Using the Gran1 Program in Mathematics Lessons

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Abstract: The article considers the mathematical software package Gran. There are described examples of solving problems by the graphical method using the Gran1 program, which can be used in the process of teaching mathematics at school. The issues under consideration are quite complex, and their solving without software such as Gran1 for graphical analysis of various issues is quite time-consuming. The range of tasks that can be solved using the Gran software package, in particular, the Gran1 program, is quite wide and needs a creative approach. Their analysis and solution can have a positive effect not only on the mental but also on the general cultural development of students. The purpose of the study is to consider examples of the effective use of the GRAN complex in mathematics lessons for solving issues of different levels of complexity.

1 INTRODUCTION

On February 26, 2021, Ukrainian science suffered a heavy loss. A well-known scientist in Ukraine and the world, the founder of a powerful scientific school, Myroslav Ivanovych Zhaldak, Doctor of Pedagogical Sciences, professor, academician of the National Academy of Pedagogical Sciences of Ukraine, passed away. One of the most significant contributions of Myroslav Ivanovich to science and education is the Gran software complex conceived by him and developed under his leadership. The Gran program complex has received an author's certificate, it is recommended by the Ministry of Education and Science of Ukraine for use in secondary and higher education institutions of Ukraine, and is also known abroad, for example in Poland.

Myroslav Ivanovych Zhaldak began active scientific and pedagogical activities at a time when computerized means of searching, collecting, storing, processing, presenting, and transmitting various data began to be introduced into the educational process. He emphasized that this opens broad perspectives for humanitarian education and human learning, contributes to the deepening and expansion of the theoretical foundations of knowledge, gives practical significance to learning results, creates conditions for revealing the creative potential of children, taking into account their age characteristics and experience, individual requests, and abilities. At the same time, the teacher is not forced to use any specific method of presentation, consolidation of knowledge and control, any specific content, methods, forms of organization and teaching tools, to maintain a certain balance between independent preparation of students and group work, etc. (Zhaldak, 1989).

Teacher should determine all these aspects according to his own preferences, with specific conditions of work, with individual features of pupils and of whole class. It's clear that it's impossible and unnecessary to teach all the children equally, to form equal knowledge in various subjects for all the children, to claim reaching equal level of logical and creative thinking development, equal perception of reality. It also relates to teaching mathematics, methods of problems solving, plotting and analysis of mathematical models for various processes and phenomena, results in-

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terpretation, and generalization of results of this analysis. At the same time there arise a lot of difficulties dealing with the content, methods, organization forms and means of study, necessary knowledge levels in various subjects for every pupil (Bobyliev and Vihrova, 2021).

Of particular importance is the teaching of natural sciences and mathematics, during which students must consider and build models of various processes and phenomena, and then explore them, analyzing their various features and characteristics, possibly using different information and communication models to perform calculations or experiments, and based on the results of such analysis, synthesizing the relevant conclusions. This approach to learning allows students to effectively develop logical, critical, creative thinking, scientific worldview, creative approach to solving various problems, their correct vision and ability to explain their nature and essence (Zhaldak et al., 2021).

The specified educational aspects can be effectively implemented using the pedagogical software complex GRAN, created under the creative leadership of Myroslav Ivanovich Zhaldak.

We consider the advantages of the specified software complex to be:

- convenient user interface, in particular, the possibility of using most services without a keyboard;
- availability of English and Polish localizations;
- methodical support developed over several decades, including teaching aids recommended by the Ministry of Education and school textbooks;
- confirmation of the effectiveness of the program by the results of numerous candidate and doctoral dissertations on the methodology of teaching mathematics and computer science.

2 THEORETICAL BACKGROUND

For the study, scientific publications and supporting sources in the following areas were analyzed: theory and practice in mathematics teaching development (Jaworski, 2006), development of mathematical and logical thinking (Astafieva et al., 2019), mathematical speech of students (Semenikhina and Drushlyak, 2015), using the Gran software package in mathematics lessons (Zhaldak et al., 2012, 2020) and other.

Since 2003, the Gran software complex has been used in some educational institutions in Poland (Smirnova-Trybulska, 2003; Smyrnova-Trybulska, 2004).

The widespread introduction of modern ICT tools in the educational process makes it possible to significantly strengthen the links between the content of education and everyday life, to give the results of training practical significance, applicability to solving everyday life problems, satisfying practical needs, which is one of the aspects of the humanization of education.

At the same time, the informatization of the educational process should be based on the creation and widespread introduction into everyday pedagogical practice of new computer-oriented methodological teaching systems for all academic disciplines without exception on the principles of gradual and non-antagonistic, without destructive restructurings and reforms, embedding information and communication technologies into existing didactic systems, a harmonious combination of traditional and computer-oriented learning technologies, not denying and discarding the achievements of pedagogical science of the past, but, on the contrary, their improvement and strengthening, including through the pedagogically balanced and expedient use of achievements in the development of computer technology and communications.

