ContourVerifier: A Novel System for the Robustness Evaluation of Deep Contour Classifiers

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Abstract: DNN certification using abstract interpretation often deals with image-type data, and subsequently evaluates the robustness of the deep classifiers against disturbances on the images such as geometric transformations, occlusion and convolutional noises by modeling them as an abstract domain. In this paper, we propose ContourVerifier, a new system for the evaluation of contour classifiers as we have formulated the abstract domains generated by rigid displacements on contours. This formulation allowed us to estimate the robustness of deep classifiers with different architectures and on different databases. This work will serve as a fundamental building block for the certification of deep models developed for shape recognition.

1 INTRODUCTION

Deep neural networks have been widely used in various applications fields. Recently, they have been embedded in safety critical systems such as autonomous driving (Bojarski et al., 2016), (Li et al., 2021), collision avoidance systems (Julian et al., 2016) and medical image analysis (Shen et al., 2017). Despite their widespread use, these methods are not yet trusted to perform reliably and as expected for making critical decisions. For example, it has been proven by (Goodfellow et al., 2014) that neural networks are sensitive to small perturbations and exhibit non-robust conduct at times. For instance, two very similar inputs with a dissimilarity in a single pixel or brightness could result in different labels. This is due to their instability. So, it is often necessary to evaluate the robustness of Deep Neural Networks. To address this need, many DNN verification systems have been proposed in the last few years. They can be categorized as either complete verifiers (Ehlers, 2017; Katz et al., 2017; Tjeng et al., 2017; Wang et al., 2018b) or incomplete verifiers (Dvijotham et al., 2018; Raghunathan et al., 2018; Gehr et al., 2018) according to whether the verification may or may not result in a false positive. To choose between the two classes of methods, a compromise between completeness and scalability must be considered. In spite of this, the community still lacks an analyzer that supports multiple architectures with distinct types of activation and different input formats. Indeed, most of the proposed methods deal with image type data. However, several DNN-based solutions have been proposed for the shape classification (Droby and El-Sana, 2020), (Lu et al., 2021)(Abeßer and Müller, 2019). Therefore, it turns out to be useful to study the robustness of deep contour classifiers. These latter can be disturbed mainly by geometrical transformations unlike image classifier whose brightness can be also perturbed. In this paper, the focus is on the evaluation of robustness of deep contour classifiers under euclidean transformations and for this, we use the theory of abstract interpretation. Consequently, we have defined a new abstract domain of contours type data in the case of rotation and translation. Figure 1 presents the designed system to verify the robustness propriety. The remainder of this paper is organised as follows: In the next section, some related works are presented including neural networks verification and the theory of abstract interpretations. We describe ContourVerifier, our proposed method in section 3 and define the Lower and Upper bounds in the case of 2D contour translation and rotation. In sections 4 and 5 the experimental settings and results are presented and finally, a conclusion at section 6.

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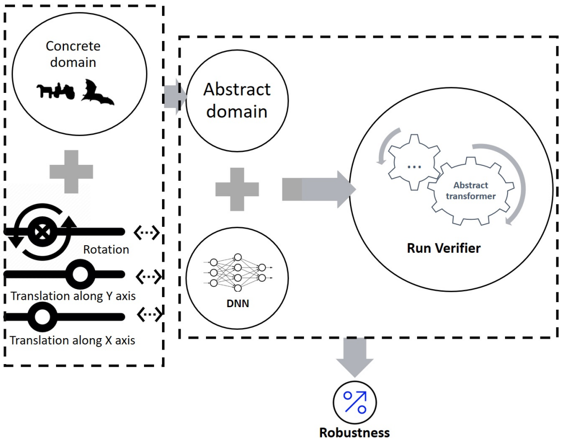


Figure 1: ContourVerifier robustness analyzer for deep contour classifiers.

