

Per-flow Packet Loss Ratios Induced by an Overflowed Buffer

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Keywords: Buffer Overflow, Packet Loss, Loss Ratio, Per-flow Analysis.

Abstract: Packet losses are common in TCP/IP networks. The main reason of that is the statistical multiplexing of flows at routers' output buffers, resulting in random overflows of these buffers and dropping of packets. Although the losses caused by an overflowed buffer have been widely studied, these studies were mainly devoted to the aggregated traffic. Namely, the total loss ratio was analyzed, for all the packets arriving to the buffer, without distinction between separate flows. In this paper, we study the packet loss ratios suffered by distinct flows arriving to a common buffer. We first observe that the loss ratios of particular flows may differ significantly from each other, and from the total loss ratio. Then we single out the properties of the flow that may influence the per-flow loss ratio. We study also possible mutual dependencies between the flows, i.e. possibilities that the properties of one flow influence the loss ratios of other flows. Finally, we study the impact of the buffer size and the distribution of the service time (packet size) on the per-flow loss ratios, as well as their relations to each other.

1 INTRODUCTION

In an IP router, all the flows bound to a particular output interface have to share a common buffer. As the packets from all these flows are statistically multiplexed upon arrival to this buffer, some random bursts may occur, overflowing the buffer and making it temporarily unavailable. Other packets, arriving to the buffer during such overflow periods, are deleted and lost.


As long as the network operates in the "best effort" manner, without allocation of resources, such losses are unavoidable. In fact, they are common in most TCP/IP networks and the Internet.


The main characteristic of the loss process is the loss ratio, L , defined as the number of lost packets divided by the total number of packets, in a long time interval. This characteristic has been widely studied using mathematical models (Takagi, 1993; Yajnik et al., 1999; Sanneck and Carle, 1999; Yu et al., 2005; Hasslinger and Hohlfeld, 2008; Chydzinski et al., 2007; Chydzinski and Adamczyk, 2012) and network measurements (Bolot, 1993; Coates and Nowak, 2000; Benko and Veres, 2002; Duffield et al., 2001; Sommers et al., 2005). The loss process has been also studied with regards to its statistical structure,

i.e. the occurrence of losses in groups, one after another (Cidon et al., 1993; Bratiychuk and Chydzinski, 2009). Especially useful in such studies is the packet burst ratio parameter (McGowan, 2005), which expresses directly the tendency of losses to group together, in long series. The packet burst ratio has been analyzed via mathematical models (Rachwalski and Papir, 2014; Rachwalski and Papir, 2015; Chydzinski et al., 2018; Chydzinski and Samociuk, 2019) and actual network measurements (Samociuk et al., 2018; Samociuk and Barczyk, 2019).

Naturally, packet losses can be studied globally (in the whole network), or in the end-to-end manner, or at the particular output interface of a networking device. In this paper we deal with the latter case, i.e. we study the loss ratio caused by a single overflowed buffer, in which many flows are statistically multiplexed.

The previous studies of the loss ratio induced by an overflow buffer lack the distinction between the loss ratios experienced by particular flows traversing the common output interface. It is not clear, whether all the flows sharing a common buffer suffer from the overflow events in the same way. In other words, it is not clear, if their private loss ratios are the same as the total loss ratio, counted for the aggregate traffic. Or, maybe the loss ratio of an individual flow can be much higher, or lower, than the total loss ratio, depending on the properties of that flow?

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In this paper, we show firstly that the latter is true. Indeed, the per-flow loss ratios may differ significantly from each other, and from the total loss ratio, depending on the statistical properties of flows. Surprisingly, this can happen even if the arrival rates of all the flows are exactly the same, or even if the inter-arrival time distributions of all flows are of the same type.

Secondly, we show the influence of the standard deviation of the interarrival time of a flow on its private loss ratio. As we will see, the per-flow loss ratio, and the total loss ratio, grow with the standard deviation of the considered flow. What is more surprising, is that the loss ratios of other flows may stay virtually unaltered, even if the total loss ratio and the ratio of one flow change by an order of magnitude.

