

# Digital Device and Mathematics: Multilevel vs Machine Learning Models for Value-added Ranking in Italy

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Abstract: With the COVID-19 pandemic and the development of distance education programs, digital learning is popular and strategic in many learning fields. The deployment of Information and Communications Technology and its impact on both national and international learning programs are becoming increasingly significant. This study seeks to explore in the Italian context both the effectiveness of digital learning in Mathematics Education and which features and how affect value-added at the classroom level. To explore Information and Communications Technology contribution and value-added scoring, the study takes into consideration the analytical power of classical multilevel models concerning the predictive power of different types of machine learning models. The study aims to investigate how Information and Communications Technology, and related concepts, impact the Weighted Likelihood Estimates in Mathematics for students in the lower secondary school, using data from the INVALSI of the school year 2017/2018. The main finding is that Personal Computer ownership at home plays an important role in mathematical learning. Finally, a machine learning model incorporated in the educational domain can be an interesting starting point for developing class-predictive policies.

## 1 INTRODUCTION

The idea of digital learning has recently caught the attention of the general public to the gap created by online learning throughout the COVID-19 pandemic for lots of students around the world. Distance or on-line learning needs careful exploration to develop an overall read of the actions taken and to be undertaken at the government and school levels. At the sunrise of the digital age, it's essential to know what proportion and how digital learning is affirmed in terms of the possession and use of technologies, to avoid new types of exclusion from the numerous areas of information and knowledge society. The strategic objective formalized with the investments in hardware and software made by Italy in recent years and in the function of the EU2020 Lisbon Strategy, which in the school began through the Digital School Plan with particular attention to some regions of Southern Italy, which had the aim of faster and wider dissemination of the Information and Communications Technology (ICT) among students at all school levels. If Europe is moving towards the so-called gigabit society, starting not by a chance from the places where the knowledge of children and young people are formed to achieve

an increasingly interconnected society, Italy remains in the latter places, after a small leap forward in 2019, with the 25th place in 28th of the DESI 2020 ranking (Commission, 2020). Exploring the involvement of macro-territorial areas, moreover, is a key issue to describing and understanding the emergence of differentiated ICT practices. The Istat data from the "Multipurpose survey on households and ICT" 2018 undoubtedly has highlighted the existence of a different acceleration in the digitization process that runs through the country and with Italian families.

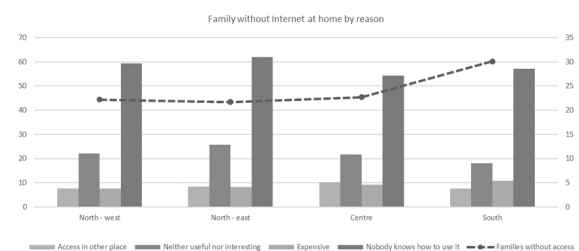


Figure 1: Family without the Internet at home by reason (ISTAT, 2018).

In southern Italy, 41.6% of households say they do not own a computer at home (compared to an average of about 30% in other areas of the country) and

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only 14.1% have at least one computer available for each component. Among the families who declared that they did not have an Internet connection at home, about 30% of cases reside in the South, declaring that the absence is determined by economic reasons in 10% of cases, while more than 60% declare that they do not have an Internet connection because no one would have been able to use it (in a greater percentage in the northeast) and about 20% because they do not find it useful or interesting.

The purpose of this study was to examine the hierarchical relation between students, class, and the territorial context about the school learning of Mathematics in the last grade of lower secondary school for Italian context regarding the use of technology devices (please see below the research question Q1). An in-depth study is needed to improve discussions on the impact of digital technologies in the classroom and at home on mathematics education in Italy. Moreover, the focus of the policy should also be on the opportunity to have access to digital resources provided by ICT for learning and territorial development. At the same time, the purpose is to evaluate the value added by the classroom and the predictive potential of machine learning models is explored, comparing them with the traditional multilevel regression approach (please see below the research question Q2).

