Formative Feedback in Mathematics Teacher Education: An Activity and Affordance Theory Perspective

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Abstract: The increasingly high number of students’ enrolment has necessitated the recent attention on the use of computer-based assessment systems for feedback delivery to students for mathematical learning, such as Numbas. However, little is known about the affordances of Numbas in the research literature. The purpose of this study is to investigate the affordances of Numbas, their perception and actualization by students and teachers, and their effects on mathematical learning from an activity and affordance theory perspective. The study follows a qualitative research design using semi-structured interviews of six students and two teachers. The results reveal the perception and actualization of several affordances at the technological, mathematical, and pedagogical level. Conclusions and future work are drawn from the results to promote Numbas formative feedback for teaching and learning mathematics.

1 INTRODUCTION

In recent years, emergent technologies like computer-based assessment systems are gaining more attention in mathematics education because they provide a resource-efficient way to providing the much-needed timely feedback to the students. Computer-based assessment systems provide new learning potentials for a large cohort of students by means of formative and summative assessment. However, research on computer-based assessment systems is still in its infancy, especially in the area that assesses the added value, affordances and constraints of such systems (Csapó et al., 2012; Hadjerrouit & Nnagbo, 2021).

This study proposes a framework that captures the affordances and constraints of Numbas in a technology-based course at the University of Agder. This study relates to previous research work on affordances of Numbas in mathematics education (Hadjerrouit & Nnagbo, 2021; Nnagbo, 2020). In specific terms, the study aims to address the following research questions:

1. What affordances of Numbas are perceived by students and teachers?

2. How are the perceived affordances of Numbas actualised by students and teachers?

3. What are the constraints for the actualisation of Numbas affordances by students and teachers?

2 NUMBAS

Numbas is a computer-based assessment system for mathematics and mathematics-related courses with emphasis on formative assessment and feedback (Lawson-Perfect, 2015). The primary use of Numbas is to enable students to enter a mathematical answer in the form of an algebraic expression, and then see how Numbas feedback can impact students’ mathematical learning. Numbas allows several question-and-answer types such as mathematical expression, number entry, matrix entry, match text pattern, choose one or several from a list, match choices with answers, gap-fill, information only, are supported by Numbas. The system shows the notation instantly beside the input field, so as students are inputting their answers, simultaneously they see how...
the tool understands their expressions. Numbas provides several capabilities to users.

**Ease of Integrating Rich Content Materials:** Numbas supports videos and interactive diagrams to be embedded on the editor before they are distributed along with the final questions. The videos can be uploaded directly, while the interactive diagrams could be included in Numbas questions either by embedding a GeoGebra applet or use JSXGraph.

**Marking:** Numbas uses marking to mark mathematical expressions. For example, in factorizing a quadratic equation, expected answers are often in this form \((x+a)(x+b)\) and not \(x^2+ax+b\), but Numbas marking algorithm is capable to understand the later form, mark correctly and give feedback accordingly.

**Feedback:** Numbas makes its feedback immediate. In order to make the feedback effective, there are multiple ways Numbas gives feedback to both students and instructors. These include the following options: submit answer, show steps, reveal answers, try another question like this one (Figure 1).

**Submit Answer:** Students get feedback when they submit an answer. The feedback indicates with a green color ‘good’ sign if the answer is correct, with red color ‘bad’ sign indicating that the answer is wrong, or partially correct. The students will also be shown the maximum attainable score for each question, and their own score for the question after they have submitted the answer. The teacher may choose to disable these feedback options.

**Show Steps:** When “show steps” is chosen, Numbas will give the general solution to that task. This is a way of reminding the student to have a look at the general solution and retry solving the task. This does not give the exact solution to the particular task.

**Try Another Question Like This One:** With this option, students have the opportunity to attempt similar questions many times until they feel confidence to move to the next question.

**Reveal Answer:** This option provides a step-by-step solution that is personalized to the question, but the students lose all the marks and cannot re-attempt the exact question. This option may be disabled by teachers.