Thirdly, we study the impact of the buffer size and the distribution of the service time (packet size) on the per-flow loss ratios. It is known that the total loss ratio grows with the standard deviation of the packet size, and shrinks with the buffer size. It will be shown that the same is true for the per-flow loss ratios. More interesting, however, is the relation between the per-flow loss ratios, when the standard deviation of the packet size and the buffer size change. For instance, imagine we have 2 flows with the per-flow loss ratios of $L_1 = 1\%$ and $L_2 = 3\%$. Therefore, we have the following proportion: $\frac{L_2}{L_1} = 3$. Will this proportion be kept, if we double the buffer size? Will it be kept, if we double the standard deviation of the packet size? As we will see, one of these answers is positive, while the other is negative.

The per-flow loss ratios are, obviously, of great importance in the QoS context. Namely, each individual flow may transport the content of a different type, e.g. data, audio, video. If the per flow-loss ratios differ between flows, and the differences depend on the statistical properties of flows, then we may expect different QoS parameters for data, audio and video flows sharing the same output buffer.

Unfortunately, there are no mathematical theorems on the per-flow loss ratios in queueing models with a single buffer and multiple flows arriving to it. The known formulas concern only the total loss ratio, for the aggregated traffic (some of them will be recalled in Section 3). Therefore, a discrete-event simulator Omnet++ is used for computing the per-flow loss ratios in this study. Due to the high efficiency of the simulator, programmed directly in C++ language, it is possible to simulate, in a rather short time, tens of millions of packets passing through the buffer. This is more than enough for the purpose of the loss ratio analysis carried out in this paper.

The rest of the paper is organized in the following manner. In Section 2, the queueing model of a buffer serving multiple flows is formally defined. In Section 3, some known formulas on the aggregated loss ratio are recalled. In Section 4, the simulation results are presented and discussed. In particular, nine different scenarios with different flow types, service time distributions and buffer sizes are considered. The final conclusions are gathered in Section 5, and accompanied by some future work suggestions.

2 THE MODEL

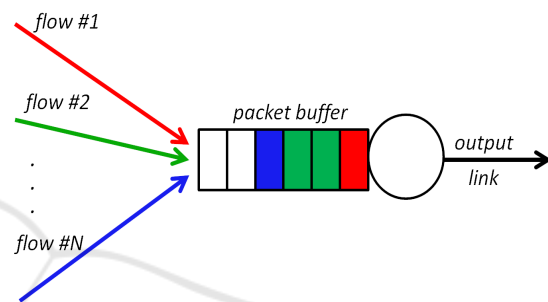


Figure 1: The model of the queue.

The scheme of the queueing model analyzed herein is depicted in Fig. 1. Namely, there are N flows arriving to a common buffer of size b packets (including the service position). The packets from all the flows are placed serially in the buffer, in the arrival order.

Each flow has a form of a separate renewal process. Namely, within the i -th flow, all the packet interarrival times are identically distributed, with distribution function G_i . Different flows, however, may have different interarrival time distributions. Thus in general it can be $G_i \neq G_j$.

The buffer is served in the usual FIFO discipline. The service time of a packet has some distribution given by distribution function F . Note that in networking the service time of a packet is simply proportional to the packet size, due to the constant capacity of the physical output link. Therefore, the service time distribution is proportional to the packet size distribution and has the same shape.

Finally, we need the following notations.

The total loss ratio is denoted by L . It is the long-run number of lost packets, divided by the total number of arriving packets, no matter what flow they belong to. Similarly, L_i denotes the per-flow loss ratio, i.e. the long-run number of lost packets, belonging to the i -th flow, divided by the total number of packets

belonging to the i -th flow. $\mathbb{E}(F)$ denotes the average service time, while $\mathbb{D}(F)$ – the standard deviation of the service time. $\mathbb{E}(G_i)$ stands for the average interarrival time of the i -th flow, while $\mathbb{D}(G_i)$ for its standard deviation. The arrival rate of the i -th flow is denoted by λ_i and we have:

$$\lambda_i = \frac{1}{\mathbb{E}(G_i)}. \tag{1}$$

The load of the queue is defined as:

$$\rho = \mathbb{E}(F) \cdot \sum_{i=1}^N \lambda_i. \tag{2}$$

3 THEORETICAL BACKGROUND

The problem of finding the total loss ratio has several known solutions, obtained assuming that the aggregated traffic has some special statistical properties. We will recall now two of them, which are especially useful.

