Assortment and Cut of Defective Stocks by Bilevel Programming

Claudio Arbib[1][a], Fabrizio Marinelli[2], Mustafa Ç. Pınar[3] and Andrea Pizzuti[2]

[1] *Dipartimento di Ingegneria/Scienze dell'Informazione e Matematica, Università degli Studi dell'Aquila, L'Aquila, Italy*

[2] *Dipartimento di Ingegneria dell'Informazione, Università Politecnica delle Marche, Ancona, Italy*

[3] *Department of Industrial Engineering, Bilkent University, Ankara, Turkey*

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Abstract: In this paper we deal with the problem of deciding the best assortment and cut of defective bidimensional stocks. The problem, originating in a glass manufacturing process, can arise in various industrial contexts. We propose a novel bilevel programming approach describing a competition between two decision makers with contrasting objectives: one aims at fulfilling production requirements, the other at generating defects that, damaging the products, reduce yield as much as possible. By exploiting nice properties of adversarial optimal solutions, the bilevel program is rewritten as a one-level 0-1 linear program. Computational results achieved on random instances with realistic features are discussed, showing the quality and the benefits of the proposed approach in reducing the yield loss from defective material in a worst-case perspective.

1 INTRODUCTION

This study finds its initial motivation in the optimization of a float glass manufacturing process. A process of this type is organized in two main stages. The first employs a float furnace to produce large rectangular glass sheets (*large items*): these are obtained in various measures, ranging from 12.6 to 19.6 sqm, by widening/narrowing a ribbon of molten glass, and then sent to warehouse. In the second stage, small rectangular sheets (*small items*), sizing between 0.21 and 3.3 sqm, are cut from the large ones previously manufactured, and then sent to downstream departments.

Mathematical programming models (Arbib and Marinelli, 2007; Arbib and Marinelli, 2009) were originally designed (and assessed in practice) for the simultaneous optimization of large sheet *assortment* and cut in the above-mentioned process. The assortment issue refers to the number of distinct large sizes stored in the warehouse at any time: this number (potentially large up to 6500) must be limited to a certain amount p (typically ≤ 20) so as to contain holding cost and setups, and also to facilitate handling operations. The cutting issue requires on the other hand to fulfill a known demand of distinct small sizes. The minimization of the *trim loss*, i.e., the difference between the total area of used glass and the total area of required small parts, is sought on the whole.

The two quoted papers study the problem in a deterministic context, assuming perfect knowledge of all the necessary data. However, glass fabrication naturally carries the burden of imperfections (e.g. bubbles) that, occurring in a substantially unpredictable way, may compromise the quality of small items and hence reduce yield. The estimated loss due to defects is a non-negligible cost term, sometimes even larger than trim loss. This, in the end, motivated us to investigate ways to model and compute *defect-reconfigurable* cutting patterns.

Approaching the issue of faultiness, one can either assume an a-priori knowledge of defect positions in raw material (as in most literature), or capture the stochastic nature of defects by modeling their occurrence as a random process which generates faults in the large items. The approach which best fits the observed situation depends on both practical and problem-specific elements. In our case, no prior knowledge of defects is available before the end of large items fabrication in the first stage. In addition, the decision process simultaneously defines both large items assortment and cutting patterns, and therefore penalizes on-line strategies in which defect-free patterns are redesigned after spotting the faults. This policy would in fact completely consume the yield gain with additional material handling, machine setups and operation scheduling for downstream depart-

[a] https://orcid.org/0000-0002-0866-3795

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ments.

In practice, though imperfections are spotted by visual inspection and marked on the large items, they are not considered before the cutting stage, and small items affected by faults are simply discarded before the following stages of the production process. Modelling problem uncertainty is therefore preferable and, in any case, is a step beyond the straightforward reaction of just reckoning defects at the end of the cutting stage, and then simply discard faulty items.

That said, a production policy in presence of defects is liable of two complementary visions: (*i*) prioritize demand fulfillment (as in just-in-time philosophy) and overproduce minimizing the need of raw material; (*ii*) given some availability of raw material, maximize the expected faultless production on can obtain from it, covering possible order backlog by safety stocks.