## 2 THE DIGITAL DIVIDE IN EDUCATION

The literature suggests that having an Internet connection and a Personal computer at home can have positive effects on school performance (Wittwer and Senkbeil, 2008). However, the more recent studies of the impact of instructional computer use in school settings have found mixed results and that vary across countries (Eickelmann et al., 2012; Hu et al., 2018; Carstens et al., 2021). Indeed, it is not the simple access to home or school to ICT that can influence in positive terms school learning (which instead would give negative outcomes) as much as the relative use and control that is provided. From this point of view, inequalities in access to Personal computers and the Internet connection and their implications have been known for some decades with the debated term of the digital divide (NTIA, 1995; DiMaggio et al., 2001). There are differences in their origin and are attributable, at the same time, to those who argue that these lacks are linked to the structural nature of the students, such as the socio-economic and cultural status of the original family, and those who argue that the absences are subjective; hence, all these

facts are partly linked to the different ways in which knowledge is processed and internalized by the different subjects. Overcoming this dichotomy, in the perspective of the study of digital inequalities, the focus is on the uses that are introduced and the skills that can be exploited (DiMaggio et al., 2001; van Dijk, 2005). There is no doubt, that technology contributes to a better distribution of knowledge. The complexity of its use and fruition, as well as the resulting costs, can intensify existing social inequalities, or, as the author reasons, large groups of misfits, people who are ill-adapted to the information society. Among the individual variables, beyond gender and the migratory background, the prevailing language spoken in the family will be considered because of what van Dijk also argued that for non-natives (p. 177): “The future is considerably less bright for migrants and ethnic minorities with low education in a network society dominated by natives and ethnic majorities. Usually, they lack digital skills and, what is worse, they do not speak or command the native or dominant language sufficiently. So, they run the risk of missing out on the technical and communicative skills required in a network society. The major handicap is having insufficient command of the dominant language. The only exception is to be able to speak and write in English”(van Dijk, 2005).

### 2.1 What Is the Relationship between Digital Learning and Learning Mathematics in the Classroom?

From the review of the international literature, about the relationship between digital learning and learning in the classroom, the students have greater difficulty using the computer equipment effectively when teachers do not work in the classroom to develop shared practices in the use of technology (Kozma, 2003; Balanskat et al., 2006; Burns, 2013; Drijvers, 2015; Bray and Tangney, 2017; Viberg et al., 2020). In the present work, the exploration will start with the creation of operationalized indicators from the frequency of PC usage in the classroom with the supervision of the Mathematics teacher (INVALSI, 2017; Rutkowski et al., 2013). The purpose is to distinguish between established practices of PC use and their connotations (positive and/or negative) on individual Mathematics learning and the relative influence on classrooms/peers. For the cooperative learning, the students tend to enjoy Mathematics and this fun motivates them to learn (Davidson, 1990). The importance of computer-supported learning is an emerging branch of pedagogical sciences that deals with studying how people can learn with the help of computers.

### 2.1.1 Machine Learning Models in Education

As reported in the work "Contrasting Classical and Machine Learning Approaches in the Estimation of Value-Added Scores in Large-Scale Educational Data" (Levy et al., 2020), the computational sciences and social sciences have been collaborating for some time to achieve greater and better results in the approaches of machine learning in the educational field, because of the growth potential in a wide range of areas of the society of supervised and unsupervised machine learning models. The school and didactic learning outcomes were evaluated with linear and/or nonlinear regression methods, such as Support Vector Machine, Random Forest, Extreme Gradient Boosting, Neural Network, etc. Despite the wide variety of regression models, there is still no consensus on which model is the "best" (Papadogiannis et al., 2020). Even with this, it may be possible to intervene with institutions in advance to limit and/or manage the phenomenon of students with a lesser chance of effective learning. This includes their socio-economic and cultural status, territorial and scholastic features, thereby improving the school system's effectiveness in terms of student performance. There are several possible applications of prediction data derived so far for machine learning in the educational context (Papadogiannis et al., 2020). Due to the implementation of policy actions in lower and upper secondary schools, machine learning models will be developed: to prevent students from leaving school too early; to provide feedback to assist at-risk students; to differentiate didactic planning at the classroom/peer level in terms of mobile technology, etc.