**Statistics:** Numbas stores data of students’ performance. Teachers can track how well the students understand the topic through their performances, and they can equally identify the tasks students perform below expectations and reemphasize on them in the next class if necessary.

### 3 THEORETICAL FRAMEWORK

Activity Theory (AT) is coupled with affordance theory to form the theoretical framework of this study. AT is found to be a source of useful concepts for describing how Numbas interacts with other elements of the learning context, including students, teachers, and the physical environment (Day & Lloyd, 2007).

AT is combined with affordance theory (Volkoff & Strong, 2017) to explicate the concepts of emergence, perception, actualisation, and effects of Numbas affordances on teaching and learning mathematics. More precisely: (a) The emergence or existence of Numbas affordances; (b) The perception of Numbas affordances; (c) The actualisation of Numbas perceived affordances; and (d) The effects of Numbas affordances on learning and teaching.

#### 3.1 Activity Theory (AT)

AT has its root in the cultural-historical psychology work of Vygotsky, Leont’ev, and Engeström. The primary ideas of the theory rests on the social-cultural perspective of learning in which learning is conceived as an offshoot of a dynamic relationship between the learner and the environment. With other words, learning is an appropriation of knowledge through a feedback relation between the learner and the environment (Vygotsky, 1978).

A fundamental concept in AT is the word ‘activity’ itself (Engeström, 2014). Leont’ev (1978) defines an activity as any purposeful interaction between a subject (which could be an individual or collective), and an object. Leont’ev (1978) further describes activity as the most basic unit of life; that subject and object have no noticeable properties if there is no activity. Thus, when activity is not studied and understood, it may be difficult to deduce how an
artefact affords a subject. The underlying assumption of the theory is that an artefact or tool mediates the interaction between subject and object to give the desired outcome.

3.2 Affordance Theory

The term ‘affordance’ was proposed by James J. Gibson to describe what the environment offers the animal (Gibson, 1986, p. 127). He argues that affordances (henceforth, in plural or singular form) can be seen from the properties of the environment that are relative to the animal in question. He further stresses that affordances must be peculiar to the animal they afford; not just any property of the environment or whatever the environment can offer.

In the world of Human-Computer Interaction, the term “affordance” (Norman, 1988) refers to a goal-oriented action potential that emerges as result of interaction between subjects (e.g., students and teachers) and an object (e.g., Numbas). Affordance is neither the property of an object in isolation nor that of the subject. Instead, it emerges as an offshoot of a dynamic relationship between the subjects (students and teachers) and the object (Numbas). It is perceived (i.e., students and teachers are aware of the existence of the action potential of Numbas) in many ways and actualized (i.e., students are able to turn the potential of Numbas into action) to produce effect (i.e., feedback delivery) depending on many factors that include Numbas platform, its user interface, capability of the students and their level of preparedness. Moreover, the actualization of Numbas affordance is either facilitated by some enabling conditions or mitigated by some constraints.

Given the emergence of Numbas affordances, it is important to ask how the affordances are perceived. As such, when students interact with Numbas to facilitate feedback delivery on some mathematics concepts they do so conveniently with the aid of the technological features of the tool. During this process, they become aware of the affordances that emerged during the interaction in terms of feedback delivery. The next issue is how they can actualize these affordances. Affordance actualization is a process of turning action potentials (affordances) into real actions to bring an effect in using a particular tool (Anderson & Robey, 2017; Bernhard et al., 2013). To turn a possibility into an action, it is expected that the user has the ability and capability to harness the potential and there are enabling conditions to facilitate the process. Affordance actualization may vary from one individual to another because it is goal-oriented and a process of specificity. Two or more students may interact with Numbas and actualize (or not) different affordances of the tool depending on their respective individual differences and choices.

Moreover, it is expected that following the actualization of Numbas affordances are some effects, which may be “intended by the user and/or those by the original creator of the artefact as well as unintended effects” (Bernhard et al., 2013, p.6). Thus, it is expected that when affordances are perceived and actualized, then some effects are generated in terms of feedback delivery to students.