Policy (*i*) was addressed in (Arbib et al., 2021). In this paper we follow policy (*ii*) and define a novel bilevel mathematical formulation of an assortment-and-cut problem to maximize the total value of faultless small item production: defect occurrence is modeled as an optimization problem solved by an adversarial follower that tries to place defects in a way that reduces production value as much as possible. The worst-case perspective of the adversarial model is subject to the following assumptions:

- the adversary can place at most one defect at random in each large item;
- the total number of defects the adversary can place in the planning horizon is a model parameter;
- yield reduction is measured by the expected value of small item loss, computed in turn by the probability that no cut reconfiguration exists, which allows defect avoidance.

The first of these assumptions is indeed quite restrictive, yet is compliant with some real productive processes, where defects are sufficiently spread on raw material, i.e., where defects do not form clusters and rather appear according to a spatial Poisson point process. Moreover, it help us give a first clear formalization in bilevel terms under mild hypothesis, indicating an original robust optimization methodology to be hopefully extended in the future.

2 LITERATURE REVIEW

Since the seminal paper of Gilmore and Gomory (Gilmore and Gomory, 1961), a vast amount of scientific contributions were dedicated to cut optimization, focusing on both the theoretical and practical side of the problem, see (Wäscher et al., 2007). Although cutting problems with defects were considered very early (Hahn, 1965), quite few papers address this issue. Most contributions assume defect sizes and locations known in advance: to quote just an example, (Aboudi and Barcia, 1998) study the MINIMUM DEFECTIVE SUBSET SUM which asks to find a new pattern layout avoiding the largest possible number of defects in a one-dimensional stock. To the best our knowledge, only (Sculli, 1981) and (Ghodsi and Sassani, 2005) consider instead stochastic settings. In the former, to handle fringe defects in insulating tape production, roll size is treated as a normal random variable that models damages caused by winding. The latter addresses a real-time wood cutting process where each strip to be cut is subject to random variations of quality along its length.

Due to the wide range of manufacturing contexts where cutting is involved, the theme of defectiveness is characterized by a multiplicity of features: holes and stains in leather or paper sheets, bubbles in glass, knotholes in wooden boards. This variety originated a corresponding richness of defect models, from point-shaped to convex areas, from imperfections compromising the quality at various grades to faults completely ruining the items.

In (Carnieri et al., 1993) a two-stage decision model is proposed for application in a one-dimensional lumber cutting problem, in which each stock item is affected by a single defect. Under the same restriction of one defect per stock item, (Neidlein et al., 2009) address a two-dimensional cutting problem and propose an AND/OR-graph approach for the defective case.

Multiple defects per stock item are also considered. In (Ozdamar, 2000) a concurrent scrap minimization/profit maximization one-dimensional textile cutting problem is discussed and dealt with, using a mutative simulated annealing. A genetic algorithm is presented by (Wenshu et al., 2015) for cutting one-dimensional wood board with possibly decayed portions, which affect product appearance and material strength. In a two-dimensional setting, (Afsharian et al., 2014) investigate a dynamic programming-based heuristic that aims at maximizing the value of the small items produced.

Among papers that consider product values conditioned to stock faults, (Sarker, 1988) studies the optimal use of defect-free areas and devises a dynamic programming procedure for the maximization of total sales value. Similarly, (Rönnqvist and Åstrand, 1998) optimize a wood-board cross cutting process by a mathematical formulation and dynamic programming. Finally, (Durak and Tüzün, 2017) presents an on-line optimization setting in glass manufacturing where the float line is directly equipped with sensors: hence, cutting patterns can be redefined on the fly to avoid defects, but only within few seconds after detection and according to several physical constraints that limit pattern configurations. Solutions are computed by dynamic programming and MILP-based algorithms. To the best of our knowledge, no reference addresses the combined assortment-and-cut problem discussed here.

3 THE ASSORTMENT-AND-CUT PROBLEM

In this section we briefly review the 0-1 Linear Program presented in (Arbib and Marinelli, 2009) to model an assortment-and-cut problem arising in a real manufacturing plant.

Technical constraints of the cutting machines and organizational rules imposed by the management impose several restrictions to cutting patterns:

- Only guillotine cuts are admitted, the first one always horizontal.
- A large item can only be cut into single-size equally oriented small items (always choosing the most productive item orientation).
- Cutting patterns are built from top-left, hance trim loss is always positioned at the bottom-right corner of the large item.

Let L (let S) be the set of all feasible large (of required small) sizes that can be produced in the first (second) stage. Let W_j and H_j (w_i and h_i) respectively be the width and height of $j \in L$ (of $i \in S$) and $A_j = W_jH_j$ ($a_i = w_ih_i$) be the item area. Suppose also $i \in S$ required in d_i copies in the planning horizon.

