

# Online Metric Facility Service Leasing with Duration-Specific Dormant Fees

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**Abstract:** Inspired by the COVID-19 pandemic, a new online facility model, known as the *Online Facility Service Leasing* problem (OFSL), has been recently introduced. In OFSL, services at different (health) facility locations are leased for different durations and costs. Each service at each facility is associated with a dormant fee that needs to be paid for each day on which the service is not leased at the facility. Clients arrive over time, each requesting a number of services, and need to be served by connecting them to multiple facilities jointly offering the requested services. The aim is to decide which services to lease, when, and for how long, in order to serve all clients as soon as they appear with minimum costs of leasing, connecting, and dormant fees. In this paper, we study a generalization of OFSL in which we are additionally given a parameter  $d$ , such that, should the service be not leased for more than  $d$  consecutive days, a dormant fee is to be paid ( $d = 0$  in the case of OFSL). We call this variant the *Online Facility Service Leasing with Duration-Specific Dormant Fees* ( $d$ -OFSL). We particularly focus on the metric version of the problem in which facilities and clients reside in the metric space. We refer to it as metric  $d$ -OFSL and design the first online algorithm for the problem. The latter is a deterministic algorithm based on a primal-dual approach. We measure its performance by comparing it to the optimal offline solution for all instances of the problem. This performance analysis is known as competitive analysis and is the standard to evaluate online algorithms.

## 1 INTRODUCTION

The COVID-19 pandemic has been a wake-up call to most communities around the world. Many have been striving to provide adequate timely health-care services to their patients as they ran out of resources. As a result, new temporary strategies were adopted, including *leasing* facility services at different locations to satisfy the needs of patients as fast as possible. Challenging decisions had to be made in regards to leasing contracts, budgeting, and distributing patients to health centers. Consequently, a significant number of works addressing such decisions from various perspectives appeared in the literature (Choi, 2021; Queiroz et al., 2020; Ivanov, 2020; Howard, 2021; Nikolopoulos et al., 2021). Recently, a new online facility model, known as the *Online Facility Service Leasing* problem (OFSL), has been introduced (Markarian and Khallouf, 2021). The latter was motivated by the following optimization problem. Imagine a company that has made contracts to

lease resources at a number of facility locations, each offering some services. These services are reserved for the company for as long as the corresponding contract states. The company is given a number of lease types for leasing the services. Each type is characterized by a duration and cost. Lease prices respect the economy of scale such that a longer lease type costs more but cheaper per unit time. Each day a service is not leased at a facility, a *dormant fee* needs to be paid. This fee is the cost the company pays for reserving the service. Clients arrive over time. The company does not know in advance how many will come and when will they come. Each would request a number of services, such as testing, treatment, and vaccination. The goal is to decide when to lease which services at which facility locations such that each client is served by connecting it to multiple facilities jointly offering the requested services, at minimum possible costs of *leasing*, *connecting*, and *dormant fees*.

In this paper, we study a generalization of OFSL in which we are additionally given a parameter  $d$ ,

such that, should the service be not leased for more than  $d$  consecutive days, a dormant fee is to be paid ( $d = 0$  in the case of OFSL). We call this variant the *Online Facility Service Leasing with Duration-Specific Dormant Fees* problem ( $d$ -OFSL). We particularly focus on the metric version of the problem in which facilities and clients reside in the metric space and respect the triangle inequality. We refer to it as metric  $d$ -OFSL and design the first online algorithm for the problem. The latter is a deterministic algorithm based on a primal-dual approach. We measure its performance by comparing its output to the optimal offline solution for all instances of the problem. This performance analysis is a worst-case analysis known as *competitive analysis* and is the standard to evaluate online algorithms (Borodin and El-Yaniv, 2005). An online algorithm is said to have a *competitive ratio*  $r$  where  $r$  is the worst-case ratio of the cost of the online algorithm to that of the optimal offline solution, for all instances of the problem. The latter is referred to as  $r$ -competitive online algorithm. The challenge is to design online algorithms that can be proven to have a small  $r$ .