However, it is important to understand that not all tasks require the use of a computer. Scientific analysis of creative, productive thinking shows that the main thing in the process of thinking is not so much operational and technical procedures and programs for solving already set tasks, but rather building a model of a problem situation, putting forward a hypothesis, guessing, formulating a problem, setting a task. The modern development of computer software has reached a level where in many cases the algorithm for achieving the goal can be built automatically. In this case, instructions to the computer can be given in terms of the desired results, and not in descriptions of the processes leading to such results. The main difficulty is to characterize the desired results in a qualified and accurate manner, which puts forward appropriate requirements for the overall rigor and logic of the user's thinking.

3 RESULTS

The latest version of the software Gran1, Gran2D, Gran3D, as well as some training manuals can be obtained on the website: https://ktoi.npu.edu.ua. All materials posted on this site are distributed free of charge (Gran, 2021).

Let's consider some examples of solving some math problems using the cloud version of Gran1.

Note that the names of services, help, tips, messages, etc., depending on the settings provided in the program, can be provided in one of four languages: Ukrainian, Russian, English, Polish.

Suppose it is necessary to solve an equation $f(x)=0$, i.e., in domain of dependence $y=f(x)$ find all the values of the argument x that their corresponding values $f(x)$ are equal to zero.

When the dependence $y=f(x)$ is represented graphically, to find a solution of the equation $f(x)=0$ means to find all the points on the graph of dependence $y=f(x)$ that have zero ordinates. In other words, it is necessary to find points that lie both on the graph of dependence $y=f(x)$ and on the axis Ox that is described by the equation $y=0$. That is, one should find points that lie on the line (straight or curve) that has equation $y=f(x)$ as well as on the line, that has equation $y=0$.

Plotting graph of the dependence $y=f(x)$ with the help of the command "Graph /Plot" and setting cursor in corresponding points for getting their ordinates makes it easy to determine abscises of all the points on the graph of dependence $y=f(x)$ that also lie on the axis Ox.

Examples

1. Find solutions of the equation $x^2-2=0$.

Plot a graph of the dependence $y=x^2-2$ and set cursor so that the cursor's abscissa coincides with the intersection point of the graph and the axis Ox. The result is as follows $x_1 \approx -1.4$, $x_1 \approx 1.4$ (figure 1).

If it is necessary to precise the roots one can enlarge a part of the graph or change the segment of function determination and plot the graph in quite small areas of the points defined before, with the help of enlarged zoom.

2. Find solutions of the equation

$$|x-1|+|x+1|-3=0.$$

Plot a graph of the dependence

$$y=abs(x-1)+abs(x+1)-3$$

and make sure that any point on the axis Ox of the segment $[-1,1]$ lies on the graph of a considered dependence (figure 2). Thus, for the equation exists unlimited set of solutions and any value $x \in [-1,1]$ is a solution of the equation.

3. Find solutions of the equation

$$\sin x+2-\ln x=0.$$

Plot a graph of the dependence

$$y=\sin(x)+2-\ln(x)$$

on the segment $[-1,40]$ (figure 3) and make sure (considering properties of functions $\sin x$ and $\ln x$), that out of the segment $[-1,40]$ there aren't roots of the equation.

While considering the graph of dependence

$$y=\sin(x)+2-\ln(x),$$

represented in the figure 3, one can suppose that the equation $\sin(x)+2-\ln(x)=0$ has 6 solutions:

$$x_1 \approx 3.9;\, x_2 \approx 6.1;\, x_3 \approx 9.2;$$

$$x_4 \approx 13.2;\, x_5 \approx 14.9;\, x_6 \approx 20.25.$$

If high accuracy of calculation is not necessary, such conclusion can be accepted.

However, if higher accuracy of results is required one should enlarge the zoom of plotting in quite small areas of the points $x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6$ (figure 4, figure 5) to sure that the equation has 5 solutions:

$$x_1=3.851,\, x_2=6.088,\, x_3=9.203,$$

$$x_4=13.184,\, x_5=14.928$$

It should be noted that precise analytical solution of the equation cannot be found, while the search of its approximate solutions without graphical plotting requires laborious calculations and careful analysis of the results.

The calculus mathematics investigates special methods of search approximate solutions of equations of the form $f(x)=0$ on given segment $[a,b]$ (bisection method, chord method, tangent method, iteration method etc.).

Sometimes it is convenient to represent the equation $f(x)=0$ in the following form:

$$f_1(x)-f_2(x)=0$$

where $f_1(x)-f_2(x)=f(x)$, or a problem leads to searching solutions of the equation of the form

$$f_1(x)=f_2(x).$$

In this case it is convenient to plot graphs of the dependencies $y=f_1(x)$ and $y=f_2(x)$, then set cursor in intersection points of the graphs and determine coordinates of the points lying on both graphs simultaneously. Abscissas x of the points are solutions of the equation $f_1(x)=f_2(x)$. If the values x are found in a such way, the values $f_1(x)$ and $f_2(x)$ are equal.

4. Find solutions of the equation:

$$\sqrt[3]{x}+\frac{1}{8}\sin(10x)=\log_{\frac{1}{2}}(x+3.5).$$

Plot graphs of the dependencies

$$y=\sqrt[3]{x}+\frac{1}{8}\sin(10x)$$

and $y=\log_{0.5}(x+3.5)$ and make sure that the equation has unique solution. Set cursor in the intersection point of the graphs to get $x \approx -1.3$ (figure 6).