Firstly, if the aggregated traffic can be approximated by the Poisson process of rate λ , then the total loss ratio can be computed following (Takagi, 1993) p. 202:

$$L = 1 - \frac{1}{\pi_0 + \rho}, \tag{3}$$

where

$$\pi_0 = \frac{1}{\sum_{k=0}^{b-1} \beta_k}, \tag{4}$$

$$\beta_0 = 1, \quad \beta_1 = \frac{1 - a_0}{a_0}, \tag{5}$$

$$\beta_{k+1} = \frac{1}{a_0} \left[\beta_k - \sum_{i=0}^{k-1} a_{k-i+1} \beta_i - a_k \right], \quad k \geq 1, \tag{6}$$

$$a_k = \int_0^\infty \frac{e^{-\lambda u} (\lambda u)^k}{k!} dF(u), \quad k \geq 0. \tag{7}$$

Secondly, if the aggregated traffic is autocorrelated, it can be modeled by the Markov-modulated Poisson process (MMPP). There are several well-known procedures for fitting the aggregated traffic to the MMPP process (Yoshihara et al., 2001; Salvador et al., 2003). Having the matrices Q and Λ of the MMPP process fitted to the aggregated traffic, we can calculate the total loss ratio using the following formula (Chydzinski et al., 2007):

$$L = \lim_{s \rightarrow 0^+} \frac{s^2 \delta_{b,1}(s)}{\lambda}, \tag{8}$$

where $\lambda = \pi \Lambda \mathbf{1}$ is the total arrival rate of the MMPP and

$$\delta_b(s) = [\delta_{b,1}(s), \dots, \delta_{b,m}(s)] = M_b^{-1}(s) y_b(s), \tag{9}$$

while

$$M_b(s) = (I - Z(s)) [R_{b+1}(s) A_0(s) + \sum_{k=0}^b R_{b-k}(s) B_k(s)] - E(s) [R_b(s) A_0(s) + \sum_{k=0}^{b-1} R_{b-1-k}(s) B_k(s)], \tag{10}$$

$$y_b(s) = E(s) \sum_{k=0}^{b-1} R_{b-1-k}(s) v_k(s) - (I - Z(s)) \sum_{k=0}^b R_{b-k}(s) v_k(s), \tag{11}$$

where

$$Z(s) = \left[\frac{(\Lambda_{ii} - Q_{ii}) p_{ij}}{s + \Lambda_{ii} - Q_{ii}} \right]_{i,j}, \tag{12}$$

$$p_{ij} = \begin{cases} 0 & \text{if } i = j, \\ Q_{ij} / (\Lambda_{ii} - Q_{ii}) & \text{if } i \neq j, \end{cases} \tag{13}$$

$$R_0(s) = \mathbf{0}, \quad R_1(s) = A_0^{-1}(s), \tag{14}$$

$$R_{k+1}(s) = A_0^{-1}(s) (R_k(s) - \sum_{i=0}^k A_{i+1}(s) R_{k-i}(s)), \quad k \geq 1, \tag{15}$$

$$A_k(s) = \left[\int_0^\infty e^{-st} P_{i,j}(k,t) dF(t) \right]_{i,j}, \tag{16}$$

$$B_n(s) = A_{n+1}(s) - \bar{A}_{n+1}(s) (\bar{A}_0(s))^{-1}, \quad \bar{A}_n(s) = \sum_{k=n}^\infty A_k(s), \tag{17}$$

$$E(s) = \left[\frac{\Lambda_{ij}}{s + \Lambda_{ii} - Q_{ii}} \right]_{i,j}, \tag{18}$$

$$v_k(s) = \bar{A}_{k+1}(s) (\bar{A}_0(s))^{-1} c_b(s) - c_{b-k}(s), \tag{19}$$

$$c_k(s) = \frac{1}{s} \sum_{i=b-k}^\infty (i-b+k) A_i(s) \cdot \mathbf{1} + \sum_{i=b-k}^\infty (i-b+k) \bar{D}_i(s) \cdot \mathbf{1}, \tag{20}$$

$$\bar{D}_k(s) = \left[\int_0^\infty e^{-st} P_{i,j}(k,t) (1 - F(t)) dt \right]_{i,j}. \tag{21}$$

In this notation, $\mathbf{0}$ is a square matrix of zeroes, I is an identity matrix, $\mathbf{1} = (1, \dots, 1)^T$, π is the stationary vector for the MMPP, which can be obtained from:

$$\pi Q = (0, \dots, 0), \tag{22}$$

$$\pi \cdot \mathbf{1} = 1, \tag{23}$$

while $P_{i,j}(n,t)$ the counting function for the MMPP.

