Keywords: Social Network Analysis, Data Mining, Graph Mining, Frequent Conceptual Links, Frequent Itemset Mining.

Abstract: In the domain of social network analysis, the frequent pattern mining task gives large opportunities for knowledge discovery. One of the most recent variations of the pattern definition applied to social networks is the frequent conceptual links (FCL). A conceptual link represents a set of links connecting groups of nodes such as nodes of each group share common attributes. When the number of these links exceeds a predefined threshold, it is referred to as a frequent conceptual link and it aims to describe the network in term of the most connected type of nodes while exploiting structural and semantic information of the network. Since the inception of this technique, a number of improvements were achieved in the search process in order to optimise its performances. In this paper, we propose a new algorithm for extracting frequent conceptual links from large networks. By adopting a new compressed structure for the network, the proposed approach reaches up to 90% of gain in the execution time.

1 INTRODUCTION

Complex networks are a set of many connected nodes that interact in different ways. In the context of network theory, a complex network is a graph (network) with non-trivial topological features that do not occur in simple networks such as lattices or random graphs but often occur in networks representing real systems (Mata, 2020).

In the last decades, the study of complex networks has become a very active research field with a strong interdisciplinary character. Many different phenomena in the physical, biological and social world can be understood as network based, i.e. a collection of objects connected by a number of links. (Barabási, 2002). Indeed, these different kinds of networks share fundamental properties that allow researchers to study various problems such as influence, propagation, terrorism and the spread of infectious diseases in the same way (Barabasi, 2002). They are based on a complex and evolving pattern of bilateral connections between entities, whereby the overall performance of the system is largely determined by the intricate architecture of these connections.

Among the most studied complex networks in recent years, we find social networks. A social network is defined as a network of interactions or relationships, where the nodes consist of actors or entities, and the edges consist of the relationships or interactions between these actors (Aggarwal, 2011). Social Network Analysis (SNA) studies the underlying conditions of such social networks to identify patterns of interaction between the network’s actors to analyse and explain social phenomena using graph theory metrics (Tabassum et al., 2018). This analysis has been referred to as structural as it focuses on the structure (links) of the network in order to highlight some patterns and features, nodes with high concentration of links, densely or weakly connected regions, etc.

Nowadays, social networks, especially online one, get larger and richer both with relational and content data. This situation makes the SNA handling new real world applications and facing new challenges like scalability, dynamism and streaming. Thus, data mining techniques appear as a very...
efficient tools in the social data analysis and provide researchers with large opportunities for the knowledge data discovery (Adedoyin, 2014).

Data mining functions include, inter alia, clustering, classification, link prediction, and frequent pattern mining, which explicitly consider links when building predictive or descriptive models of the linked data (Getoor, 2005). Moreover, they have the ability to exploit information about nodes (attributes) or relationships between nodes (links) to extract the maximum knowledge from the network.

One of the most recent approaches combining both information on the structure and attributes of nodes is the frequent conceptual links (FCL) (Stattner, 2012; Stattner, 2012c). The FCL is a descriptive data mining technique which aims to extract, from a social network, knowledge about the most connected type of nodes. It gathers, for accomplishing this task, concepts from the community detection, the frequent pattern mining and the formal concept analysis techniques (Stattner, 2012b). In fact, this new pattern exploits the community structure property of networks and starts by grouping nodes into clusters or modules with homogenous attributes (Fortunato, 2009). Then, as for the frequent pattern mining problem (Agrawal, 1994; Luna, 2019; Agrawal, 2014; Cafaro, 2019), it defines a support threshold and eliminate all the groups (clusters) but those with a number of links between their nodes more than a predefined threshold. The result is a set of links called Frequent Conceptual Links, connecting groups of nodes such as nodes of each group share common attributes. Hence, each part of the conceptual link is composed by a set of attributes and the nodes verifying these attributes, this is what is called a concept by the concept theory (Kumar, 2011; Sumangali, 2017) in the formal concept analysis (FCA). The advantage of this kind of analysis is that the set of the conceptual links of a network forms a concept lattice as defined by the formal concept analysis and preserves all its properties (Stattner, 2012b).

Moreover, a conceptual view of the network is extracted. It consists of a reduced representation of the original network which facilitates the task of extracting knowledge from the network, chiefly for larger networks. The nodes, or meta-nodes in this view are groups of nodes in the original network sharing common attributes and a link between two meta-nodes represents the whole links between these two groups (Stattner, 2012b).

The conceptual view is a key aspect of the frequent conceptual links pattern, as it synthesizes and summarizes the knowledge acquired from the network in one visualisation tool allowing the researcher to directly read the most connected features of the network.

Figure 1 presents an extract of the conceptual view extracted by the application of the FCLs approach to the amazon co-purchasing network (Leskovec, 2007). This summarized visualisation depicts the type of products frequently co-purchased by customers relatively to a support threshold. Every node in the conceptual view corresponds to a group of nodes in the original network characterised all by the attributes that label the node. For instance, the node (book, science) designates all the books of the science category. Furthermore, a link between a couple of nodes in the conceptual view represents the whole links between the nodes of each group in the original network. For instance, the link between the nodes labelled (book, science) and (book, computers & internet) designates all the books of the science category that are co-purchased with the books of the computers & internet category. The appearance of this link on the conceptual view means that these types of products are frequently co-purchased by the amazon customers.