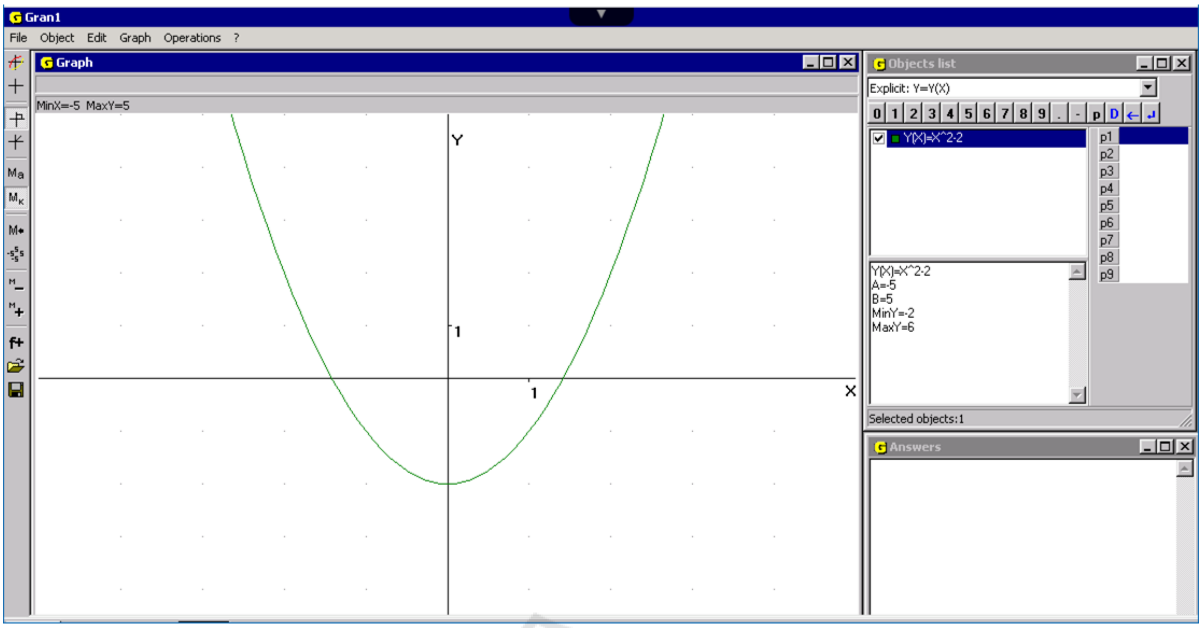

Figure 1: Graph of a given function $y = x^2 - 2$.

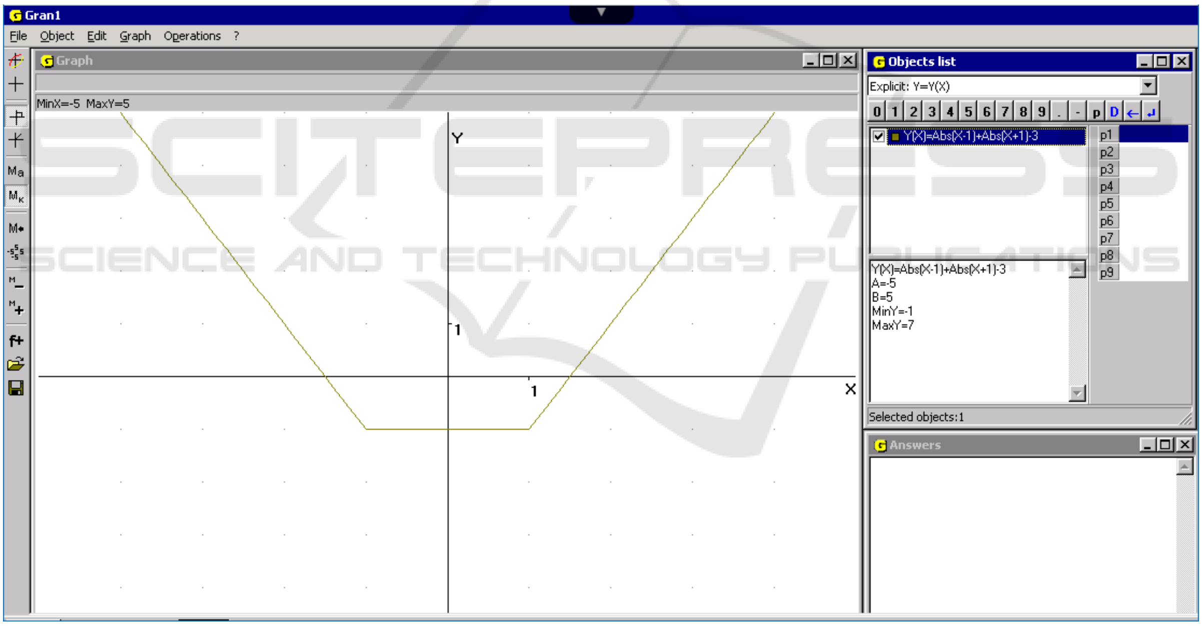

Figure 2: Graph of a given function $|x-1| + |x+1| - 3 = 0$.