## 3 RESEARCH PROBLEM & OBJECTIVES

Due to the global economic crisis COVID-19 is experiencing, the use of ICT and distance learning has become a necessary teaching method. This implies the application of new didactic strategies and pedagogical approaches to improve strategic skills. Furthermore, it is just as important to make sure no one is left behind in this digital competition and to explore at the classroom context to understand which one of the exogenous factors (from the scholastic) may contribute to widening the gap. The educational process is improved by exploring whether and to what extent digital know-how developed at home and at school have an equal impact on mathematical ability. It is important to distinguish between existing practices and connotations (positive and/or negative) concern-

ing individual mathematics learning, as well as its influence on classes and peers, in collaboration activities with the mathematics teacher, supported by the computer or in teacher continuing education. Specifically, this work uses data from the National Large-Scale Computer-Based Survey conducted by the National Institute for the Evaluation of the Education System (INVALSI) to assess the knowledge and skill of eighth grade students in mathematics for the school year (SY) 2017/2018. The following research questions are addressed in this study: Q1. Combined with "exogen" features to the scholastic institution, for example, gender, socio-economic and cultural status, linguistic background and other contextual aspects, how and how much ICT features contribute to the value-added ranking? Q2. For the Italian context, could be the analytic power of traditional multilevel models relaunched by the predictive rule of different types of machine learning approaches?

## 4 MATERIAL AND TOOLS

Although the National INVALSI test produces census data, it also extracted a two-stage probabilistic sample: in the first stage, the schools are sampled and in the second one, two classes for each school selected from the previous stage (INVALSI, 2018). The sample extracted for the SY 2017/2018 is 29 359 lower secondary school students, representative of the general population of 567 986 students. It is decided to proceed with the analysis of the sample rather than the entire population for the need to analyze the features of teachers, contained in the questionnaire for Mathematics teachers administered only to sample classes participating in the National Surveys (INVALSI, 2017). What's more, it was necessary to study to the s.y. 2017/2018, instead of the most recent, because that teacher's questionnaire was not administered (neither to the population nor to the sample). Which exogen variable is considered to evaluate value-added score is a long-term question. First of all, it is necessary to point out that the dependent variable and the independent variables adopted in this paper have different nature, detection, and synthesis procedures. The data is collected through different sources: the school administration, teacher and the student questionnaire and the standardized INVALSI test. The independent variables about students are:

- the school career (here "late\_enrolled\_student");
- the measure of socio-economic and cultural status (here "ESCS", corrected by the exclusion of the two items of a computer and internet ownership at home);

- PC availability at home and Internet availability at home (here "PC/Internet\_availability\_at\_home")
- the immigration background (here "native", "I\_gen\_imm" and "II\_gen\_imm");
- the gender (male),
- the language is spoken at home (italian, here "lang"),
- the math score in grade 7 (the dichotomized written scoring in Mathematics at grade 7, where insufficient value is equal to 1 otherwise 0, here "math\_score\_grade\_7").