Although the presented formulas are long, they are rather easy to apply, as it was demonstrated by numerical examples in (Chydzinski et al., 2007).

Unfortunately, the formula for the total loss ratio in the case when the arrival process has the form of the general renewal process (i.e. for the $G/G/1/b$ queue), is unknown and believed to be very hard to find. Similarly, there are no analytical formulas for the per-flow loss ratios, when the arrival traffic is split to several separate flows. Therefore, we had to use simulations to study the per-flow loss ratios.

4 RESULTS AND DISCUSSIONS

The results presented in this section were obtained using the Omnet++ simulator (www.omnetpp.org) version 5.6. Namely, the model presented in Section 2 has been implemented in Omnet++, with a configurable number of flows, interarrival time distribution in each flow, service time distribution and the buffer size. Nine different simulation scenarios were considered in total. In each simulation run, 10 million packets were passing through the buffer, to make the results statistically reliable. In all the scenarios, the queueing system was fully loaded, i.e. $\rho = 1$. If not stated otherwise, the buffer size was 50, while the service time was exponentially distributed with $\mathbb{E}(F) = 1$.

4.1 The Same Interarrival Time Distributions

In the beginning, we considered several flows of exactly the same interarrival time distributions and rates. If there is no statistical distinction between the flows, there is no reason, why the per-flow loss ratios should be different among the flows. Furthermore, if all the flows have the same loss ratio, then the total loss ratio must be exactly the same as all the per-flow loss ratios. We confirmed this in two scenarios.

In the first scenario, there were $N = 3$ flows, each Poisson with the same rate, i.e. $\lambda_1 = \lambda_2 = \lambda_3 = \frac{1}{3}$. The following loss ratios were obtained:

L	L_1	L_2	L_3
0.0197	0.0197	0.0197	0.0198

As we can see, all loss ratios are practically the same, including the total loss ratio.

In the second scenario, we used different interarrival time distribution, to exclude the possible influence of special properties of the Poisson distribution. Moreover, more flows were involved. Namely, there were $N = 10$ identical flows, each with $\Gamma(10, 1)$ distribution of the interarrival time within the flow. As it is easy to check, we had $\lambda_1 = \dots = \lambda_{10} = \frac{1}{10}$. The following loss ratios were obtained:

L	0.0112
L_1	0.0113
L_2	0.0113
L_3	0.0112
L_4	0.0111
L_5	0.0112
L_6	0.0113
L_7	0.0112
L_8	0.0113
L_9	0.0110
L_{10}	0.0113

As expected, all the loss ratios are the practically the same, with minor statistical fluctuations, which can be expected in a simulation.

4.2 The Same Distribution Types, Different Rates

In the third scenario, we had the same type of the interarrival time distribution, but different arrival rates between flows. Namely, it was $N = 3$, G_1 was the $\Gamma(10, 10)$ distribution, G_2 was the $\Gamma(1, 10)$ distribution, while G_3 was the $\Gamma(0.11235955, 10)$ distribution. The resulting per-flow arrival rates were $\lambda_1 = 0.01$, $\lambda_2 = 0.1$ and $\lambda_3 = 0.89$.

The following loss ratios were obtained:

L	L_1	L_2	L_3
0.0808	0.0247	0.0256	0.0876

As we can see, the loss ratios may differ significantly when the interarrival distribution type is common among flows, but their rates differ. This might be a little surprising. Moreover, the differences between the per-flow loss ratios are significant. For instance, L_3 is more than 3 times greater than L_1 and L_2 . As for the total loss ratio, it is dominated by the losses of the most intense flow, which is L_3 in this scenario. Therefore L and L_3 have similar values.

4.3 The Same Rates, Different Distribution Types

In the fourth scenario we reversed the assumptions from the third one. All the flow rates were the same,

but the interarrival time distributions were different. Namely, we had $N = 3$ flows of the same rates, i.e. $\lambda_1 = \lambda_2 = \lambda_3 = \frac{1}{3}$. However, G_1 was constant and equal to 3 (a CBR flow), G_2 was exponential with the average of 3, while G_3 was $\Gamma(0.03, 100)$.

The following loss ratios were obtained:

L	L_1	L_2	L_3
0.0879	0.0172	0.0222	0.2244

As we may observe, the loss ratios of different flows may differ by an order of magnitude, even if the arrival rates are exactly the same (compare L_3 with L_2).