Now solve the system of equations of the form

$$\begin{cases} G_1(x,y) = 0, \\ G_2(x,y) = 0, \end{cases}$$

where $G_1(x,y)$ and $G_2(x,y)$ are some expressions of two variables x and y.

Set the type of dependence $G_1(x,y) = 0$ and plot graphs of the dependencies $G_1(x,y) = 0$ and $G_2(x,y) = 0$, then set cursor in intersection points of the graphs and determine coordinates of the points that meet both equations

$$G_1(x,y) = 0 \text{ and } G_2(x,y) = 0$$

i.e. coordinates of intersection points of the lines described by the equations

$$G_1(x,y) = 0 \text{ and } G_2(x,y) = 0.$$

5. Solve the system of equations $\begin{cases} x^2 + y^2 = 16, \\ \lg(xy) = 0.1 \end{cases}$

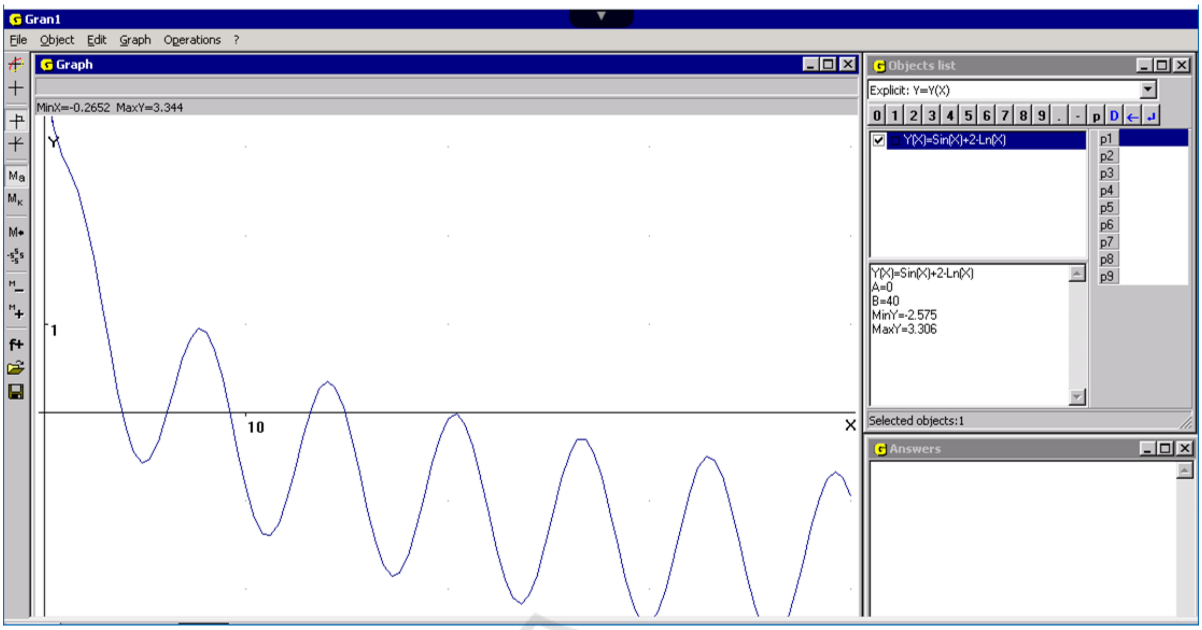

Figure 3: Graph of the dependence $y = sin(x) + 2 - ln(x)$.