For all these variables, macro variables have also been prepared at the class level by aggregations (with mean) of the subset of students that make up the same class. By the teacher questionnaire, seven variables are administered for detecting the frequency of ICT use by the Mathematics teacher in the classes, such as Computer, multimedia interactive whiteboard (MIW), educational and computer software, digital camera, tablet, and smartphone. These variables are synthesized through Principal Component Analysis (PCA). The first component is responsible for high scores in the variables related to teaching activities (the MIW, educational and computer software, here "PC\_use\_Classroom\_didactics"); while the second one is linked to high scores in other activities (use of digital camera, tablet and smartphone, here "PC\_use\_Classroom\_other"). Other ten variables are administered through the items of the Likert scale for the updating activities in the last 2 SY (2016-17 and 2017-18) declared by the classroom Mathematics teacher. Also, these variables are summarized through PCA. Only one component is relevant for the present study and considers the updating of teachers in the last two school years for Didactics and the Integration of Information Technologies in the teaching of Mathematics (here "teacher\_update\_ICT"). Furthermore, also a dummy variable related to the use in class of peer activities is selected from the teacher questionnaire. Non-response data is more common with self-administered questionnaires when specific questions are left unanswered. In this study, albeit with a reduced incidence, these missing data occurred for data such as administrative and/or teacher or student questionnaires. The missing data are concentrated among the categorical variables and have a percentage of less than 10%. In this regard, it was deemed best to use the single imputation technique, which focuses on substituting each missing value using mode as a statistical method.

#### 4.1 Analysis of the WLE Distribution

The dependent variable considered is the mathematical Weighted Likelihood Estimates (WLE) score at the INVALSI scale (with Mean = 200 and Standard Deviation = 40). The estimate is based on the concurrent calibration of the INVALSI data from the Main Study based on the Rasch model of measurement (Rasch, 1960). As described in the INVALSI technical report (INVALSI, 2018), standardized weakly parallel multiple test-forms were assembled from a large item bank, developed by INVALSI and based on the Rasch unidimensional model of measurement. Taking into account the main international research on mathematics education, the theoretical framework is aligned with the National System of Evaluation (SNV). Further details on grade 8 assessment design, item bank characteristics, test-form assembly, and psychometric properties are reported in the corresponding chapter of the INVALSI technical report (Desimoni, 2018). For the inferential models, the data of the unweighted sample will be used. This is because the main goal of the machine learning model is different from weighted analyses obtained with the adjusted population estimates. Indeed, the machine learning techniques were not developed to explain relationships, but for predictive purposes.

## 5 RESULTS

This work seeks to identify which machine learning regression model is closest to the results obtained by the multi-level hierarchical model to improve the educational system with more innovative and proactive predictive, organizational, and institutional developments. For this evaluation, the first step was either the adaptation to the INVALSI data and the previously exposed hypotheses of the procedure exposed by Levy and colleagues (Levy et al., 2020) through the R Project 4.1.0 software (with the "lmer4", "caret", "nlme", "kernlab", "nnet", "xgboost" libraries). Before moving on to the analysis of the results, it is necessary to point out that in the far and the fitting training we have opted for a reduction to 10 k-fold validation for each regression and tree model. As noted in the literature, repeated cross-validation of "k-folds" has the advantage of improving the estimation of the average performance of the model at the cost of adopting and evaluating many more models (James et al., 2013). It is also recalled that for the implementation of machine learning models, intending to predict Mathematics performance, this work has drawn on the set of independent variables of the multilevel regres-

sion model described above, preferring the simplified multilevel regression model to random intercept. From a first analysis, as shown in Figure 2, the different models provide results very close to each other with a better-predicted error, as well as for the multilevel model (abbreviated as hlm), for the Extreme Gradient Boosting (boostin) and the Random Forest (rf). The peculiarity of these last two "tree" regression models is precisely to consider exhaustively the presence of relationships between the variables belonging to different hierarchical levels of a data matrix and their breakdown of the variance in the terminal nodes (e.g., classes), considering both the net effect on statistical units (ad.es. students) and the interactions present in them. Among the models that have less reliable results in terms of  $R^2$  are the polynomial (pol) and linear regression (lm), the linear Support Vector Machine (svmlin) and the Neural Network (nn).

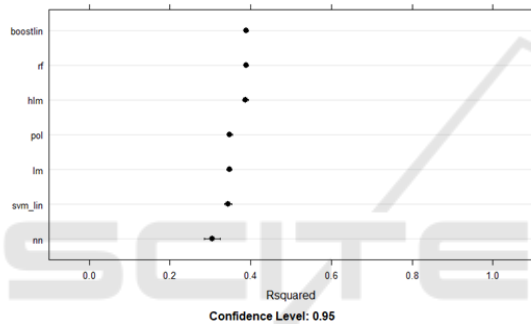


Figure 2: R2 distribution by the statistical model.