2 BACKGROUND AND RELATED WORKS

In the past few years, DNN verification topics have been explored and researched extensively. Among the well-known frameworks, we cite Reluplex (Katz et al., 2017), PLANET (Bunel et al., 2018), ERAN (Singh et al., 2018a), DeepPoly (Singh et al., 2019b), DeepSymbol (Li et al., 2019), DeepG (Balunovic et al., 2019) and PRODeep (Li et al., 2020). Using linear programming (Tjeng et al., 2017), linear approximations (Weng et al., 2018) or abstract interpretation(Singh et al., 2019a), (Singh et al., 2018a), (Singh et al., 2019c), formal approaches are the key technical insight behind the majority of those NN verification's system. The effectiveness of this class of methods has been proved through several research projects. However, despite progress there remains serious challenges, not least in terms of supporting more NN architectures, input format and increase the application scope to real-world problems. In table 1, we list some verifiers dealing with different data formats. While there has been considerable interest in certifying the robustness of image data type network classifiers, less attention has been given to other models input types. The most common used image datasets are MNIST and CIFAR10. On the other hand, few verifiers deal with audio datasets and among these methods, we cite RnnVerif, Propagated-output Quantified Robustness for RNNs (POPQORN) and Polyhedral Robustness Verifier of RNNs (Prover). To the best of our knowledge, there is **no** previous work done on evaluating the robustness of deep 2D planar closed contour classifiers. As a result, we propose ContourVerifier, based on the abstract interpretation.

2.1 Abstract Interpretations for Neural Network Certification

The abstract interpretation (Cousot and Cousot, 1977) is a general theory that allows the approximation of a potentially infinite set of behaviors with a finite representation. This theory has been widely used over the last decades to build large-scale automatic code analyzers (Blanchet et al., 2003). Analyzers in fact are verification tools whose common point is the prediction of disturbed input model using an approximate neural network behavior. The formulation of neural network verification problem is as follow:
Let denote by $R_{\bar{X},\varepsilon}$ the original inputs $\bar{X}$ perturbed by ε. Verifying the robustness property for $R_{\bar{X},\varepsilon}$ is checking the property over the whole possible perturbation of $\bar{X}$.
Let C_L be the set of outputs having the same label L. $\bar{Y}$ denotes the set of each prediction for each element in $R_{\bar{X},\varepsilon}$.

$$C_L = \{\bar{y} \in \bar{Y} \,|\, \arg\max \bar{y}_i = L\} \tag{1}$$

The ($R_{\bar{X},\varepsilon}$,C_L) robustness property is verified only if the outputs O_R of $R_{\bar{X},\varepsilon}$ are included in C_L. However, we have no knowledge about O_R since we cannot control the behaviour of hidden layers. By considering a new abstract domains α_R, which is an abstraction of $\bar{X}$, the ($R_{\bar{X},\varepsilon}$,C_L) property is checked:

- If the outputs O_R of $R_{\bar{X},\varepsilon}$ are included in C_L.
- If the outputs α_R^O of α_R (the abstraction of $R_{\bar{X},\varepsilon}$) are included in C_L.

2.2 Lower and Upper Bound for Contrast and Geometrical Attacks

(Henry, 2014) defines the upper and lower bounds as the longest execution time in the case of abstract interpretation for computer science. In AI2, (Gehr et al., 2018) defines the lower bound (LB) and upper bound (UB) as the limits of the disturbance. For instance, if the image brightness is perturbed, the (LB) represents the minimum brightness value and (UB) is the maximum brightness value. The LB and the UB enable the definition of abstract intervals. If we apply a 2D rotation to the image, the contribution of the neighboring pixels to the intensity of the perturbed pixel is proportional to the distance from the initial pixel. This approximation enables the estimation of the possible LB and UB. Together, they give us the polyhedron where each rotated pixel is going to end.

Table 1: Examples of state of the art verifiers dealing with different dataset formats.