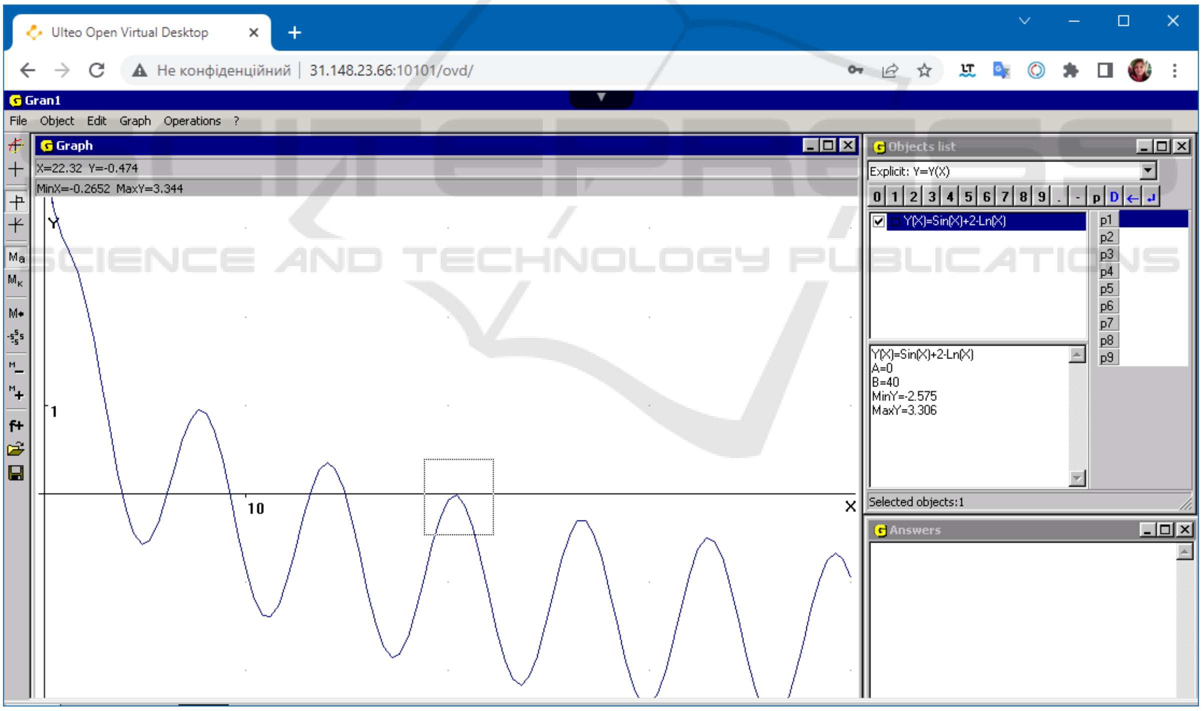

Figure 4: Increasing the scale of graphic constructions in rather small points.

Represent the equations in the following form: $0 = x^2 + y^2 - 16$, $0 = \lg(xy) - 0.1$ and plot graphs of the dependencies (figure 7).

Set cursor in each of intersection points of the graphs and obtain:

1. $x \approx -3.99$, $y \approx -0.31$;
2. $x \approx -0.31$, $y \approx 3.99$;
3. $x \approx 0.31$, $y \approx 3.99$;
4. $x \approx 3.99$, $y \approx 0.31$.

For more precise determination of coordinates of the intersection points of graphs one should enlarge the zoom of plotting, i.e., use the command

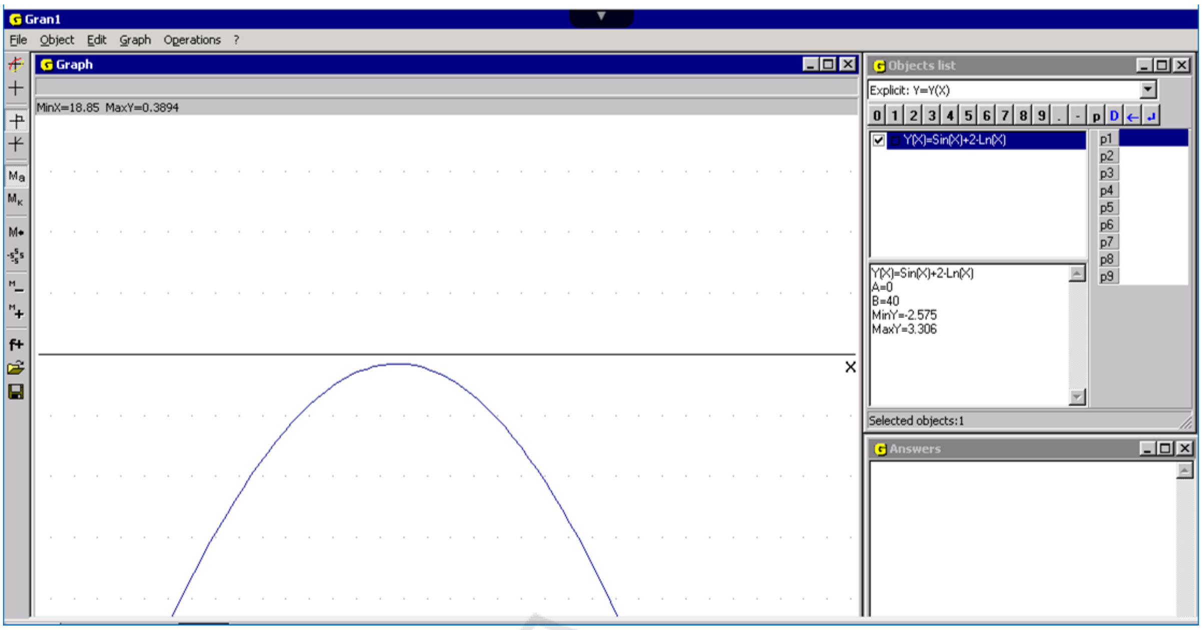

Figure 5: Enlarged scale of graphic construction $y = sin(x) + 2 - ln(x)$.