We develop the first online algorithm for metric  $d$ -OFSL, with an  $O((L + \frac{d}{l_{min}}) \cdot \log l_{max})$ -competitive ratio, where:

- $L$  is the number of lease types available
- $d$  is the maximum number of days after which a dormant fee needs to be paid
- $l_{min}$  is the shortest lease duration
- $l_{max}$  is the longest lease duration

In addition to generalizing OFSL, metric  $d$ -OFSL generalizes two well-known online optimization problems, the *Parking Permit* problem (PP) (Meyerson, 2005) and the *Metric Online Facility Location* problem (metric OFL) (Meyerson, 2001). There is a lower bound of  $\Omega(L)$  on the competitive ratio of any deterministic algorithm for PP, where  $L$  is the number of lease types available. Moreover, there is a lower bound of  $\Omega(\frac{\log n}{\log \log n})$  on the competitive ratio of any randomized algorithm for metric OFL (Fotakis, 2003), where  $n$  is the number of clients. These imply a lower bound of  $\Omega(L + \frac{\log n}{\log \log n})$  on the competitive ratio of any deterministic algorithm for metric  $d$ -OFSL, where  $n$  is the number of clients.

We note here that the algorithm for OFSL in (Markarian and Khallouf, 2021) can't be extended to our problem, since the latter is for the non-metric variant. In general, results for the metric variants, including the one in this paper, exploit the metric properties of the problem to achieve better bounds on the competitive ratio in comparison to the non-metric variants. Moreover, similar techniques as those

in (Markarian and Khallouf, 2021) could have been used to achieve a non-trivial competitive ratio for the non-metric variant of  $d$ -OFSL, and hence our motivation in this paper to target the metric version rather than the non-metric version of the problem.

**Outline.** The rest of the paper is structured as follows. In Section 2, we present a summary of works related to metric  $d$ -OFSL. In Section 3, we give a formal description of metric  $d$ -OFSL. In Section 4, we give a graph formulation of metric  $d$ -OFSL. Following this graph formulation, we give, in Section 5, a primal-dual program for metric  $d$ -OFSL. In Section 6, we present our online algorithm and prove its competitive ratio in Section 7. We conclude in Section 8 with some remarks and future work.

## 2 RELATED WORK

Meyerson (Meyerson, 2005) introduced the first online leasing framework, with a simple problem known as the *Parking Permit* problem (PP), for which he gave upper and lower competitive bounds. He presented an  $O(L)$ -competitive deterministic algorithm and an  $O(\log L)$ -competitive randomized algorithm along with matching lower bounds.

Many network optimization problems were formed based on this framework (Anthony and Gupta, 2007; Markarian and Kassar, 2020; Nagarajan and Williamson, 2013; Abshoff et al., 2016). Later, a number of extensions to the original framework were introduced, including lease prices changing over time, clients with deadlines, and lease types with dimensions (Feldkord et al., 2017; Li et al., 2018; Markarian, 2018; De Lima et al., 2017b; De Lima et al., 2020).

Online Facility Location problems have been intensively studied in the metric setting. Meyerson (Meyerson, 2001) proposed a randomized  $O(\log n)$ -competitive algorithm for the *Online Facility Location* problem (OFL), where  $n$  is the number of clients. Later Fotakis (Fotakis, 2003) gave an  $O(\log n / \log \log n)$ -competitive algorithm and showed that this bound is optimal. Many other results that include other online variations were also known (Fotakis, 2007; Fotakis, 2011; San Felice et al., 2015).

A number of leasing variants of OFL were also studied (Abshoff et al., 2016; Nagarajan and Williamson, 2013; Markarian and Meyer auf der Heide, 2019; Li et al., 2018; De Lima et al., 2017a). Unlike in these variants, in  $d$ -OFSL and OFSL, *services* rather than *facilities* are leased. Moreover, unlike the case in  $d$ -OFSL and OFSL, all clients in these

variants can be served by *all* facilities.  $d$ -OFSL and OFSL generalize these variants by having *one* service offered by all facilities and setting all dormant fees to 0. Despite the differences, many of the techniques used in these variants do seem helpful in solving  $d$ -OFSL and OFSL, as we will see in the coming sections.

