ate	7.6141
mild	San Martin de Porres	7.4299
mild	El Agustino	7.0407
serious	El Agustino	6.6580
serious	San Martin de Porres	4.5564
serious	La Molina	3.7086
mild	Lima	3.5557

BN model according to the data:

$Q_6 : P \left(Shift \mid \begin{matrix} Severity = deathly, \\ District = Rimac \end{matrix} \right)$, this query helps to obtain the distribution of Shift knowing that a deathly accident at Rimac District has happened. Table 9 shows the distribution for this query, the main shift for accident is the afternoon.

Table 9: Percentage values for query Q_6 .

Shift	Percentage (%)
afternoon	38.0837
morning	31.2627
night	21.8436
early morning	8.8099

$Q_7 : P \left(Reason \mid \begin{matrix} Severity = deathly, \\ District = Rimac, \\ Shift = afternoon \end{matrix} \right)$, this query helps to obtain the distribution of Reason knowing that a deathly accident at Rimac District at afternoon has happened. Table 10 shows the distribution for this query, the main reasons are reckless driver, pedestrian on the road and speeding.

Table 10: Percentage values for query Q_7 .

Reason	Percentage (%)
reckless driver	44.7673
pedestrian on the road	21.2712
speeding	6.2850
mechanical / electrical problems	5.6429
previous accident	5.4414

$Q_8 : P \left(Reason, Type \mid \begin{matrix} Severity = deathly, \\ District = Rimac, \\ Shift = afternoon \end{matrix} \right)$, this query helps to obtain the distribution of Reason and Type knowing that a deathly accident at Rimac District at afternoon has happened. Table 11 shows the distribution for this query, the main reasons are reckless driver, pedestrian on the road and speeding.

Table 11: Percentage values for query Q_8 .

Reason	Type	Percentage (%)
reckless driver	rear collision	22.0395
pedestrian on the road	hit a pedestrian on the road	14.0416
reckless driver	side collision	8.4765
reckless driver	hit a pedestrian on the road	5.9364
pedestrian on the road	run over	4.3538

4.3 Discussion

The Q_1 query allowed us to identify the four districts in which road sections have a higher probability of fatal vehicle accidents. Based on these results, we carry out queries Q_2 , Q_3 , Q_4 and Q_5 to infer the probabilities for the reasons and types of accidents that these fatal accidents occur in these four districts.

The results of the Q_1 query also helped us to develop the last 3 queries, since it allowed us to detect the section of the Pan-American highway belonging to the Rimac district as the one with the greatest probability for fatal vehicle accidents, based on this, we carry out query Q_6 , to infer the probabilities of the shifts when these fatal accidents occur in the Rimac district, resulting in the afternoon.

Based on the result of query Q_6 , we carry out queries Q_7 and Q_8 that consider vehicular accidents whose severities are fatal in the Rimac district that occur in the afternoon, these queries let us identify the reasons and types of accidents respectively. We believe that these results will be of great relevance to carry out preventive actions and thus reduce the number of these unfortunate accidents.

5 CONCLUSIONS

A Bayesian network was obtained with one of the highest scores, using the Hill Climbing Search and Chow Liu algorithm with K2 score- After carrying out the experiments, it was possible to identify that very serious accidents occur with a high probability in the part of the Pan-American Highway belonging to the Rimac district at the afternoon shift and due to pedestrian on the road, a reckless driver or speeding.

It is recommended to place a greater police guard in the part of the Pan-American Highway, belonging to the Rimac district between 12 and 18 hours and to carry out awareness campaigns in this area, in order to reduce the speed of cars and thus reduce the number of accidents with fatalities. Due to the fact that one of

the factors is speeding, a photo ballot systems with the respective notice to drivers should be implemented.

Further works can be done with similar analysis of other critical points in the country, for instance, find other traffic problems by applying Markov chains, for example variations in driving patterns, pose estimation for road pedestrians (Fernandez-Ramos et al., 2021) and looking if car sharing may help to decrease the traffic accident rate (Vásquez-Garaya et al., 2021).

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