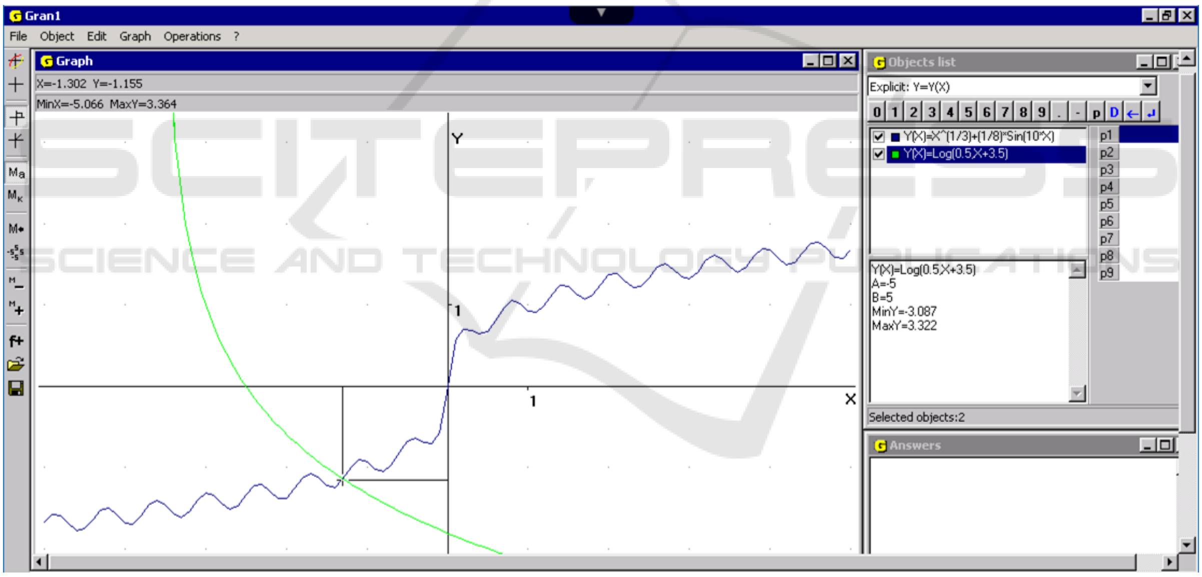

Figure 6: The point of intersection of graphs of functions.

„Zoom in” or change bounds for the variables x and y. For example, if we change the zoom by setting the bounds $MinX = -4.5$, $MaxX = -3.5$, $MinY = -0.5$, $MaxY = 0.1$ and plot the corresponding graphs, we obtain the image represented in the figure 8. Use the coordinate cursor to get $x \approx -3.988$, $y \approx -0.316$, and while cursor is moving, the third digit after the comma is changing (is defined more exactly).

One can put $MinX = -4.0$, $MaxX = -3.98$, $MinY = -0.32$, $MaxY = -0.3$ (using the command “Graph / Zoom / User zoom”) to obtain $x \approx -3.9875$, $y \approx -0.3157$, and while the cursor is moving, the fourth digit after comma is changing (is defined more exactly).

It should be noted that the problem of finding solutions of the equation $f(x) = 0$ can be also considered as the problem of solving the system of equations

$$\begin{cases} 0 = y - f(x), \\ 0 = y, \end{cases}$$

and the problem of finding solutions of the equation $f_1(x) = f_2(x)$ as the problem of solving the system of

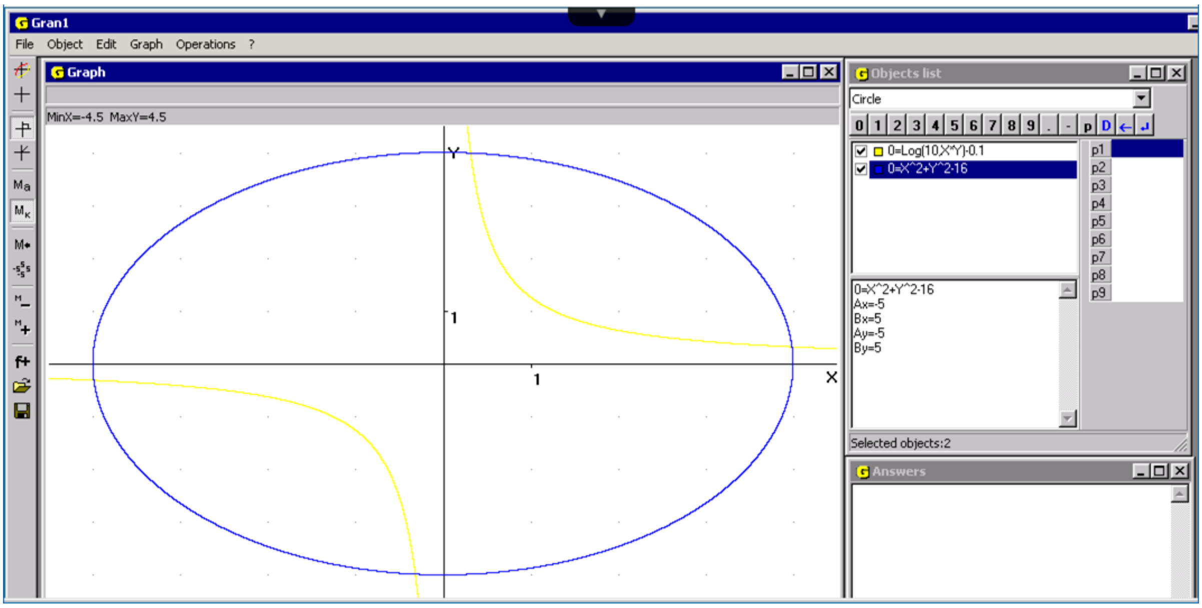

Figure 7: Two equations are graphically represented.