Noticing that, we should look for flow characteristics, which may potentially influence its loss ratio. A good candidate to start with is the standard deviation of the interarrival time of the flow. It will be studied in the next subsection.

4.4 Dependence on the Standard Deviation of the Interarrival Time

In this scenario, there were $N = 5$ flows. In every flow, the interarrival time was gamma distributed, with the average interarrival of 5, resulting in $\lambda_1 = \dots = \lambda_5 = \frac{1}{5}$. The standard deviations of interarrival times were, however, twice larger in the every next flow, namely: $\mathbb{D}(G_1) = 1$, $\mathbb{D}(G_2) = 2$, $\mathbb{D}(G_3) = 4$, $\mathbb{D}(G_4) = 8$, $\mathbb{D}(G_5) = 16$.

The following loss ratios were obtained:

L	0.0337
L_1	0.0170
L_2	0.0177
L_3	0.0196
L_4	0.0316
L_5	0.0826

As we can see, the per-flow loss ratio grows with the standard deviation of the interarrival time. This dependence is rather weak for low values of $D(G_i)$. Namely, the loss ratio is only slightly larger for $D(G_i) = 2$, than for $D(G_i) = 1$. But for larger $D(G_i)$, this dependence gets strong. Namely, the per-flow loss ratio is more than twice larger for $D(G_i) = 16$, than for $D(G_i) = 8$.

In the next simulations, we used only $N = 2$ flows, and changed continuously the standard deviation of the second flow, while keeping unaltered the first flow. Namely, G_1 was the exponential distribution with the average of 2, while G_2 was the $\Gamma(2u, \frac{1}{u})$ distribution, dependent on a positive parameter u . It is easy to check that it was $\lambda_1 = \lambda_2 = \frac{1}{2}$ and $D(G_1) = 2$. On the other hand, it was $D(G_2) = \sqrt{\frac{2}{u}}$. Therefore ma-

nipulating u , we could easily change the standard deviation of the second flow.

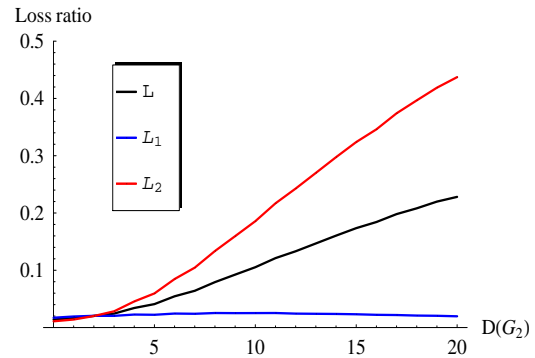


Figure 2: Dependence of the total and per-flow loss ratios on the standard deviation of one of the flows.

The results are depicted in Fig. 2. Namely, L_1 , L_2 and L are shown as functions of $D(G_2)$, which varies from 0 to 20. As can be noticed, both L and L_2 grow with $D(G_2)$, while L_1 remains virtually the same. The latter is rather surprising. In the experiment, the total loss ratio had grown by the factor of 16. Intuitively, such large growth of L should somehow affect L_1 , not only L_2 . This did not happen. Almost all growth of L was to be attributed to the growth of L_2 .

4.5 Dependence on the Standard Deviation of the Service Time

In most of the previous experiments, we did not vary the distribution of the service time. In the next scenario, we changed this distribution, with a special attention to its standard deviation. Moreover, we used two flows with very different their own standard deviations, to see how the variable service time affect such different flows. Namely, it was $N = 2$, $\lambda_1 = \lambda_2 = \frac{1}{2}$, G_1 was the uniform distribution on the interval $(1, 3)$, while G_2 was the $\Gamma(0.16, 12.5)$ distribution. The standard deviations in flows were $\mathbb{D}(G_1) = 0.577$ and $\mathbb{D}(G_2) = 5$, respectively. The service time had the $\Gamma(u, \frac{1}{u})$ distribution, where $u > 0$ was a parameter. Therefore, $\mathbb{E}(F)$ did not depend on u and was always equal to 1, while $\mathbb{D}(F) = \frac{1}{\sqrt{u}}$ depended on u , so that using u we could obtain arbitrary positive $\mathbb{D}(F)$.