The demand d_i can be fulfilled by cutting an arbitrary subset $P \subseteq L$ of large sizes. For each $j \in P$, up to $n_{ij} = \lfloor A_j/a_i \rfloor$ small items of size i can be obtained from a large item of size j (we say that n_{ij} is the outcome of a *maximal* cutting pattern). Introducing the integer variable $y_{ij} \geq 0$ to count how many large items j are employed to manufacture small size i, the total material cost c_{iP} to cut i from $j \in P$ can be computed through the following integer knapsack problem:

$$
\begin{aligned}
c_{iP} = \min \sum_{j\in P} A_j y_{ij} & & & \quad (1)\\
\sum_{j\in P} n_{ij} y_{ij} &\geq& d_i &\\
y_{ij} &\in& \mathbb{Z}^+ &
\end{aligned}
$$

Costs c_{iP} are then plugged into the following mathematical model of the assortment-and-cut problem:

$$
\begin{aligned}
\min \sum_{i\in S}\sum_{P\subseteq L} c_{iP} x_{iP} & & & & (2)\\
\sum_{P\subseteq L} x_{iP} &=& 1 & \quad i \in S & (3)\\
\sum_{P\subseteq L: P\ni j} x_{iP} &\leq& z_j & \quad i \in S, j \in L & (4)\\
\sum_{j\in L} z_j &\leq& p & & (5)\\
x_{iP},\ z_j &\in& \mathbb{Z}^+ & \quad i \in S, j \in L, P \subseteq L &
\end{aligned}
$$

The x_{iP} and z_j are implicitly forced to behave as binary variables in the model. Hence, we interpret $x_{iP} = 1$ if the whole demand of small size i is met by cutting large sizes $j \in P$, and $z_j = 1$ if at least one large item of size $j \in L$ is cut.

The objective function measures the total amount of glass employed in the process. Equations (3) ensure demand fulfillment for all $i \in S$; inequalities (4) identify the distinct large sizes $j \in L$ selected in the assortment from subsets $P \subseteq L$; finally, by (5), the assortment must not exceed p different sizes.

Since variables x_{iP} are exponentially many in the cardinality of L (i.e. $O(|L|)^p$), (Arbib and Marinelli, 2009) solve (2) by column generation. A very effective heuristic can anyway be devised by restricting subsets P to singletons $\{j\}$, so that each small size i is produced by only one large size in L. This brings to a p-median formulation that widely shrinks the size of (2) and well approximates the original problem, as its optimal value asymptotically tends to the true optimum as small item requirements increase (Arbib and Marinelli, 2009). Note, in particular, that for $P = \{j\}$ the optimal solution of (1) is immediately found as

$$
\bar{y}_{ij} \triangleq \min\{y_{ij} : n_{ij}y_{ij} \geq d_i, y_{ij} \in \mathbb{Z}^+\} = \lceil \frac{d_i}{n_{ij}} \rceil \quad (6)
$$

which gives the number of large items of size j saturated to fulfil the entire demand of small sizes i.

The nice properties of the p-median formulation allow us to refer from now on to this simplified model, that we will use as starting point for the design of our bilevel approach. The model can however be generalized to non-singleton sets by an appropriate change of variables.

4 BILEVEL MODEL

Bilevel programming provides a mathematical model of Stackelberg games. These are strategic games describing the sequential interaction of two players: a leader, or upper-level player, and a follower, or lower-level player. The leader makes decisions assuming that the follower will react in a rationally optimal way. For an introduction to bilevel optimization, see the survey by (Colson et al., 2005).

Our bilevel framework entails two decision makers D_1, D_2 that compete with contrasting goals: D_1 aims at the best usage of material to fulfill demand; D_2, the adversary, tries to impair it as much as possible by distributing f defects in the large items totally produced by D_1, at most one per large item. As said, we illustrate the model assuming that D_1 uses the approximated version of the assortment-and-cut problem described in §3, with variables x_{iP} defined for singletons only.

Let us consider the solutions of (2)-(5) that maintain the total requirement of raw material under a prescribed supply A_F. For any $i \in S, j \in L$, let then $x_{ij} = 1$ if and only if large size j is used to produce the whole demand of item size i. A pair $(i, j) \in S \times L$ for which $x_{ij} = 1$ will be called from now on a *production*.