### 3 PROBLEM DESCRIPTION

In this section, we give a formal description of metric *Online Facility Service Leasing with Duration-Specific Dormant Fees* (metric  $d$ -OFSL).

**Definition 1.** (metric  $d$ -OFSL) *Given  $m$  facility locations and  $k$  services. Each facility location offers a subset of the  $k$  services. These services can be leased with  $L$  different types, each differing by a duration and price. Given a positive integer  $d \geq 2$ . For each service at each facility location, there is a dormant fee that needs to be paid whenever the service is not leased for  $d$  consecutive days. There are at most  $n$  clients which arrive over time. Each day, a subset of the clients arrives, each requesting a subset of the  $k$  services. The algorithm serves a client by connecting it to a number of facility locations jointly offering the requested services, such that these services are leased at the time of the client's arrival. Connecting a client to a facility location incurs a connecting cost which is equal to the distance between the client and the facility location. To each day, the algorithm reacts by deciding which services to lease at which facility locations with which lease type in order to serve all arriving clients. The goal is to minimize the total leasing costs, connecting costs, and dormant fees.*

Next, we describe the *Dormant-Fee-Interval* model and the *Lease-Interval* model. These will be assumed for the dormant fees and the lease structures, respectively.

**Dormant-Fee-Interval Model.** The algorithm pays a dormant fee, if needed, only on days  $x$  where  $x \bmod d \equiv 1$ , without affecting the competitive ratio.

*Proof.* Consider an instance  $I$  of the original problem. Let  $\text{Opt}$  be an optimal solution for  $I$ . In the Dormant-Fee Interval model, we are only allowed to pay a dormant fee on days  $x : x \bmod d \equiv 1$ . Let  $i$  be an interval of  $d$  days at the end of which  $\text{Opt}$  has paid a dormant fee. Starting from day 0, we will divide the timeline into intervals of length  $d$ . Interval  $i$  crosses at most two of these intervals. We can create a feasible solution for the Dormant-Fee Interval model by paying the dormant fee associated with the first interval

crossed by  $i$ . This would not affect the feasibility of the solution constructed. Doing this for all the intervals associated with dormant fees paid by  $\text{Opt}$  would complete the proof.  $\square$

**Lease-Interval Model.** Meyerson (Meyerson, 2005) showed that the following can be assumed by losing only a constant factor in the competitive ratio.

- Leases of the same duration do not overlap.
- All lease durations are power of two.

This model has also been assumed in (Markarian and Khallouf, 2021) and many leasing optimization problems studied thus far (Nagarajan and Williamson, 2013; Abshoff et al., 2016; Li et al., 2018; Markarian, 2015).

### 4 GRAPH FORMULATION

In this section, we formulate  $d$ -OFSL as a graph-theoretic problem.

- For each client which arrives, we create a node, called *actual client node* at the location of the client. This client needs to be served as soon as it arrives. For each service it is requesting, it needs to be connected to at least one facility location offering the service.
- For each service at each facility, we create a node, called *actual service node* at the location of the facility. This *actual service node* can be leased for  $L$  different durations.
- For each service at each facility, we create a node, called *virtual service node* at the location of the facility. This *virtual service node* can be leased only for a duration of a single day and has cost equal to the dormant fee associated with the service. Moreover, it can be leased only on days  $x : x \bmod d \equiv 0$ . Figure 2 shows an example of  $d = 5$ .
- For each service at each facility, we create a node, called *virtual client node* at the location of the facility. This client appears on days  $x : x \bmod d \equiv 1$  and requests to be connected either to the *virtual service node* or to the *actual service node* associated with it. Moreover, it can be served on any day starting from the day  $y$  it appears until day  $y + d - 1$ . Figure 2 shows an example of  $d = 5$ .
- We add an edge from an *actual client node* to an *actual service node* if the client corresponding to the client node has requested the service corresponding to the service node. The weight of this edge would be equal to the connecting cost between the client and the facility location.