The results are depicted in Fig. 3. As we can see, the total loss ratio, as well as both per-flow loss ratios, grow with the deviation of the service time. Moreover, all the loss ratios grow in the same way, i.e. the curves have the same shape and approach each other. The latter has an interesting consequence: the difference between the per-flow loss ratios, caused by their different standard deviations, is mitigated when the standard deviation of the service time gets larger.

This effect is visible clearly in Fig. 4, in which the ratio $\frac{L_2}{L_1}$ is depicted as a function of $\mathbb{D}(F)$. As we may notice, L_2 is about 7 times larger than L_1 , when $\mathbb{D}(F)$ is small. But the difference diminishes, as $\mathbb{D}(F)$ grows, and for $\mathbb{D}(F) = 10$, both L_2 and L_1 are practically the same.

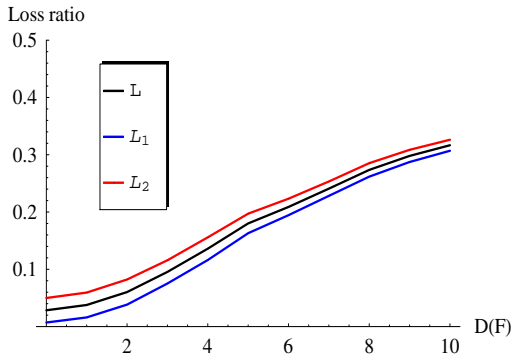


Figure 3: Dependence of the total and per-flow loss ratios on the standard deviation of the service time.

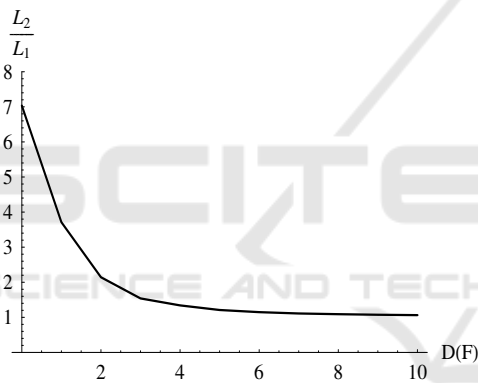


Figure 4: Ratio $\frac{L_2}{L_1}$ versus the standard deviation of the service time.

4.6 Dependence on the Buffer Size

In all the previous experiments, we did not change the buffer size. In the next scenario, we varied the buffer size from 10 to 100 and observed, how this affects the per-flow loss ratios and their relations to each other. We used again two flows which had very different their own standard deviations, to see this time how the buffer size affects such different flows. Namely, it was $N = 2$, $\lambda_1 = \lambda_2 = \frac{1}{2}$, G_1 was uniform on the interval $(1,3)$, G_2 was $\Gamma(0.16, 12.5)$, and we had $\mathbb{D}(G_1) = 0.577$, $\mathbb{D}(G_2) = 5$.

The results are depicted in Fig. 5. All three loss ratios decrease with time, which is not surprising. However, the relation between L_1 and L_2 is quite different than the one observed in the previous experiment. It can be seen in Fig. 6, in which the ratio $\frac{L_2}{L_1}$ is

depicted for various buffer sizes. For two quite different flows, the proportion $\frac{L_2}{L_1}$ is virtually the same for the buffer sizes which differ by an order of magnitude.

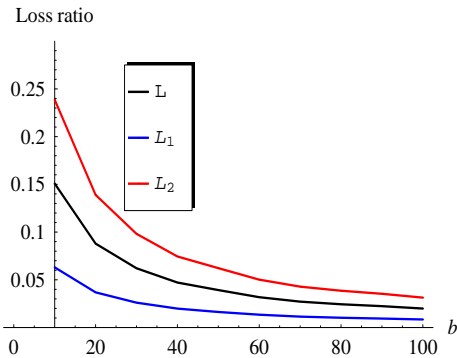


Figure 5: Dependence of the total and per-flow loss ratios on the buffer size.

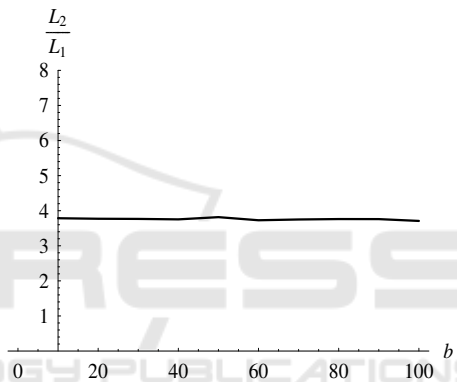


Figure 6: Ratio $\frac{L_2}{L_1}$ versus the buffer size.