For sufficiently large A_F, the following system has a solution that represents a set of productions that fulfill the whole demand:

$$
\begin{array}{lrcll}
(D_1) & \sum_{j\in L} x_{ij} & = & 1 & i \in S \quad (7)\\
& \sum_{i\in S}\sum_{j\in L} c_{ij}x_{ij} & \leq & A_F & \quad (8)\\
& x_{ij} & \leq & z_j & i \in S, j \in L\\
& \sum_{j\in L} z_j & \leq & p &\\
& x_{ij},\ z_j & \in & \mathbb{Z}^+ & i \in S, j \in L
\end{array}
$$

where c_{ij} is the optimum value of (1) for $P = \{j\}$. All such solutions have the same total value

$$a_S = \sum_{i\in S} a_i d_i$$

where a_i is the area of (or perhaps the economical value attributed to) each item of size $i \in S$.

The decision of D_2 is encoded by variables $u_{ij} \in \mathbb{N}$ that indicate the number of large items j that contain a defect and are cut to obtain item size i. Let $\tilde{x}$ be a particular feasible solution to (D_1) and π_{ij} denote the probability of existence of a faulty item of size i from a single large item of size j (this probability will be computed in §5). With these positions, the largest return of the adversarial choice is obtained by solving the following ILP:

$$
\begin{array}{lrcl}
(D_2) & \max_z \lambda(\tilde{x}, u) & = & \sum_{i\in S} a_i \sum_{j\in L} \pi_{ij} u_{ij} \quad (9)\\
& & & \sum_{i\in S}\sum_{j\in L} u_{ij} \leq f\\
& & & u_{ij} \leq \bar{y}_{ij}\tilde{x}_{ij} \quad i \in S, j \in L\\
& & & u_{ij} \in \mathbb{Z}^+ \quad i \in S, j \in L.
\end{array}
$$

with the $\bar{y}_{ij}$ given by (6). The objective function is the expected loss of D_1, that is the expected value of small items lost when at most f large items are hit by a defect. The total amount of defects distributed by D_2 is bounded by the first inequality, while the subsequent bounds ensure that no more large items than those used in production j can be affected by a fault.

Problem (D_2) is a continuous knapsack with bounded variables, therefore an optimum to (9) can be found in $O(n \log(n))$ time by initially ranking productions (i, j) in non-increasing order of weighted losses $a_i\pi_{ij}$, then sequentially saturating bounds by setting

$$u_{ij} = \bar{y}_{ij}\tilde{x}_{ij} \quad \text{while} \quad \sum_{i\in S}\sum_{j\in L} u_{ij} \leq f$$

and placing the unsaturated difference to f defects (if any) on the production with the largest of the remaining loss.

More interestingly, this helps us build a compact formulation of the bilevel problem

$$\max_{x\in D_1}\{a_S - \max_{x\in D_1, z\in D_2} \lambda(x, u)\} \quad (10)$$

By the above argumentation, for $i, h \in S$ and $j, k \in L$ such that $(i, j) \prec (h, k)$, $u_{hk} > 0$ implies $u_{ij} = \tilde{x}_{ij}$ whereas $u_{hk} = 0$ implies $u_{ij} \leq \tilde{x}_{ij}$. We can write those conditions by suitable linear inequalities, so formulating the bilevel problem (10) as a one-level 0-1 LP:

$$(BP) \qquad \min \sum_{i\in S} a_i \sum_{j\in L} \bar{y}_{ij}\pi_{ij}\eta_{ij} \quad (11)$$

subject to D_1 and:

$$
\begin{array}{rcll}
x_{ij} + \eta_{hk} - \eta_{ij} & \leq & 1 & (i, j) \prec (h, k) \quad (12)\\
\eta_{ij} - x_{ij} & \leq & 0 & i \in S, j \in L \quad (13)\\
\sum_{i\in S}\sum_{j\in L} \bar{y}_{ij}\eta_{ij} & \geq & f & \quad (14)\\
\eta_{ij} & \in & \{0, 1\} & i \in S, j \in L
\end{array}
$$

where $\eta_{ij} = 1$ if and only if production (i, j) is faulty.

The objective function (11) is the same as (9) and represents the largest loss D_2 can impose to any feasible choice of D_1. Inequalities (12) impose the discussed ranking condition: assuming $(i, j) \prec (h, k)$, if the (i, j)-th production is chosen and the (h, k)-th production is affected by defects, then the (i, j)-th production must have faults. Inequalities (13) state on one hand that a production not chosen cannot of course be faulty; on the other hand, if it is faulty, then it must be chosen and the number of its faults must be $\bar{y}_{ij}$. Finally, inequality (14) enforces D_2 to insert at least f defects: minimization will then reduce faults to the smallest possible amount.