Verifier	Dataset	Dataset type	References
Verifier with constraints	MNIST, CIFAR10	image	(Bastani et al., 2016)
Planet	MNIST	image	(Ehlers, 2017)
Reluplex	MNIST Drebin	image Multidimensional vector	(Katz et al., 2017)
MIPVerify	MNIST, CIFAR10	image	(Tjeng et al., 2017)
Neurify	MNIST Drebin	image Multidimensional vector	(Wang et al., 2018a),(Henriksen and Lomuscio, 2020)
DeepZono	MNIST, CIFAR10	image	(Singh et al., 2018a)
RefineZono	MNIST, CIFAR10	image	(Singh et al., 2018b)
RefinePoly	MNIST, CIFAR10	image	(Singh et al., 2019a)
DeepPoly	MNIST, CIFAR10	image	(Singh et al., 2019b),(Henriksen and Lomuscio, 2020)
VeriNet	CIFAR10	image	(Henriksen and Lomuscio, 2020)
POPQORN	MNIST	sequence dataset	(Ko et al., 2019)
RnnVerif	VCTK	speech data	(Jacoby et al., 2020)
DNN Robustness Guarantees on videos	UCF101	video dataset	(Wu and Kwiatkowska, 2020)
Prover	FSDD GSC v2 MNIST	audio/speech dataset audio/speech dataset Flatten each image into one dimensional vector	(Ryou et al., 2021)

3 PROPOSED METHOD

The existing state of the art methods for the evaluation of deep neural network classifiers are almost designed for models with image type input. In this research, we introduce ContourVerifier, a new ERAN-based approach for deep contour classifiers verification.

3.1 Lower and Upper Bound in the Case of Translation

For a given contour C and a fixed batch size $Batch\_size$, we define in algorithm 1 the UB and LB respectively denoted by T_{fU} and T_{fL} to verify if every contour, even perturbed by a given translation in $I = [a,b]$, is yet well classified or not. In fact, we do not use the entire interval I as it is. However, we use a partitioning technique combined with batching in order to refine the UB and LB. By subdividing I into several segments [δ1,δ2], we obtain precise intervals. Hence, for each point of the contour, T_{fL} corresponds to the minimum value of all previous translated contours and T_{fU} is the maximum value of all previous translated contours. The contour C could be translated along the x axis, y axis or both at the same time. However, in the interests of simplification, algorithm 1 illustrates only translation along the x axis such as the example presented in figure 2 (a) where I is set to be $[100,200]$.

3.2 Contours Lower and Upper Bound in the Case of Rotation

For determining the upper and lower bounds, we consider rotating the input noted by C with $\theta \in [\alpha,\beta]$. For this purpose, in algorithm 2, we start by converting the euclidean coordinates C_x and C_y of the contour into polar coordinates (r,ϕ). where:

$$r = \sqrt{x^2+y^2} \tag{2}$$

And ϕ, in $]-\pi,\pi[$, is obtained via the following formula:

$$\phi = 2\arctan\frac{y}{x+\sqrt{x^2+y^2}} \tag{3}$$

Using the polar coordinates, we perform a rotation with angles θ_1 and θ_2 respectively $\in [\alpha,\beta]$. Next, we reconvert the found rotated contours $R\_C\_with\_\theta 1$ and $R\_C\_with\_\theta 2$ from Polar to Cartesian representation and denote them C_{ϕ_1} and C_{ϕ_2} whose x and y coordinates are obtained as follow:

$$x = r\cos\phi, \quad y = r\sin\phi \tag{4}$$

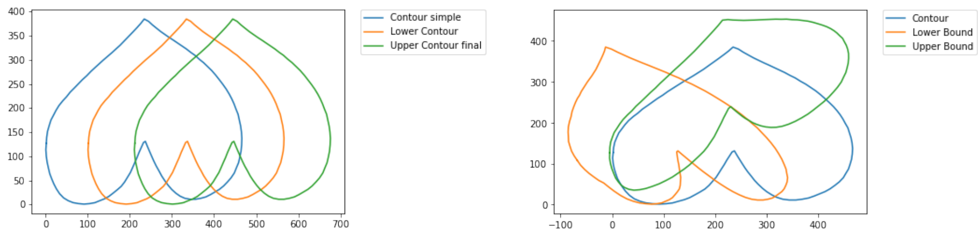
Let denote by T_{LB} and T_{UB} respectively the minimum and the maximum of C_{ϕ_1} and C_{ϕ_2}. They are used for initializing T_{fL} and T_{fU} for the first iteration. Next, LB corresponds to the minimum between T_{LB} and T_{fL} and UB corresponds to the maximum value between T_{UB} and T_{fU}.

4 EXPERIMENTAL SETTINGS

In this section, we present our experimental settings including the used contour datasets and the implementation environment.

4.1 Datasets Description

We carry out our experiments based on two contours datasets, the first is MPEG7 existing contour dataset



a　　　　b

Figure 2: UB and LB: a. Translation UB and LB in $[100, 200]$ b. Rotation UB and LB with $\theta \in [\frac{\pi}{6}, \frac{\pi}{3}]$.

Algorithm 1: Lower & Upper Bound Translate Contour with Batchsize.

procedure UB_LB_CONTOUR_TRANSLATION
Input:
$C \in 1 \times dim_C$; p_c; $Batch\_size \in N$;a, b
$step = \frac{(|b-a|)}{Batch\_size}$

for $k \in \{0, Batch\_size\}$ **do**
$\delta 1 = a + k \times step$
$\delta 2 = a + (k+1) \times step$
$T\_C\_with\_\delta 1$
$T\_C\_with\_\delta 2$
$T_{LB} = min(T\_C\_with\_\delta 1, T\_C\_with\_\delta 2)$
$T_{UB} = max(T\_C\_with\_\delta 1, T\_C\_with\_\delta 2)$
if $k = 0$ **then**
$T_{fL} = T_{LB}$
$T_{fU} = T_{UB}$
else
$T_{fL} \leftarrow min(T_{LB}, T_{fL})$
$T_{fU} \leftarrow max(T_{UB}, T_{fU})$
end if
end for
Return T_{fL}, T_{fU}
end procedure

and the second is a contour dataset generated from MNIST numbers using a mathematical morphology based algorithm.

1. MPEG-7 shape dataset consists of 70 types of object contours, each having 20 different shapes, for a total of 1400 shapes. The database is challenging due to the presence of examples that are visually dissimilar from other members of their class and examples that are highly similar to members of other classes.
2. MNIST shape dataset of handwritten digits (LeCun, 1998; LeCun et al., 1998) is a sub-set of a larger set available from MNIST. It contains 70000 samples divided into training set (60000 samples) and test set (10000 contours). 500 contours are utilized for robustness test.

Algorithm 2: Lower & Upper Bound Rotate Contour with Batchsize.

procedure ROTATION_UB_LB_CONTOUR
Input: $C \in 1 \times dim_C$; α, β; $Batch\_size \in N$;

$\phi, r = cart2pol(C_x, C_y)$
▷ convert Cartesian coordinates to Polar coordinates
step $= \frac{|\beta - \alpha|}{Batch\_size}$
for $k \in \{0, \ldots, Batch\_size\}$; **do**
$\theta_1 = \alpha + k \times$ step
$\theta_2 = \alpha + (k+1) \times$ step
$C_{\phi 1} = pol2cart(R\_C\_with\_\theta_1)$
$C_{\phi 2} = pol2cart(R\_C\_with\_\theta_2)$
$T_{LB} = min(C_{\phi 1}, C_{\phi 2})$
$T_{UB} = max(C_{\phi 1}, C_{\phi 2})$
if $k = 0$ **then**
$T_{fL} = T_{LB}$
$T_{fU} = T_{UB}$
else
$T_{fL} \leftarrow min(T_{LB}, T_{fL})$
$T_{fU} \leftarrow max(T_{UB}, T_{fU})$
end if
end for
Return T_{fL}, T_{fU}
end procedure