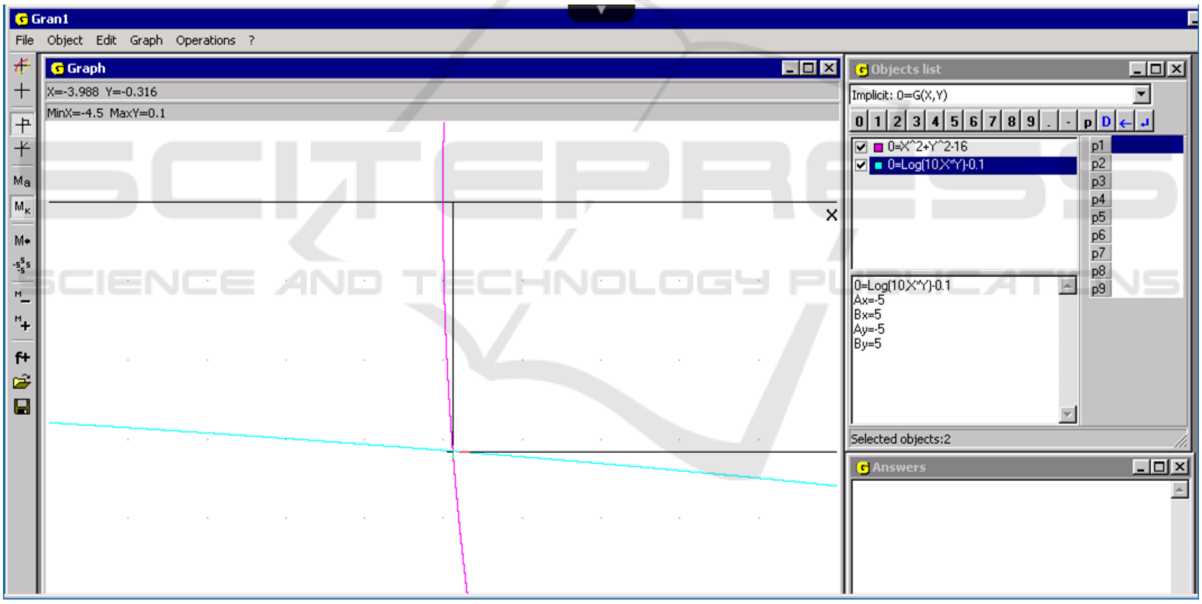

Figure 8: Graphically, two equations are presented on a modified scale.

equations

$$\begin{cases} 0 = y - f_1(x), \\ 0 = -f_2(x). \end{cases}$$

In many cases finding solutions of the system of equations $\{G_1(x,y) = 0,\ G_2(x,y) = 0\}$ with the help of plotting is unique suitable method for practical use since the method of variable exclusion or other methods are very difficult or lead to wrong results.

6. Solve the system of equations (figure 9)

$$\begin{cases} 0 = \sin(xy) + \cos(x - y), \\ 0 = \frac{x}{y} - \lg(x + y). \end{cases}$$

In this case it is impossible to exclude one of the variables x or y and it is difficult to offer any practically suitable way of solution besides the graphical method.

It is obvious that plotting can be used for determination of intersection points of lines independently of types of the dependencies. For example, if it is