An optimal solution to model (BP) may not be *really* optimal: the problem arises when (14) is fulfilled with the sign $>$. To cope with this inconvenience, it is necessary to allow the last nonzero variable η_{ij} in the ranking to get values between 0 and 1. However, we do not know in advance which variable will be the last in the ranking induced by an optimal solution, so we have to identify it using the differences between consecutive η_{ij}. We then introduce real variables θ_{ij} that optimization will set all to 0 but one, fixed to the surplus value $\sum_{h,k} \bar{y}_{hk}\eta_{hk} - f$. The new variables obey

$$\begin{aligned} 0 \le \theta_{ij} &\le \bar{y}_{ij}\eta_{ij} && i \in S, j \in L \\ \theta_{ij} &\le \bar{y}_{ij}(1-\eta_{hk}) && (i,j) \prec (h,k) \\ \theta_{ij} &\le \sum_{h\in S}\sum_{k\in L} \bar{y}_{hk}\eta_{hk} - f && i \in S, j \in L \end{aligned}$$

The former constraints allow just one nonzero, precisely the θ_{ij} associated with the last $\eta_{ij} > 0$ in the ranking; the latter constraint allows it to get up to the correct surplus. Optimization will then set it to the exact surplus as soon as we subtract the term

$$g = \sum_{i\in S} a_i \sum_{j\in L} \pi_{ij}\theta_{ij}$$

from the objective function.

Passing to variables x_{iP}, one can rewrite (D_1) as in (2), maintaining the objective function (11) and constraints (12), (13) in the x_{ij} variables, and adding the inequalities

$$x_{iP} - x_{ij} \quad \le \quad 0 \qquad i \in S, j \in P \qquad (15)$$

In this case, for $j \in P$ the $\bar{y}_{ij}$ form an optimal solution of (1) — note that if $\bar{y}_{ij} = 0$ then j can be removed from P, thus implications (15) hold only for active productions.

Note also that the number of constraints (15) grows linearly with the cardinality of S and P, but recall that in principle the x_{iP} variables are exponentially many. A formulation that avoids such a large number of constraints (and the consequent row generation) can however be devised rewriting inequalities (12), (13) as

$$\begin{aligned} x_{iP} + \eta_{hk} - \eta_{ij} &\le 1 \quad j \in P, (i,j) \prec (h,k) \quad (16) \\ \eta_{ij} - \sum_{P \ni j} x_{ij} &\le 0 \quad i \in S, j \in P \end{aligned}$$

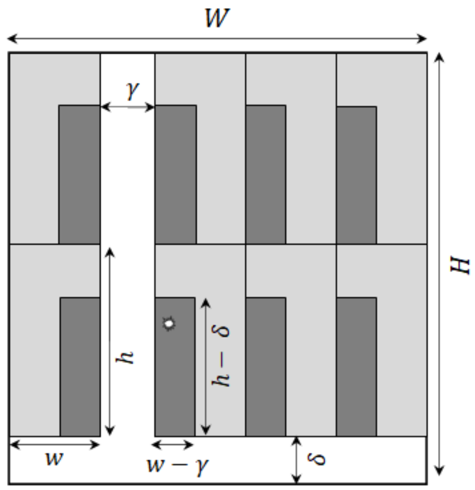

Figure 1: A defect falling in a vertical or horizontal strip (white) can be recovered; a defect in the critical area (dark grey rectangles) causes the loss of one item.

5 FAULT MODEL

The cost coefficients c_{ij} in (7), that one computes by (1), are uncertain due to defects that may alter the value of parameters n_{ij}. Suppose those defects point-shaped, statistically independent and uniformly distributed in A_F (which, we recall, is an amount of float material sufficient to meet the whole small item requirement: a lower estimate of A_F is $\sum_S a_i d_i$, a more refined evaluation can be found solving problem (2)).

Given an evaluation of A_F and an upper bound f to the total number of faults that can affect the entire float campaign, the probability that the j-th size of L exhibits a defect is

$$\varphi_j = f\left(\frac{A_j}{A_F}\right)\left(1 - \frac{A_j}{A_F}\right)^{f-1}.$$

However, the yield reduction consequent to faults is also dependent on both *where* the defect falls in the large item and *which* small item is cut from that item. We consider the most general case in which pattern layouts can be freely reconfigured, also by splitting waste strips with additional cuts, and will call *critical* a defect that causes a small item loss for any admissible reconfiguration of the pattern layout. For any production (i,j), κ_{ij} is then the probability that a single large item of size j used for manufacturing small items of size i has a critical defect. Clearly, $\kappa_{ij} = 0$ if the pattern is non-maximal. To illustrate its computation, we refer to Figure (1) and temporarily drop indices $i \in S$ and $j \in L$.