Drawing on this view, Engeström (2014) asserts that the subject of any activity system uses a combination of both physical and psychological tools. As such, the mediating artefact in the present study is Numbas. It is important to remark that there is a thin line between the mediating artefact (Numbas) and the object (feedback delivery) in this study because the former encloses the latter. Unlike, physical classroom objects such as whiteboards and pointers that are used to mediate learning content.

Therefore, it is argued that the outcome of a dynamic interaction between the subject (e.g., student), the object (feedback delivery), and the mediating artefact (Numbas) are the affordances of Numbas. In other words, Numbas affordances are not an exclusive property of the tool and not completely determined by the subject; instead, they emerge from a dynamic interaction between the tool and the subject. A key issue is that the interaction between the subject and object is considered from a socio-cultural perspective following the lines of thought of Gibson (1986).

Figure 2 shows the theoretical framework that captures the emergence, perception, and actualisation of Numbas affordances, and their effect from an activity theoretical perspective. The perception of Numbas affordances concerns its awareness by a goal-oriented user during the interaction. Affordance actualisation is a process of turning action potentials (affordances) into real actions to bring an effect in using a particular tool (Anderson & Robey, 2017; Bernhard et al., 2013). In specific terms, affordance actualisations are “the actions taken by actors as they take advantage of one or more affordances through their use of the technology to achieve immediate concrete outcomes” (Strong et al., 2014, p. 70). Moreover, it is expected that following the actualisation of Numbas affordances are some effects.

It is important to highlight that actualisation of Numbas affordances does not happen in isolation. In fact, affordances are not without constraints; these are facilitated by enabling conditions and hindered by constraints. As captioned by Hadjerrouit (2020)
affordances and constraints are inseparable because they complement each other, and not opposite.

Figure 2: Perception, actualisation, and effects of Numbas affordances from an Activity Theory perspective.

4 METHODS

A case study design approach (Yin, 2009) is chosen to understand and analyze the affordances perceived by both students and teachers while interacting with Numbas, and how they actualize the perceived affordances. Data collection was done from two set of participants: Two teachers and six students from a mathematics teacher education class of a Norwegian university. The two teachers were considered and selected because they are actively making use of Numbas for formative assessment in their respective classes. The second cohort is six out of about twelve students from one class who willingly volunteered to participate in the study. These participants are master’s degree students taking a course entitled “Digital tools in mathematics teaching”.

A thematic approach is used to analyze the data by identifying themes or codes within the data set (Bryman, 2016). The analysis takes both a deductive and inductive approach by following the pre-defined framework in search for meaningful interpretation of the empirical data. Rom is given for the data to express itself by creating new codes that emerge from the data inductively. The development of codes follows reading and rereading of the data carefully and annotating same to identify topics, which are refined and validated by checking whether these are repeated or highlighted by different participants as an important topic (Hennink et al., 2020).

5 RESULTS

Figure 3, which is an extension of figure 2, shows the results achieved so far. The figure shows both students’ and teachers’ activity systems in interaction, and the affordances (and constraints) that emerged, are perceived, and actualized, and their effects on teaching and learning. Three types of affordances are perceived: (a) Technological (e.g., ease-of-use and navigation); (b) mathematical (e.g., varied mathematical representations); and (c) pedagogical (e.g., learner autonomy, motivation, formative feedback, etc.). A subset of the perceived affordances is actualized, and some of these have an effect on teaching and learning. Space is limited to report on all affordances. Therefore, the paper focuses on the three types of affordances highlighted above.

Figure 3: Students’ and teachers’ activity systems in interaction, and the affordances that emerged, are perceived, actualized, and their effects on teaching and learning.
5.1 Technological Affordances

The findings reveal that both teachers and students globally share the same views regarding technological affordances. They perceived and actualized affordances related to technological issues such as ease-of-use and navigation, accessibility, and facility to contain mathematical contents.