Value-added (VA) is defined as the difference between the expected and the actual performance. Each VA score was computed based on average residuals per classroom. The positive sign indicates that students in each class have achieved a better-predicted result in Mathematics (net of factors that cannot be affected by a class), while a negative sign indicates that a worse result has been obtained. The comparison was made by classifying the added value scores into three quartiles, calculated based on the average residuals per classroom (the residuals represent precisely what cannot be attributed to the personal, social, and economic characteristics of the students and thus depends on belonging to a given classroom). Based on the classification obtained by three quartiles, the classes were divided into these categories of added value: "Needs improvement", "Neutral", "Highly effective". It is apparent that the central class is the one that does not have a positive or negative impact on the students within; the classes with "Highly Effective" labels are usually those that have achieved results that are significantly higher than those that are on average obtained by students attending classes

with comparable characteristics (based on the socio-demographic profile and level of the previous school year); the classes with "Needs Improvement" labels have achievement levels that are significantly below those achieved by students attending classes with similar characteristics. As can be seen from Table 3, with the multilevel classification, the maximum disagreement is 23% for Extreme Gradient Boosting Linear, 22% for Neural Network and 14% for RF.

Table 1: Crosstable of Multilevel model by Machine learning models VA classifications.

		Classified by Multilevel model as:		
		Needs improvement	Neutral	Highly effective
Linear Regression	Needs improvement	477 (96.56%)	17 (3.44%)	0 (0%)
	Neutral	17 (3.44%)	460 (93.12%)	17 (3.45%)
	Highly effective	0 (0%)	17 (3.44%)	476 (96.55%)
Polynomial Regression	Needs improvement	475 (96.15%)	19 (3.85%)	0 (0%)
	Neutral	19 (3.85%)	455 (92.11%)	20 (4.06%)
	Highly effective	0 (0%)	20 (4.05%)	473 (95.94%)
Random forest	Needs improvement	425 (86.03%)	68 (13.77%)	1 (0.20%)
	Neutral	69 (13.97%)	360 (72.87%)	65 (13.18%)
	Highly effective	0 (0%)	66 (13.36%)	427 (86.61%)
Neural Network	Needs improvement	378 (76.52%)	110 (22.27%)	6 (1.22%)
	Neutral	112 (22.67%)	271 (54.86%)	111 (22.52%)
	Highly effective	4 (0.81%)	111 (22.87%)	376 (76.27%)
Extreme Gradient Boosting Linear	Needs improvement	374 (75.71%)	111 (22.47%)	9 (1.83%)
	Neutral	113 (22.87%)	271 (54.86%)	110 (22.31%)
	Highly effective	7 (1.42%)	112 (22.67%)	374 (75.86%)
Support Vector(Machine Linear	Needs improvement	472 (95.55%)	22 (4.45%)	0 (0%)
	Neutral	22 (4.45%)	455 (92.11%)	17 (3.45%)
	Highly effective	0 (0%)	17 (3.44%)	476 (96.55%)

The results are shown highlighting the need for additional exploration for decision tree algorithms. In another word, the best models are the linear algorithms, confirming linearly separable data. The largest difference between linear regression models and multilevel model is that there is potentially a different intercept and a different slope coefficient for every level 2 variable for the classroom (Levy et al., 2019). In other words, the students are clustered in the classroom in multilevel analysis, giving more accuracy to the results, while in linear regression models it is not available.

### 5.0.1 Ensemble Methods vs. Multilevel Model

Going deeper into the analysis of the results for the ensemble method (decision tree algorithms), the Figures 3 show the variables ordered by the models of Extreme Gradient Boosting Linear and Random Forest (the best in terms of adaptation to the model). The differences in the first positions derive from the determination of the importance of the explanatory variables for which the "variants" of the basic regressor of the two models are produced. The Extreme Gradient Boosting produces variants with a greater focus on "difficult" examples and produces a reduction of the forecast error in sequential terms, while the Random Forest produces variants by introducing randomness into the tree construction process thanks to the bootstrap aggregation meta-algorithm that guides the

random choice predictors to use for each tree (James et al., 2013).