Let us assume a large item of unitary area $A = 1$ from which small items are cut. Let moreover $\alpha =$

$\lfloor \frac{W}{w} \rfloor$ and $\beta = \lfloor \frac{H}{h} \rfloor$ respectively count the columns and rows in which small items are aligned in the pattern.

Packing all items according to a top-left strategy leaves two overlapping strips: one horizontal, of height $\delta = H - \beta h$ and area δW, at the bottom of the large item; and one vertical, of width $\gamma = W - \alpha w$ and area γH, to its extreme right. Similarly, strips of the same size can be obtained by aligning part of the α small item columns (part of the β rows) to the extreme right (respectively, the bottom) of the large item. Any such re-alignment is a pattern reconfiguration that avoids a defect falling in the strips, so any such defect is non-critical.

Instead, there exists a *critical area* complementary to that covered by the above strips such that a defect falling in it will surely compromise an item, whatever the pattern layout. This critical area consists of the black identical rectangles displayed in Figure (1), which are $\alpha\beta$ and have width $w - \gamma$ and height $h - \delta$. Hence, by construction

$$\kappa = \alpha\beta(w - \gamma)(h - \delta)$$

For non-unitary large items the above formula is immediately adapted by normalizing w, γ with respect to W, and h, δ with respect to H.

Finally, the probability π_{ij} that a large item in production (i, j) has a critical defect is given by the compound probability $\varphi_j \kappa_{ij}$.

6 NUMERICAL TESTS

We did some numerical tests using an Intel® Core™ i7-7500U (2 cores) 2.90 GHz with 16Gb RAM. Mathematical formulations were implemented with AMPL and solved by IBM® CPLEX® 12.9.0.0 with default setting and a time limit of 600 seconds.

Experiments aim at assessing usability and benefits of formulation (BP) as a tool to evaluate the worst-case yield loss induced by material defectiveness. We discuss a comparison with the solutions obtained via the p-median approximation of (2), with worst-case loss measured by running the adversarial model D_2 with $\tilde{x}$ taken from the computed solutions.

Tests were made on a set of 200 randomly generated instances of limited dimensions, divided into 8 groups of 25 instances according to combinations of the number of small sizes $|S| \in \{5, 6, 7, 8\}$ and the assortment level $p \in \{|S| - 1, |S| - 2\}$. Compliantly with the production features described in (Arbib and Marinelli, 2007), we assumed large sizes (in meters) $W_j \in [2.80, 3.21]$ and $H_j \in [4.50, 6.10]$ for $j \in L$, small sizes $i \in S$ randomly generated as $w_i \in [0.37, 1.56]$ and $h_i \in [0.56, 2.11]$, with requests $d_i \in [1000, 50000]$. Critical defect probabilities were computed as in §5. To limit the size of formulations, we filtered L by the pre-processing phase devised in (Arbib and Marinelli, 2007). Finally, the number of defects is assumed proportional to the total glass volume, that is $f = \varepsilon A_F$. In our tests, we assumed a per-square-meter defect rate $\varepsilon = 1.25\%$.

Table 1 gives details for each instance group $G_1, \ldots, G_8$, and reports the number of small sizes, the assortment level, and (averaged among instances) the number of large sizes, the total demand of small items and the number of defects.

Table 1: Eight groups of random instances with realistic features.

Group	$\|S\|$	p	$\|L\|$	mean $\sum d_i$	f
G_1	5	3	17.1	130813.8	2065.7
G_2	5	4	16.6	124040.4	2034.4
G_3	6	4	21.4	167948.2	2739.6
G_4	6	5	24.0	156715.8	2471.2
G_5	7	5	24.2	177740.0	3013.0
G_6	7	6	28.9	174806.2	2933.8
G_7	8	6	33.3	195385.1	3266.8
G_8	8	7	31.7	194955.6	3074.7

Model (BP) was solved by altering the right-hand side of (8) by different amounts. In particular, we set it to $A_F(1 + \omega)$, where A_F is the optimum found via the p-median approximation of (2) and ω, varying in $\{0\%, 2.5\%, 5.0\%, 7.5\%, 10.0\%\}$, is a percentage increase of material supply to increase, in turn, solution robustness. We highlight that, in our testbed, solutions of (2) always coincide with those of (BP) for $\omega = 0$: this can be ascribed to the lack of equivalent optimal solutions, induced in turn by the relatively small $|L|$ resulting from the pre-processing phase. Thus, in the following discussion we refer to this case to indicate solutions of p-median formulation.