Regarding ease-of-use and navigation, one of the students pointed out that "(…) anything you see there is understandable; they are not complex. I think everything is ok, I don’t have any problem with it. I think the graphics are ok. It's just simple to use, there are not much confusing buttons, every icon in the interface is self-explanatory. It's just attractive". Another student added "the navigations were fairly easy, the buttons are visible with good inscriptions, just click on it and see what is inside. Like I said there are not too many icons, so anywhere you want to move to, it’s easy to find, and navigate there.”.

One of the teachers said: “it’s very much simple to use, especially when compared with MyMathLab, the main feedback from my students was that they could see the mathematical expressions when they write it in Numbas, they could see how the program understands what they feed in, unlike in other programs; so they committed fewer errors in Numbas than in MyMathLab”.

The study finding clearly shows that the perception of technological affordances such as navigation and ease-of-use supports the perception of and actualization of other affordances - which depend to a large extent on the technological features of the tool - such as learner autonomy, differentiation, collaboration, and variation. If students or teachers find the interface of Numbas difficult to use, they may likely not use the tool to achieve their pedagogical purpose. If the navigation buttons are hidden, the user might not be able to move to the feedback pages, thereby not getting the desired help.

However, reverse is the case when the teachers themselves interacted with Numbas for the purpose of creating tasks. Their responses seem to suggest that creating tasks in Numbas is difficult, especially when the task is a complex one. This can be seen from the response given by one of the teachers "I will say that could probably be better, once you start to getting the grips on, I will say that using the basic things if you want to create a simple task is quite easy, but again as soon you start on more complicated questions, on what to do more, (…), I will say it's not that intuitive then you really need to go into the guidance because there is a lot of boxes to check out if you want to do that and you could”.

Finally, the findings reveal some, mostly technological constraints both for students and teachers, such as insufficient navigation buttons, poor internet connection when solving tasks, lack of teachers’ knowledge and skills, e.g., programming skills and lack of time for teachers.

5.2 Mathematical Affordances

Both teachers and students perceived the mathematical affordance “varies mathematical presentations”. With this affordance, teachers can create formative assessment tasks using different representations - diagrams, graphs, matrices, multiple choices etc., also they can create the associated feedbacks in various forms that may cover students’ misconceptions. Formative feedbacks that Numbas give in these forms are found useful and motivating by the teachers and students.

One of the students stated: “I think the presentation of math contents in Numbas is of high quality. Many things including graphs, diagrams, videos, formulas, numbers, signs are well presented ...I think it's very nice”.

Another student suggested: “I have also come across in Numbas some questions that contain GeoGebra pages and graphs, that show how sophisticated Numbas is, and that makes presentations of mathematical contents really pleasing”. Therefore, the possibilities of increased variation, including supporting embedment of third-party software, are high in using Numbas. The tool was also found to be useful in terms of enhancing pen and paper skills of students.

Likewise, another student indicated that “yes, again as I said before, you often need your pen and paper to do the calculations on Numbas. ...you have to solve the tasks on paper especially the difficult ones, by doing so, your pen and paper skills are developing”.

The findings from the students’ perceptions are similar to teachers’ views. One of the teachers thinks that the “presentation of mathematical contents like graphs, interactive diagrams, videos, GeoGebra work well too… You can put in video and everything, or link to YouTube channels or different pages and it shows the video, you can play it within the program”.
5.3 Pedagogical Affordances

Both teachers and students perceived and actualized several pedagogical affordances, such as learner autonomy, collaboration, differentiation, and in particular formative feedback. Most perceived affordances were actualized with effect on motivation, engagement, learning and misconceptions (see Figure 3).

Basically, formative assessment requires setting learning and monitoring progress towards achieving the goals. This type of feedback provision helps to achieve learning goals. Similarly, Numbas feedback gives the students the opportunity to access the level they are in a learning process, what the learning goals are, and how to achieve them. Findings reveal that Numbas promotes formative assessment to both students and teachers in a timely fashion in four different forms:

a) It provides feedbacks to the teachers in form of the statistical report of students’ activities
b) It provides support for students to test their knowledge and exercises as much as they want
c) It helps students improve their learning, and stay on track to meet their goals
d) It gives other types of feedbacks in different forms, e.g., instant feedbacks, reveal answers, show steps, or try another question like this one

Firstly, with statistical reports, time is saved for teachers and students. From the teacher point of view, the feedback in form of statistics containing students’ problem-solving strategies and ways of thinking identifies their current performance level, areas of difficulties and strengths are useful to the teachers for conducting diagnostic teaching. Both teachers expressed satisfaction with Numbas, particularly because the tool is equipped with randomization mechanisms, which means that it can generate unlimited similar tasks with corresponding feedbacks. This saves teachers a lot of time. They do not need to spend days preparing tasks for formative assessment. It also offers students the opportunity to solve many tasks until they master the topic.