4.7 Special Case: Poisson Flows Only

In subsection 4.2 we showed simulation results, in which the per-flow loss ratios were different, even though all the flows had the same type of the interarrival time distribution (but the rates were different).

In the following experiment, we checked if this holds true if all the flows are Poisson. Namely, the following parameters were used: $N = 3$, all interarrival times exponential, with the average values of 100, 10 and $\frac{100}{89}$ in different flows, respectively. The rates were thus $\lambda_1 = 0.01$, $\lambda_2 = 0.1$ and $\lambda_3 = 0.89$, the buffer size was 50. The obtained results are:

L	L_1	L_2	L_3
0.0195	0.0194	0.0195	0.0195

As we can see, all the per-flow loss ratios are the same now. This observation, which differs from the one of subsection 4.2, is most likely caused by the special properties of the Poisson process and expo-

nential distribution. As we know, a superposition of many Poisson processes, perhaps of different rates, is again a Poisson processes. This is a consequence of the memoryless property of the exponential distribution, which is the only one among continuous distribution possessing this property.

5 CONCLUSIONS

In this paper, we presented a simulation study of the per-flow loss ratios caused by an overflowed buffer at the output interface of a networking device. We tried to find the properties of the flow that may influence its loss ratio, investigated the possible mutual dependencies between the flows, studied the impact of the buffer size and the distribution of the service time (packet size) on the per-flow loss ratios, as well as their relations to each other. Several interesting observations were made in the simulations. In particular, we observed that:

- the per-flow loss ratios were different from each other and different from the total loss ratio, even when the flows had the same type of the interarrival time distribution, but different rates;
- the per-flow loss ratios were different from each other and different from the total loss ratio, even when the flows had the same rates, but different interarrival time distributions;
- the per-flow loss ratio of a particular flow, as well as the total loss ratio, grew with the standard deviation of that flow, while the losses of other flows were practically unaffected;
- the per-flow loss ratios and the total loss ratio grew with the standard deviation of the service time;
- as the standard deviation of the service time grew, the differences between the per-flow loss ratios got smaller, even for flows of quite different types. For a large standard deviation of the service time, these differences practically vanished;
- the per-flow loss ratios decreased with the buffer size, but the proportions between them were maintained, i.e. different buffers affected the per-flow loss ratios in the same way, even if the flows were of very different types;
- the per-flow loss ratios were all the same only in two cases: when all the flows had the same interarrival time distributions and rates, or when all the flows had the exponential interarrival time distribution (in the latter case, the rates might not be the same).

As for the future work, it would be great if we could solve analytically the model presented in Section 2, so that all the results and conclusion obtained here via simulations could be obtained from formulas. Unfortunately, this seems to be far beyond the state of the art of the queueing theory. The model considered herein, is clearly a generalization of the classic $G/G/1/b$ model, i.e. we can obtain the $G/G/1/b$ model by putting $N = 1$ to the model of Section 2. It is well known, however, that the exact solution of the $G/G/1/b$ is very hard to obtain, and nobody have succeeded so far in finding it. Naturally, solving the model of Section 2 would be even harder.

Therefore, we are left with the following two possibilities. Firstly, we can restrict the analysis to some special cases of the model, for instance to some special distributions of the interarrival time. The $G/G/1/b$ model has several known solutions for special distributions, like exponential, Erlang and other Markovian distributions. Secondly, we may search for approximate solutions of the model of Section 2. There are many successful approaches to solve the $G/G/1/b$ model approximately, so maybe the same is possible in the case of the multi-flow arrival model.

Finally, it is recommended by Internet Engineering Task Force, (Baker and Fairhurst, 2015), that the classic finite-buffer queueing at routers' output interfaces are replaced by some active queue management algorithms (Chrost et al., 2009; Chrost and Chydzinski, 2013). In these algorithms, the losses occur before the buffer gets full and are caused by the algorithm itself. It would be interesting to extend to the per-flow loss analysis to models of such algorithms (Chydzinski and Mrozowski, 2016).

ACKNOWLEDGEMENTS

This work was conducted within project 2020/39/B/ST6/00224, funded by National Science Centre, Poland.

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