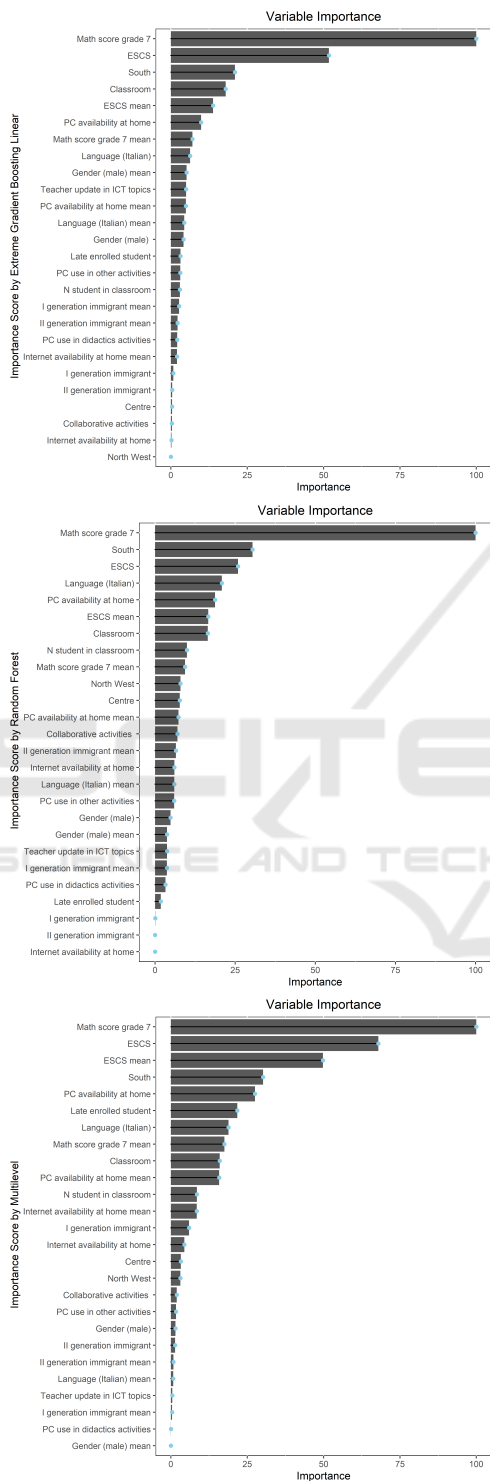


Figure 3: Variable importance score by Extreme Gradient Boosting, Random Forest and Multilevel models.

Which of these produces a more "accurate" importance of the variables is difficult to pinpoint with certainty (Breiman et al., 1984). However, the importance of each independent variable is calculated based on the reduction of the "impurity" compared to the average of all decision trees: it is nothing more than the weighted average of each explanatory variable in the creation of the node. As usual in machine learning algorithms, the importance of the predictors is calculated with training that reduces the error propagated in each subsequent training step and the important values of the predictors vary from 1 to 0 (and transformed into a percentage below). Focusing on the analysis of variables with greater relative importance in the training process, observing the jump in the Figures 3, the most important predictors are the average class ESCS, the class, the computer ownership at home, the prevailing language is spoken at home (Italian), the ESCS, the belonging to a class of Southern Italy, insufficient scoring in grade 7 in Mathematics, both at individual and class level. The insufficient scoring in grade 7 is predominant, both at the individual and class level because it is the student's entry rating according to the evaluation of the teacher's performance. The effects of all other characteristics diminish, even at the class level, indicating how the favourable class context, in terms of prior preparation in Mathematics, is crucial on individual outcomes of the Standardized Invalsi test. As expected, obviously along with the socio-economic and cultural status of the household, computer equipment at home at the individual level is important on individual outcomes of the test, while one must scroll through the graphs a bit to discern that of the class. Hence, ICT plays an important role, specifically in the PC ownership at home or to belonging to a class with a high average of PC ownership at home. What's more, the language spoken in the household is particularly influential on outcomes across models, along with the class size and collaborative classroom activities (for the RF model). The different models also highlight the lower importance of the immigrant background (I and II generation immigrant) and Internet connection at home, in addition to gender (male) and classroom activities with a personal computer.