Tables 2 summarizes results for the different ω. For each group of instances, we report the average expected loss (Λ) for $\omega = 0$ and the percentage mean reduction of loss ($\tilde{\Lambda}_{\%}$) in the other cases, the percentage mean surplus of raw material cut ($\tilde{S}_{\%}$), the mean CPU time in seconds (T) and the number of instances for which CPLEX reached the time limit (#lim). In particular, $\tilde{\Lambda}_{\%}$ and $\tilde{S}_{\%}$ are defined as

$$\tilde{\Lambda}_{\%} = 100 \cdot \frac{\Lambda_0 - \tilde{\Lambda}}{\Lambda_0} \qquad \tilde{S}_{\%} = 100 \cdot \frac{\tilde{A}_F - A_F}{A_F},$$

averaged on each instance group, where Λ_0 is the expected yield loss when $\omega = 0$, $\tilde{\Lambda}$ and $\tilde{A}_F$ are respectively the expected yield loss and the material employed in the production plan given by the solution of (BP). As $\tilde{A}_F = A_F$ for $\omega = 0$, we neglect the corresponding column in Table 2. The 'Overall' row shows the aggregate values on the average across instance groups, except for column '#lim' that counts the total number of time limits occurred.

Let us first comment $\omega = 2.5\%$. With respect to $\omega = 0$, the expected yield loss has a consistent decrease: $\tilde{\Lambda}_{\%} = 19.7\%$, ranging from 13.4% to 23.0%. This reduction is obtained by paying an average $+2.1\%$ raw material to meet item demand, a value that slightly increases with the amounts of small sizes to cut. For $\omega = 5.0\%$, increasing raw material by 4.4% brings a mean yield loss reduction of 31.3%, from a minimum 21.5% to a maximum 35.8%. The trend continues up to $\omega = 10\%$, halving yield loss on average (from 40.3% to 54.7%) while $\tilde{S}_{\%} = 9.4\%$.

These result shows that, in the worst-case perspective described and as far as decision makers consider the use of additional material, the solutions of (BP) provide the fabrication process with increasing resilience to defects. Indeed, with a mere count of raw material, the reduction of defective parts does not compensate the float glass increment: with $\omega = 2.5\%$, in front of an average $+2.1\%$ raw glass usage, only 0.4% is recovered in terms of faultless semi-finite glass, so leaving a 1.7% material surplus unsettled. Similarly, $\omega = 5\%, 7.5\%, 10\%$ respectively give $+3.7\%, +6.0\%, +8.4\%$ net raw glass usage. This performance is not surprising, and can generically be attributed to the discrepancy between a continuous parameter (the amount of raw material supplied) and discrete actions (sheet cuts). Instead, one must observe that raw material (siliceous sand, glass cullet, additives) is only one component of semi-finite cost (which includes among others energy and workforce), and an algebraic sum of volumes is thus unfair. In fact, the reduction of product defectiveness is definitely worth the cost of the additional material, not only considering the added value of semi-finite items, but also looking at the whole manufacturing process, which is improved by (i) less item inspection and handling, and (ii) downstream operation schedules less vulnerable to delays.

On the computational side, finding optimal solutions through (BP) requires more CPU time as the size of $|S|$ grows and constraint (8) is relaxed, taking from 2.4 seconds to 123.2 seconds on average (last row of the tables). In detail, T evidently increases on instance groups G_5-G_8 in all cases, that is, those with the largest amount of small sizes. The time limit is sporadically reached up to $\omega = 2.5\%$, and occurs 16 times for $\omega = 10\%$, that is, only in 8% of the instances. In all the 47 cases where an optimum could not be certified, we observed very large gaps: they were under 30% in 4 cases only (with a minimum of 9.7%) and above 90% in 38 cases. Tests highlighted that CPLEX struggled to tighten the dual bound and in 21 instances computation halted with a lower bound equal to zero. Nevertheless, good primal bounds were found in relatively few iterations and usually corresponded to actual optima, when certified. All in all, though the time response is not a critical aspect (being a mid-term planning problem), the picture so obtained motivates us to investigate valid inequalities to strengthen model (BP) or to identify combinatorial dual bounds.