Secondly, teachers think that students have shown motivation by asking for an opportunity to do more exercises in Numbas, even when they have reached the threshold. This can be seen from one teacher’s response: "(...) I think the instant feedback is motivating for the students". The other teacher suggested that "for most of them, at least for the way I do it with this kind of programs they need (...) to pass a certain amount of test to be able to attend exam, ... and most of them will do it again even though they have passed the test, because they want to improve...I have got students that write to me asking can you open the test again, I want to get 100%".

Thirdly, in terms of quality feedback, one of the students responded: "...with the two equations, there was a movie, and it was sort of helping because it assured me that I was doing it in the right way. The third one, it was helping because it was the rule you were supposed to use". Another student explained that "... it gives you a lot better feedback, than most of that kind of programs ...So that feedback is good, and as I said, when you write, the next box shows you how the program interprets, that program is really good". Students seem to appreciate the feedbacks, including the video hints. The response from one student does not only show that the video helped her, but it also encouraged and motivated her to solve the task. As a result, her confidence increased. Another student tried to compare the feedback to that of other similar tools and she found it better than other programs she had used. She was particularly overwhelmed that Numbas could instantly show how it understands her answers.

Finally, in terms of instant or immediate feedbacks, hints, and reveal answers, findings show that the students equally found Numbas feedback helpful and motivating. Teachers state that their students “do get stuck” and when they do, that “most of them chose to show hints and the tips, (and) the other feedback options from the program”. They also think that the feedbacks motivate the students.

Findings also reveal that engagement in Numbas enhances students’ motivation. Students identified among others, the instant feedback to be very motivating. However, they believe that bulk of the job lies on the teachers’ ability to create tasks that would take into consideration students’ misconceptions about a particular task.

They further expressed concerns that the feedbacks, no matter how good it may be, may never be sufficient to get some students going, especially the low achieving students. This can be seen from one of the teachers’ responses “I would say that the feedback does help them but again for the strongest students, it’s helpful for them but the weaker students, I think they need the teacher actually to tell them what they have done wrong, it’s not enough for them to see the feedback or the examples.”

6 DISCUSSION

The purpose of this paper is to explore how Numbas promotes formative assessment for mathematics teaching and learning by analyzing the affordances
and constraints that emerge from interactions between teachers/students and Numbas.

The main essence of formative assessment according to Weeden et al. (2002) is to identify students’ current performance that will hopefully lead to improvement in learning and teaching. Therefore, formative feedback is vital to improving mathematics education (Pereira et al., 2016).

Feedbacks from teachers to students regarding their performances, challenges and difficulties are aimed at encouraging and helping them to identify their misunderstandings and misconceptions regarding the topics, concepts, and ways to improve. Many studies have linked feedback as one of the most powerful ways to increase students learning and achievement (e.g. Hattie & Clarke, 2018; Hattie & Timperley, 2007). However, delivering it on time is often challenging to the teachers.

This is the reason why formative feedback is done while Numbas is on-going. It is to identify how far teaching and learning goals have been achieved. Teachers and their students mostly undertake this kind of assessment to obtain vital information in form of formative feedback that they will apply to modify and improve the ongoing teaching and learning activity (Black & Wiliam, 2010).

Figure 2 and 3 show that achieving the goal (formative feedback delivery), which is needed to improve teaching and learning of mathematics subject depends on the perception and actualization of the emerged affordances of Numbas by students or teachers. If they fail to actualize the affordances, the intended goal may not be achieved.