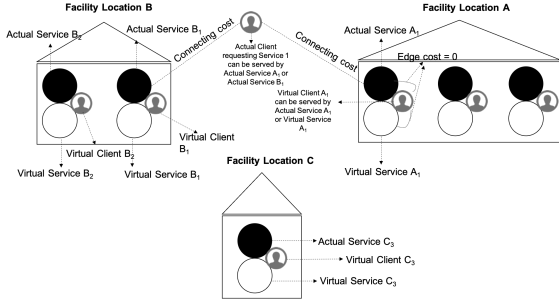


Figure 1: Three facility locations and a client requesting one service.

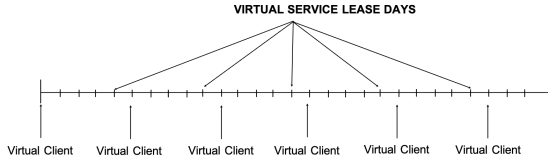


Figure 2: Virtual client, virtual service lease days for  $d = 5$ .

- From each *virtual client node*, we add two edges, of weight 0, one to its corresponding *actual service node* and another to its corresponding *virtual service node*.

Figure 1 shows an example of three facility locations, each offering one, two, and three services, respectively, and one client requesting one service.

Initially, the algorithm knows all about the facility locations, the services, and the lease prices. The client locations and their requests are revealed over time when clients show up. Each day, the online algorithm reacts to the client nodes created by purchasing from the available leases. Edges correspond to the connecting costs that will be paid upon connecting a client to a facility location. Notice that, a virtual client in our formulation is associated with a deadline that represents whether or not a dormant fee will be paid. Each service at each facility is associated with such a client that appears every  $d$  days to ensure that every  $d$  days, the algorithm checks whether it is required to pay a dormant fee for the service or not. Figure 1 illustrates the days on which virtual client nodes appear and virtual service nodes are leased.

## 5 PRIMAL-DUAL FORMULATION

In this section, we present an integer linear program and the corresponding relaxed dual program for the graph-theoretic problem described above. Fig. 3 illustrates this formulation.

$$\begin{aligned} \min \quad & \sum_{(i,k,t) \in \mathcal{S}} c_{ik} x_{ikt} + \sum_{(js,t,d) \in \mathcal{R}} \sum_{i \in S_{js}} c_{ijs} y_{i,jstd} \\ & \sum_{i \in S_{js}} y_{i,jstd} \geq 1 \quad (js,t,d) \in \mathcal{R} \\ & \sum x_{ikt'} - y_{i,jstd} \geq 0 \quad (i,k,t') \in S_{js} \\ & y_{i,jstd} \in \{0,1\} \quad i \in S_{js}, \\ & \quad \quad \quad (js,t,d) \in \mathcal{R} \\ & x_{ikt} \in \{0,1\} \quad (i,k,t) \in \mathcal{S} \end{aligned}$$

$$\begin{aligned} \max \quad & \sum_{(js,t,d) \in \mathcal{R}} \alpha_{jstd} \\ & \alpha_{jstd} - \beta_{i,jstd} \leq c_{ijs} \quad i \in S_{js} \\ & \quad \quad \quad (js,t,d) \in \mathcal{R} \\ & \sum_{(js,t',d) \in \mathcal{R}} \beta_{i,jstd} \leq c_{ik} \quad (i,k,t) \in S_{js} \\ & \beta_{i,jstd} \geq 0 \quad i \in S_{js}, \\ & \quad \quad \quad (js,t,d) \in \mathcal{R} \\ & \alpha_{jstd} \geq 0 \quad (js,t,d) \in \mathcal{R} \end{aligned}$$

Figure 3: Linear Programming Formulation of.

The objective function has two parts. The first part represents the costs incurred by leasing services. We denote each service at each facility as a triplet  $(i, k, t)$ , where  $i$  is the service type,  $k$  is the lease type, and  $t$  is the starting day of the lease. A variable  $x_{ikt}$  is assigned to each  $(i, k, t)$  indicating whether it is bought or not.  $c_{ik}$  is the cost of leasing service  $i$  with type  $k$ .

A *request* is characterized by a client-service pair, such that for each service requested by a client, we generate a request  $(js, t, d)$  referring to client  $j$  requesting service  $s$ , arriving at time  $t$ , and having deadline  $t + d$ .  $c_{ijs}$  is the cost of connecting  $j$  to  $i$ .