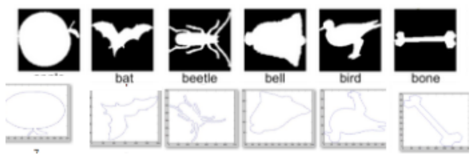


Figure 3: MPEG7 Dataset: On the top some samples from MPEG7 image dataset; On the bottom the corresponding contour.

4.2 Datasets Processing

In this work, we assume that contours are represented by their x and y Cartesian coordinates. We propose to re-parametrize them using the arc-length reparameterization given by formula 5. We suggest setting the number of points to 120 points for the inves-

Table 2: Different deep neural network architectures for MPEG7 and MNIST contours classification.

Dataset	Model	Type	#Units	#Layers	Accuracy
Contours Mpeg7	3×100	fully connected	51,471	3	61.42%
	3×150	fully connected	92,171	3	65.23%
	6×100	fully connected	81,771	6	66.19%
	1_Conv	Convolutional	41,402	4	70%
	1Conv_MaxPool	Convolutional	51,502	5	70.5%
	2_Conv1_MaxPool	Convolutional	65,408	5	74.5%
	3_Conv	Convolutional	41,436	6	73.8%
Contours MNIST	3×100	fully connected	45,310	3	94.03%
	3×150	fully connected	45,310	3	93.97%
	6×100	fully connected	75,610	6	93.96%
	1_Conv	Convolutional	35,213	4	92.4%
	1_Conv_MaxPool	Convolutional	45,341	5	93.7%
	2_Conv1_MaxPool	Convolutional	35,244	5	85.7%
	3_Conv	Convolutional	35,247	6	94.66%

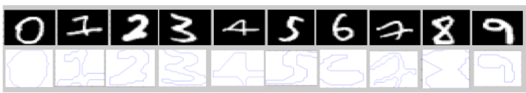


Figure 4: MNIST Dataset: On the top some samples from the MNIST image datest; On the bottom corresponding MNIST contour dataset.

tigated datasets.

$$s(t) = 1/L \int \sqrt{x_t(u)^2 + y_t(u)^2} du, t \in [0,1] \quad (5)$$

Where L represents the total length of the contour.

4.3 Implementation

We use Python for the implementation of the abstract domain in both cases: translation and rotation. As abstract interpretation analyzer, we use DeepPoly solution. It is based on two main libraries: ERAN and ELINA, coded in respectively Python and C programming languages. The pretrained models presented in table 2 are implemented, where fully-connected layers and convolutional models are evaluated using MNIST and MPEG7 datasets. We measure the robustness of these models and compare the obtained results in section 5. This criterion is calculated as the number of verified contours over the total number of well classified instances by the neural network. The robustness metric is set to:

$$Robustness = \frac{\text{Verified contours}}{\text{Well classified contours}} \quad (6)$$

5 RESULTS

The introduced DeepPoly analyser adapted for measuring the robustness of deep contour classifiers uses the abstract interpretation through UB and LB introduced in sections 3.1 and 3.2. It takes as input the different contours described in section 4.1 and processed as mentioned in section 4.2 as well as the different models detailed in table 2. Hence, we measure the robustness of these models in presence of two studied attacks: rotation and translation. We consider rotation intervals of 3° and translation intervals of 0.01 along the x axis. This choice of such little intervals is justified by the fact that the contours in our study are normalised during the training process. Using ERAN for computing the robustness values in presence of each attack with ($batch\_size = 100$), we obtain Figure 6 (resp Figure 7) that shows an example of robustness variation function computed using equation 6 on MPEG7 dataset (resp MNIST dataset) in case of rotation and translation attacks. The results of the Mpeg7 data show that the $2Conv\_Maxpool$ model is more robust against rotation and translation attacks while the $fully\_connected\_6 \times 100$ model is the most vulnerable. Often convolutional models are more robust than $fully\_connected$ because such models contain a features extraction block. This block gradually extracts invariants which makes it possible to describe each input so that it is subsequently classified through the fully connected part.