The object is the mathematical knowledge in the form of formative feedback while the subject is the student/teacher, and the mediating artefact is Numbas. Then, the outcome of a dynamic interaction (activity) between the subject (student/teacher), the object (formative feedback), and the mediating artefact (Numbas) is the affordance of Numbas. Thus, the goal of students is to receive formative feedback from Numbas. However, the desired goal (formative feedback delivery) does not manifest straight away. In fact, it manifests as an effect of the actualized affordances of Numbas.

In an activity system, teachers and students are the subjects, and the goal of the teachers in their relationship with students is to give feedback to the students or receive feedback about the students’ performance through Numbas. While the goal of students is to receive feedback from teachers through Numbas. Therefore, formative feedback delivery is the common goal, but the ultimate goal, which is the effect of the formative feedback delivery is to improve teaching and learning of mathematics. According to the theoretical framework, the desired goal (formative feedback delivery) does not manifest itself directly, but as an effect of actualization of Numbas affordances. Moreover, the emergence of Numbas affordances is viewed as an offshoot of a dynamic relationship between students/teachers and Numbas, and the perception of the emerged affordances concerns its awareness by students/teachers. Whereas actualization is the action taken by the students/teachers to take advantage of the perceived affordances.

When students and teachers actualize some required affordances, then the effect will lead to achieving the goal (formative feedback delivery) and by extension improves teaching and learning. For example, when a student wants to solve mathematical problems at home using Numbas, her/his goal is to achieve formative feedback through the mediation of Numbas. However, she/he must first of all actualize the affordance of accessibility (amongst other affordances needed). If the student faces constraint of internet connection, then the effect will be that she/he will not achieve her/his goal (formative feedback delivery) because she/he could not actualize an important affordance required. But if the student actualizes the affordance by accessing the internet, she/he may achieve the goal (formative feedback delivery), however this is subject to actualizing other affordances (like ease of use, navigate, etc.) she/he might also need to successfully achieve the goal.

7 CONCLUSIONS

The main contribution of this paper is the development of a theoretical framework drawing on a combination of AT and affordance theory. AT has proved to be useful for arguing that the emergence of Numbas affordances is a result of a dynamic relationship between a goal-oriented user and the assessment tool. Likewise, affordance theory has shown to be a useful in explaining the distinctiveness of the perception and actualisation processes of affordances. However, the framework as presented in this study is not intended to map all affordances and constraints, but it is open enough to capture potential affordances. This is the reason why the deductive-inductive approach to data analysis is so important for the emergence of affordances. Moreover, Csapó et. al. (2012) posited that large-scale implementation of computer-based assessment systems still needs further investigations in real education settings.
Summarizing, the findings show that Numbas is basically a useful tool for assessing mathematical concepts and problem-solving. However, there are issues related to the feedback, which can act as a source of motivation for a few students while demotivating other students. Numbas may be included in the Norwegian curriculum with the sole intention of identifying possible problems and effecting necessary modifications along with improving the learning of students and teachers. For teachers, it is important to ascertain their role in using their skills and expertise for adding new tasks of formative assessment, and identifying students’ learning progress, while for students, it is important to focus on using Numbas as a practice, learning, and feedback tool. However, the role of Numbas should be clearly defined along with the role of teachers.

From a practical point of view, the study has two limitations. Firstly, the participants (N=8) are master’s students and their teachers (N=2) from a teacher education program of one university. A larger number of participants from several universities could have been more desirable to make better generalization. Nevertheless, the chosen number of participants with a large set of information seems to be justifiable for addressing the research questions.

The second limitation is that the participative students are not the ‘end users’ of Numbas. Though they have sufficient knowledge of Numbas, and used the tool for assessment, but in a limited form. However, it is difficult to generalize their views to encompass students using Numbas regularly in their studies. Students from other study programs using Numbas for day-by-day activities may have a different perspective about perception of affordances and actualization processes. Future research studies involving such set of students would be relevant to compare with findings of the present study to achieve more reliability and validity of the results.

REFERENCES