The second part of the objective function represents the costs incurred by connecting each request to a service, such that variable  $y_{i,jstd}$  indicates whether request  $(js, t, d)$  is connected to service  $i$ . Recall that all requests associated with actual client nodes have deadline 0.

The first primal constraint guarantees that each request is connected to at least one service. The second constraint makes sure that each request is only connected to a service that is leased within the arrival time and the deadline of the request. We denote by  $\mathcal{S}$  the collection of all service triplets and by  $\mathcal{R}$  the collection of all request triplets. We denote by  $S_{js}$  the collection of service triplets that can serve request  $js$ . We call these triplets *nominees*. Let  $H_n$  be the  $n$ th harmonic number  $1 + \frac{1}{2} + \dots + \frac{1}{n}$ .

## 6 ONLINE PRIMAL-DUAL ALGORITHM

In this section, we present an online deterministic primal-dual algorithm for metric  $d$ -OFSL.

The main idea of the algorithm is that whenever a client arrives and a request is formed, as long as (i) the dual constraints associated with it are not violated and (ii) the request's dual variable is not equal to the distance to a purchased nominee, the algorithm keeps increasing its dual variables. Algorithm 1 illustrates the steps that react to each request formed.

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Algorithm 1: Online Primal-Dual Algorithm for metric  $d$ -OFSL.

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When a request  $(js, t, d)$  is generated, we increase its variable  $\alpha_{jstd}$  and the variables  $\beta_{i, jstd}$  corresponding to its nominees while maintaining  $\alpha_{jstd} - \beta_{i, jstd} \geq 0$ , until:

(i) either the dual constraint of some nominee  $(i, k, t') \in S_{js}$  becomes tight:

$$\sum \beta_{i, jstd} = c_{ik}: (js, t, d) \in \mathcal{R}$$

So, we buy  $(i, k, t')$  (i.e., we set its primal variable  $x_{ikt'}$  to 1).

(ii) or  $\alpha_{jstd} = c_{ijs}$  for some bought nominee  $(i, k, t') \in S_{js}$

We connect  $(js, t, d)$  to the closest bought nominee.

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## 7 COMPETITIVE ANALYSIS

In this section, we give a competitive analysis of our algorithm, based on dual fitting arguments (Freund and Rawitz, 2003; Jain et al., 2003; Jain and Vazirani, 2001).

The proof ideas are based on our previous result in (Li et al., 2018). We partition the timeline into rounds  $\tau_j := \{(i-1)l_{\max}, \dots, il_{\max} - 1\}$  of length  $l_{\max}$  and conduct the analysis on the first  $l_{\max}$  time steps only. This has been proven to be sufficient to conclude the competitive ratio of the algorithm (Abshoff et al., 2016).

Note that according to the primal-dual formulation, the dormant fees are embedded in the primal-dual program as leasing costs. Hence, they will not appear in the analysis.

Notice that our algorithm outputs a *feasible* primal solution and an *infeasible* dual solution. Consequently, the proof will be composed of two parts. In the first part (Lemma 1), the cost of the primal so-

lution will be bounded by  $O(L + \frac{d}{l_{\min}})$  times the cost of the dual solution. In the second part (Lemma 2), the infeasible dual solution constructed will be scaled down by  $O(H_{l_{\max}})$  to make it feasible. Using Weak Duality Theorem, we will imply the competitive ratio of the algorithm.

**Lemma 1.** *The cost of the primal solution constructed by the algorithm is at most  $(L + \frac{2d}{l_{\min}}) \cdot \sum_{(js, t, d) \in \mathcal{R}} \alpha_{jstd}$ .*

*Proof.* We first show that the sum of the connection costs is at most  $\sum_{(js, t, d) \in \mathcal{R}} \alpha_{jstd}$  and then show that the sum of the leasing costs is at most  $(L + \frac{2d}{l_{\min}}) \cdot \sum_{(js, t, d) \in \mathcal{R}} \alpha_{jstd}$ .