## 6 CONCLUSION AND DISCUSSION

This paper examined the contributions of Information and Communication Technology to mathematics education (Q1) and the process for analyzing the multilevel value-added current approach, which included

several statistical improvements (Q2). It started by giving a formal problem description of digital device and its definition related to mathematics, some related literature was discussed in which the more recent studies of the impact of instructional computer use in school settings has found mixed results and vary across countries (Eickelmann et al., 2012; Hu et al., 2018; Carstens et al., 2021), while further studies on the Internet connection and a Personal computer at home shown positive effects on school performance (Wittwer and Senkbeil, 2008). It is not simply having access to an ICT at home or in school that can influence in positive terms school learning (which instead would provide negative outcomes) but rather the relative ability to use and control it. From the review of the international literature, about the relationship between digital learning and learning in the classroom, the students have greater difficulty using the computer equipment effectively when teachers do not work in the classroom to develop shared practices in the use of technology (Kozma, 2003; Balanskat et al., 2006; Burns, 2013; Drijvers, 2015; Bray and Tangney, 2017; Viberg et al., 2020). According to the research question Q1, confirmed from the machine learning models, the most important predictors for mathematics education at the classroom level are the ESCS, both at individual and classroom level, the Personal Computer ownership at home, the language is spoken at home (Italian), the belonging to a classroom of Southern Italy (maybe in a pejorative connotation), the math score at grade 7, both at individual and class level. Here, Information and Communications Technology plays a part in the value-added approach, specifically in terms of Personal Computer ownership at home or belonging to a class with high Personal Computer ownership at home. However, the role of Information and Communication Technologies is a minor part compared to the ascribed characteristics. Indeed, it can occasionally discourage students' trouble and logical thinking if education systems don't borrow technology to meet their tutoring needs. Therefore, it is essential to develop a more articulated model in the Italian context, possibly taking cues from international efforts (Fraillon et al., 2014), which discusses self-efficacy and the use of Information and Communication Technology also in the classroom setting. In addition, the value-added score analysis suggests that technology-based professional development by teachers is not so essential. However, for a future where technologies are crucial, school principals cannot ignore their importance (Karakose et al., 2021). The capability to break fine problems with group conditioning in the school class tends to induce a disadvantage in the per-

formance of the individual test. The calculation test score gap between first-generation immigrants, natives and second-generation emigrants has verified the OECD framework (Peña-López et al., 2017)) that's the language walls the key to explaining differences in performance between these two groups of students, as well as a major disadvantage in terms of implicit perceptivity with access from home to a particular computer. It remains to be explored how peer collaboration activities are structured and how they interact in the use of new information technologies. Recent studies show that the effectiveness of peer group activities is strongly correlated with the strategies introduced by the teacher to make it work, such as role assignment, group contracts, anonymous peer assessments, etc. (Chang and Brickman, 2018). Finally, the multi-level hierarchical model is fully superimposable with machine learning models that are recommended for ranking added value. It should therefore be noted that if a hierarchical model is not applicable, and there are many other applications, the Extreme Gradient Boosting models and the Random Forest could prove complementary to hierarchical analysis to discover and test complex relationships between variables in the educative field also exploiting their predictive potential. However, it must be pointed out that it is still challenging, especially for concepts such as value-added scores in classes, where the goal is still to interpret and know in depth what are the factors that influence the effectiveness of one class rather than another, to use machine learning models without a consolidated conceptual scheme.

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