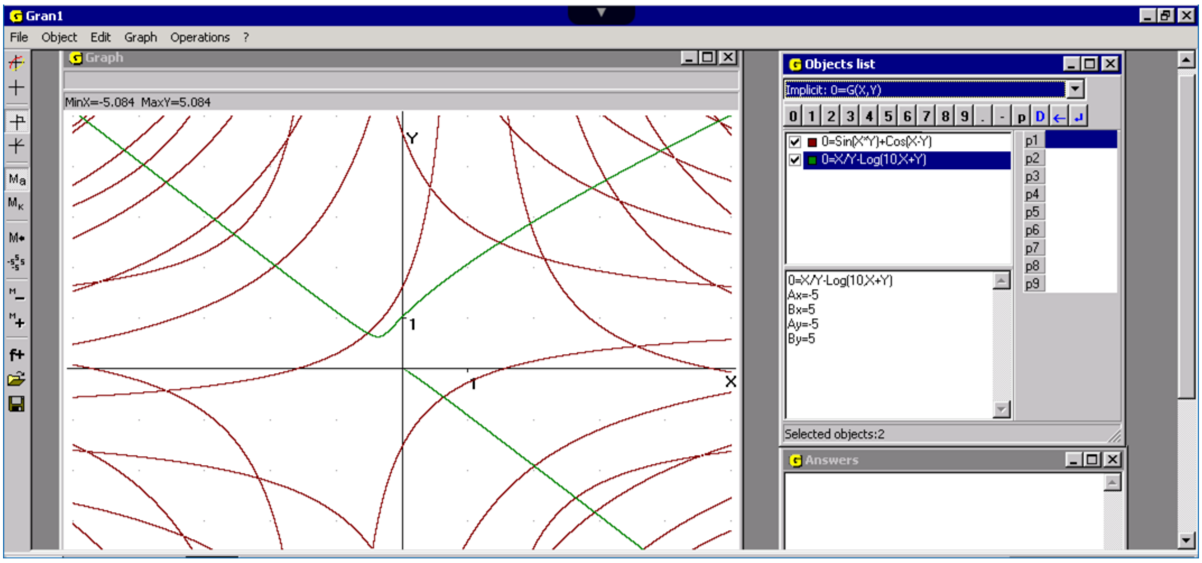

Figure 9: The system of equations $\begin{cases} 0 = \sin(xy) + \cos(x-y), \\ 0 = \frac{x}{y} - \lg(x+y). \end{cases}$ is presented graphically.

required to determine coordinates of points of the circle $x^2 + y^2 = 9$, that lie on the parabola $y = \frac{x^2}{7} - 2$ or on the five-petalled (pentapetalous) rose (5φ) (figure 10), one should plot the graphs and obtain coordinates of the required points (with accuracy up to hundredths) with the help of the coordinate cursor:

1. $x = -2.89, y = -0.81; x = -2.89, y = 0.39;$
2. $x = 2.89, y = -0.81; x = 2.89, y = 0.39;$
3. $x = -2.18, y = -2.06; x = 2.18, y = -2.06;$
4. $x = -2.63, y = 1.44; x = 2.63, y = 1.44;$
5. $x = -1.29, y = -2.71; x = 1.29, y = -2.71;$
6. $x = -0.55, y = 2.95; x = 0.55, y = 2.95.$

In this case it is impossible to exclude one of the variables x or y and it is difficult to offer any practically suitable way of solution besides the graphical method.

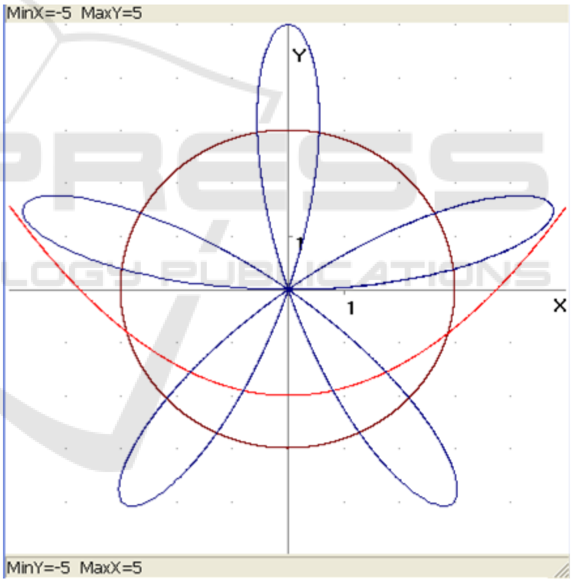

Figure 10: Dependence graphs.

4 CONCLUSIONS

The research examines the GRAN software complex, known in Ukraine and abroad.

Its expediency and effectiveness of use in the educational process of secondary and higher schools is confirmed by many years of practice, numerous research results for obtaining scientific degrees of doctors and candidates of sciences in the field of teaching mathematics and informatics (Vlasenko et al., 2020).

The approach to the study of mathematics, proposed by Myroslav Ivanovych Zhaldak, gives a clear idea of the concepts being studied, develops figurative thinking, spatial imagination, allows one to penetrate deeply enough into the essence of the studied phenomenon, to solve the problem informally. At the same time, clarification of the problem, formulation of the problem, development of the appropriate mathematical model, and material interpretation of the results obtained with the help of a computer come to the fore. All technical operations related to processing the built mathematical model, implementation of the solution search method, design, and presentation of the results of input data processing rely on the computer. It was to implement this approach that the GRAN software complex was developed.

Some selected problems proposed in the study demonstrate the expediency of using the GRAN complex for in-depth study of mathematics. Namely: the ability to conduct the necessary numerical experiment; quickly perform the necessary calculations or graphic constructions; to test the hypothesis; check the problem-solving method; to be able to analyze and explain the results obtained with the help of a computer; find out the limits of use. When learning mathematical methods, the use of a computer or the chosen method of solving a problem is extremely important.

The problems presented in the work were successfully used by the authors of the study during classes in various informatics disciplines, including computer modeling and computer mathematics.

A significant improvement of the GRAN complex has recently been its transfer to the cloud, which provides access to its services from any platform through a browser (Zhaldak and Franchuk, 2020). To get to the virtual desktop on a remote server, you must access the browser services and in the input line above the desktop enter the address https://gran.npu.edu.ua, then press the Enter key on the keyboard (or in the list of appropriate symbols on the screen “press” the label with the word “Go” in the case when using a smartphone or other laptop computer where there is no keyboard). As a result, a virtual desktop will open on which in the line “Username” you should select from the proposed list one of the available names, such as “gran”, and then in the line “Password” enter password “gran” (Zhaldak et al., 2021).

Currently, the expediency and possibility of transferring the entire GRAN complex of individual services into a mobile application is being investigated.

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