Figure 7 shows the results obtained on MNIST contours. The model with a single convolutional layer is more robust against geometric translation and rotation attacks. However, $fully\_connected\_6 \times 100$, $fullly\_connected\_3 \times 150$ are the most vulnerable against rotation. We notice that the convolutional models are more robust for the two types of attacks on the MNIST contours. In this case, the translation attack is stronger than the rotation, indeed the robustness decreases more quickly in the case of translation. Models trained on MNIST contours are more resistant against rotation. This may be due to the fact that the

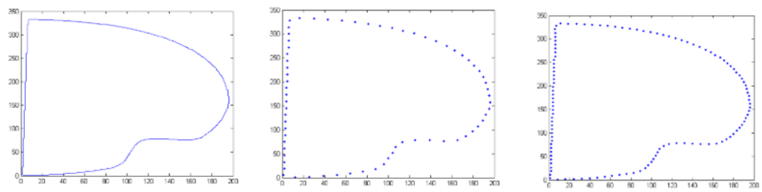
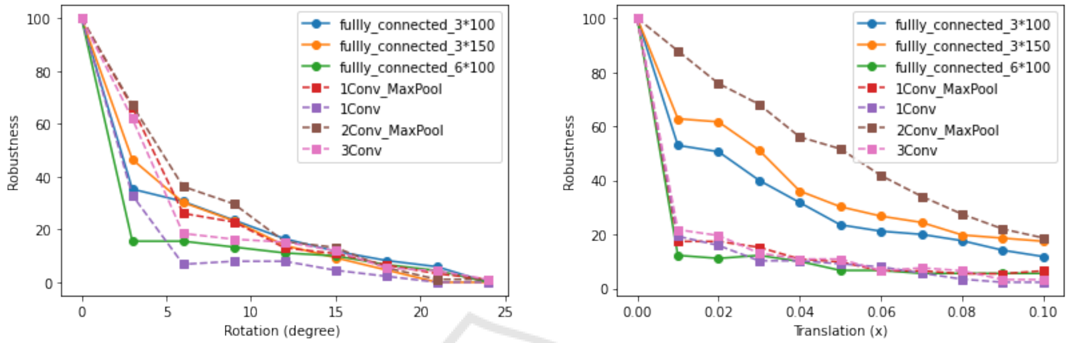
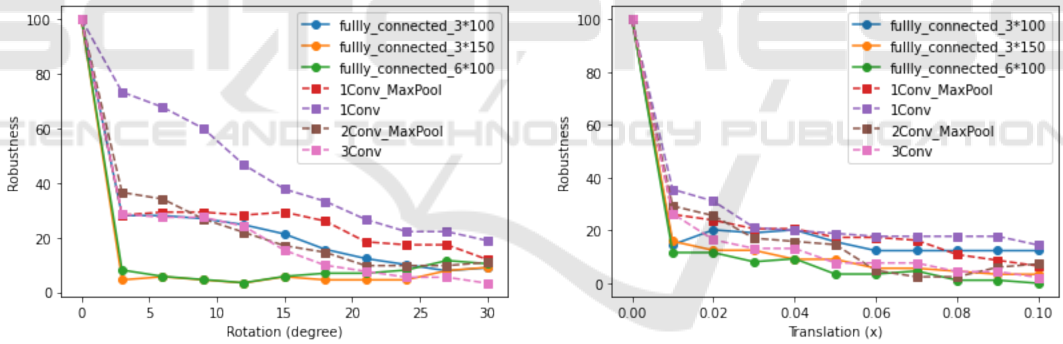