A request  $(js, t, d)$  is either assigned to an already leased service or it leads to leasing a new service. If it is the first case, then the request has increased only the variable  $\alpha_{jstd}$  until  $\alpha_{jstd} = c_{ijs}$ . If it is the second case, then the request has increased both  $\alpha_{jstd}$  and  $\beta_{i, jstd}$  as long as  $\alpha_{jstd} - \beta_{i, jstd} \leq c_{ijs}$ , while maintaining  $\alpha_{jstd} - \beta_{i, jstd} \geq 0$ . Thus  $\alpha_{jstd} \geq c_{ijs}$ . We can sum up over all requests and get a total connection cost of  $O(\sum_{(js, t, d) \in \mathcal{R}} \alpha_{jstd})$ .

As for the leasing costs, we say a request *contributes* to the leasing cost of a service of type  $k$  if it has caused such a service lease to be purchased. The total contribution of request  $(js, t, d)$  to service leases of type  $k$  can be upper bounded by  $\alpha_{jstd} \cdot \frac{d}{l_k}$ . This is because the number of nominees of type  $k$  does not exceed  $\frac{d}{l_k}$  (the maximum is for the virtual clients case). Summing up over all  $L$  lease types yields:

$$\sum_{j=1}^L \left\lceil \frac{d}{l_j} \right\rceil \leq L + d \left[ \frac{1}{l_1} \left( \frac{1-(1/2)^L}{1-1/2} \right) \right] = L + d \left[ \frac{2}{l_1} (1 - (1/2)^L) \right]$$

Since  $L \geq 1$ , we have:

$$L + d \left[ \frac{2}{l_1} (1 - (1/2)^L) \right] \leq L + \frac{2d}{l_{\min}}$$

The sum of all clients' contributions implies the total leasing costs:

$$\sum_{(js, t, d) \in \mathcal{R}} \alpha_{jstd} \cdot \left( L + \frac{2d}{l_{\min}} \right)$$

□

The following Lemma has been proven in (Abshoff et al., 2016; Li et al., 2018; Nagarajan and Williamson, 2013). It shows that it is sufficient to divide the infeasible dual solution by  $2(H_{l_{\max}} + 1)$  to yield a feasible dual solution. Its proof is based on repeatedly exploiting the triangle inequality. Moreover, the bound is not based on the number of clients but

rather on the number of time steps. That is why the additional number of clients resulting from the virtual client nodes does not appear in the analysis.

**Lemma 2.** For any service  $(i, k, t) \in S_{js}$  and  $\mu = \frac{1}{2(H_{l_{\max}}+1)}$ , it holds that:

$$\sum_{(js, t', d) \in \mathcal{R}} \mu \cdot \alpha_{jst'd} - \beta_{i, jst'd} \leq c_{ijs}$$

Notice that both bounds do not depend on the number of services and that is why the additional number of clients resulting from the virtual service nodes does not appear in the analysis too.

By combining the two lemmata, we obtain the following theorem.

**Theorem 1.** There is an online deterministic  $O((L + \frac{d}{l_{\min}}) \cdot \log l_{\max})$ -competitive algorithm for metric  $d$ -OFSL, where  $L$  is the number of lease types available,  $d$  is the maximum number of days after which a dormant fee needs to be paid,  $l_{\min}$  is the shortest lease duration, and  $l_{\max}$  is the longest lease duration.

## 8 CONCLUDING REMARKS

In this paper, we have introduced a natural generalization of the well-known facility location problem in the online setting. The latter appears as a sub-problem in many real-world optimization scenarios involving serving clients, as they appear over time, by leased resources.

The first research direction would be to close the gap between the upper and lower bounds for metric  $d$ -OFSL. This can be done by either designing another algorithm, or by improving the analysis of the current one. Proving a better lower bound would also be worth trying.

Furthermore, we have considered in this paper a fixed parameter  $d$  for all our facility services. It is important to note here that our algorithm does extend to the case where this parameter differs between one service and the other. Yet, it is not clear whether the same can be said if we consider other variations of the parameter. That is, it could be that we have to pay a small fee the first time we leave a service unleased and then a higher fee in the next times. It would be interesting to investigate about these variations, by observing their connection to actual real-world examples.

This brings us to the next research direction, which would be to actually implement the proposed algorithm and evaluate it under real-world or simulated instances of the optimization problem.

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