7 CONCLUSIONS AND FUTURE RESEARCH

In this paper we discussed an assortment-and-cut problem derived from a glass manufacturing process where raw material is affected by imperfections that can compromise the efficiency of the production. Defects are modeled as statistically independent points uniformly-distributed on material, and we limit their occurrence to at most one defect per large glass sheet. Due to the simple form of cutting patterns, a critical area can be identified, where a defect causes a small item loss whatever the pattern layout. Based on this observation, the probability that a defect induces a yield loss is easily computed.

Following a worst-case perspective, we developed an original bilevel approach where two decision makers operate with contrasting goals: while the leader optimizes assortment and cuts to fulfill as much demand as possible with a given amount of material, the follower tries to impair it by distributing a given amount of defects in a way that maximizes faulty parts. We then rearranged the bilevel program into an equivalent one-level 0-1 LP, whose optimal solutions give an expectation of the yield reduction and a measure of robustness against defectiveness.

We tested the formulation on a set of random instances, limited in size but generated with a parameter setting that reflects real-world production. When fed with the minimum amount of raw material required to fulfill demand in absence of defects, our model returns a worst-case estimation of losses identical to that achieved with defect-free optimal solutions obtained as in (Arbib and Marinelli, 2007). Employing some amount of extra glass leads to solution that are much more robust against defect occurrence: for instance, just supplying the system with 2.5% extra glass, one can reduce mean yield loss by about 20%.

Although computational evidence on the largest

Table 2: Results of (BP) for diffent values of ω.

	$\omega = 0\%$			$\omega = 2.5\%$				$\omega = 5\%$				$\omega = 7.5\%$				$\omega = 10\%$			
Group	Λ	T	#lim	$\tilde{\Lambda}_{\%}$	$\tilde{S}_{\%}$	T	#lim	$\tilde{\Lambda}_{\%}$	$\tilde{S}_{\%}$	T	#lim	$\tilde{\Lambda}_{\%}$	$\tilde{S}_{\%}$	T	#lim	$\tilde{\Lambda}_{\%}$	$\tilde{S}_{\%}$	T	#lim
G_1	3798.3	0.4	0	13.4	1.7	1.6	0	21.5	3.7	2.6	0	33.6	6.3	3.3	0	40.3	8.6	3.8	0
G_2	3656.6	0.4	0	20.3	1.7	4.3	0	28.4	3.8	8.4	0	43.0	6.6	13.5	0	49.5	9.4	9.4	0
G_3	4906.6	0.9	0	17.4	2.1	7.0	0	29.0	4.4	13.6	0	39.0	6.7	26.5	0	48.3	8.8	24.2	0
G_4	4156.2	1.3	0	19.1	2.0	16.6	0	35.8	4.5	31.6	0	45.3	7.0	39.0	0	54.7	9.6	60.2	0
G_5	5646.4	1.9	0	23.0	2.3	42.8	0	32.3	4.4	96.2	2	42.1	6.9	87.1	2	50.5	9.6	119.8	2
G_6	6147.4	2.5	0	20.1	2.2	47.5	0	33.1	4.7	126.3	2	44.5	7.3	160.4	2	52.0	9.7	191.8	2
G_7	6420.0	6.1	0	22.0	2.2	97.7	1	35.3	4.6	214.6	5	43.9	7.0	259.9	6	53.1	9.9	296.3	7
G_8	6277.9	5.7	0	22.7	2.3	133.4	2	34.7	4.7	158.2	3	45.3	7.2	256.0	6	54.7	9.7	281.1	5
Overall	**5126.2**	**2.4**	**0**	**19.7**	**2.1**	**43.9**	**3**	**31.3**	**4.4**	**81.4**	**12**	**42.1**	**6.9**	**105.7**	**16**	**50.4**	**9.4**	**123.2**	**16**

instances suggests that our model hardly provides valuable dual bounds, the model appears suitable to get good primal solutions in reasonable time. This encourages to explore the possibility of strengthening the model, e.g. by valid inequalities.

Further investigation is required to observe the model response to noise-contaminated or partially missing inputs (e.g., by simulation), as well as to generalize the assumptions made and devise a bilevel approach suitable for processes with (i) more than one fault per large item; (ii) more general cutting patterns, as in the standard cutting stock or two-dimensional knapsack problems.

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