Figure 5: Contour arc-length re-parameterization. (a) Extracted contour from a shape that belongs to MPEG7 dataset. (b,c) Contour arc-length re-parameterization with respectively 70 points and 120 points.



a b

Figure 6: a) Robustness variation according to the rotation computed on 100 contours from MPEG7 contour dataset with different models. b) Robustness variation according to the translation computed on 100 contours from MPEG7 contour dataset with different models.



a b

Figure 7: a) Robustness variation according to the rotation computed on 500 contours from MNIST contour dataset with different models. b) Robustness variation aaccording to the translation computed on 500 contours from MNIST contour dataset with different models.

shapes in the database already have different orientations. There exists an infinity of possible models, our objective is not to test them all or find the most robust one in case of contour classification; We aim through the models given by table 2 to test our verification system. We conclude that the robustness varies as a function of the attack and it is not necessarily correlated with the model accuracy (Performance). In figure 8, we present these two metrics for different models tested on MPEG7 and MNIST datasets. The last two bars on the right show respectively the accuracy (in blue) and the robustness (in orange) of a deep neural network containing six layers each composed of one hundred neurons. The model is trained and tested on contours from MNIST dataset in presence of a rotation attack in the interval [0°,3°]. Despite the height of the accuracy which reached 93.96% , the model robustness is equal to 8.14%. Even Though this model performs well in terms of accuracy, it has a low robustness. To sum up, the evaluation of contour classifiers based on deep neural nets must take in consideration both metrics: accuracy and robustness.

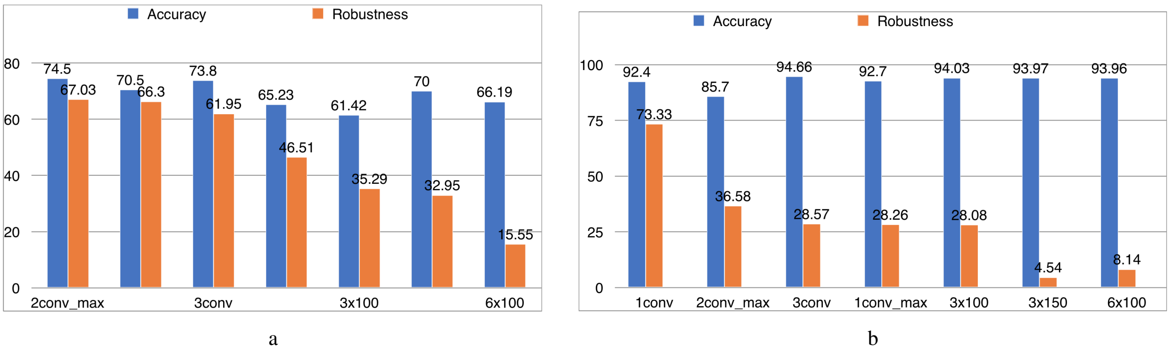


Figure 8: a) Accuracy & Robustness variation (%) as a function of rotation attack of [0°,3°] tested on contours from MPEG7 dataset with different models. b) Accuracy & Robustness variation (%) as a function of rotation attack of [0°,3°] tested on contours from MNIST contour dataset with different models.

6 CONCLUSIONS

This paper presents ContourVerifier: a novel system for the robustness evaluation of deep contour classifiers. Unlike the existing methods which deal with only image, video, time series or audio data types, our approach enables the verification of deep classifiers designed for shape recognition and considering contour information as a 2D closed planar shape. We define the appropriate Upper and Lower bounds of the shape perturbed with a translation or rotation. Given this abstract domain, and a set of test contours, ContourVerifier computes the robustness value of the given pre-trained model using DeepPoly analyser. As an initial step, we have considered rigid transformations of the contours. In further work, we aim to extend ContourVerifier to support more perturbations such as nonlinear and projective transformations.

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