The FCL problem has been proven NP-hard (Stattner, 2012a) since the search process depends on the number of attributes, their valence (the sum of possible number of attribute values), and the network size (Stattner, 2012b) which leads to the explosion of the search space over large networks. The challenge is, then, to get the solution in a reasonable time. Thus several attempts were driven in the literature to optimize the search process, each trying to exploit properties of the network in order to prune a part of the search space. Accordingly, several efficient and tailored exhaustive search methods are proposed to overcome this difficulty. Each of them tries to achieve
a good trade-off between the runtime and the number of extracted FCLs relatively to the predefined threshold. Indeed, the number of the FCLs depends on the threshold set in the beginning of the algorithm. A higher threshold allows extracting the FCLs in a short time, but the result is not significant enough, since most of the links will be rejected and significant latent information may be omitted in this case, while a lower threshold will extract more frequent links at the expense of performance.

FLMin (Stattner, 2012d), uses a bottom-up search and the Apriori principle (Samatova, 2014) by browsing only the itemsets that all of their subitemsets are frequent. Using the same principle as well as the frequency property (Stattner, 2012b), MFCLMin (Stattner, 2012c) looks for the maximal frequent conceptual links i.e. those which are not included in other frequent conceptual links. Subsequently, authors in (Stattner, 2013) and (Tabatabaei, 2017) proposed respectively the algorithms H-MFCLMin and D-MFCLMin that implement the concepts of filtering threshold and itemset dependency to reduce the search space, thus significantly improving the performance of the search process with the trade-off of loss of searched patterns. Comparing with the results of the complete research process, the authors have shown that the loss is admissible from a certain support threshold. Finally, PALM (Stattner, 2017) is a parallel implementation that tries to improve performance of the extraction process by simultaneously exploring several parts of the search space.

To the best of our knowledge, these are exhaustively the list of works addressing the FCL extraction problem. While the last one constitutes a parallel implementation, the former are sequential and they adopt an apriori based approach, i.e., scanning the database for each FCL candidate and computing the relative support. Furthermore, each one of the sequential implementations improves the performances of the previous, by exploiting more properties of the network. At this stage, we should notice that despite that the solution space given by the MFCLMin and the D-MFCLMin algorithms is smaller than that obtained by FLMin, this doesn’t cause any loss in the solution space because as for the itemset mining problem, from the maximum FCLs we can reach all the FCLs in the network.

Contrariwise, the H-MFCLMin sacrifices some solutions for a performance gain. Finally, MFCLMin and D-MFCLMin remain the only sequential implementations that list all the maximal FCLs for a given network. Despite all the properties exploited by these two algorithms (frequency property, downward-closure property and the dependency property) in order to maximize the extraction process performances, the main problem of them and of any apriori based algorithm is the multiple scan of the database. Indeed, MFCLMin and D-MFCLMin proceed in a breadth first manner, generate all conceptual links candidates of size k, scan the network for each candidate and eliminate all but those frequent before moving to larger candidate conceptual links. This may induce heavy charge on the process for large networks.

In this paper, we present a new solution for the maximum FCLs extraction problem, namely the Bin-MFCLMin, it constitutes a sequential implementation that looks for all the maximum FCLs within a social network, and uses a compressed binary representation of the social network in order to reduce the time of extracting frequent conceptual links. As we will see through this paper, the compressed representation transforms the input network data into an integer matrix whose size is reduced by a factor more than 60 than the original network, which allows a gain in run time up to 91%. The paper is organized as follows: section 2 gives details about the problem modelling, section 3 explains the proposed solution and section 4 shows and discusses the obtained results, we finally conclude and present our perspectives in the last section.

2 PROBLEM MODELLING

In order to model the FCL extraction problem, we consider a social network represented by a graph $G = (V,E)$ where $V$ is the set of nodes and $E$ is the set of relations between the nodes.

We use a set of attributes $A (a_1, ..., a_m)$ and a set of attribute values $\{a_11, ..., a_1j_i, a_21, ..., a_2j_i; ..., a_{m1}, ..., a_{mj_i}\}$ where $j_i$ the number of values that can take the attribute $a_k$.

Each node is described by a set of pairs (attribute, value), each attribute $a$ = value pair is said to be an item and the set of (attributes, values) describing a node(s) constitutes an itemset.

An itemset which contains one pair (attribute, value) is called 1-itemset, while an itemset containing $t$ pairs (attribute, value), it is called $t$-itemset. If $m_1$ and $m_2$ are two itemsets, then the set of ties linking the nodes satisfying the itemset $m_1$ and the nodes satisfying the itemset $m_2$ constitutes a conceptual link noted $(m_1, m_2)$:

$$\{(e \in E, \ e = (a, b) \ a\text{ satisfy } m_1 \text{ and } b\text{ satisfy } m_2, \ a, b \in V\}$$  (1)
In the conceptual link \((m_1, m_2)\), \(m_1\) is called the left itemset while \(m_2\) represents the right itemset.

The support of a conceptual link \((m_1, m_2)\) represents the number of links connecting the nodes verifying \(m_1, m_2\):

\[
\text{Support}((m_1, m_2)) = | \{ e \in E, e = (a, b) a \text{ satisfy } m_1 \text{ and } b \text{ satisfy } m_2, a, b \in V \} |
\]  

(2)

Let be a social network where the nodes are constituted of individuals and a link between two nodes describes a relation between the corresponding individuals. Furthermore, each individual is described by three attributes: age class \((\text{age}/10)\), gender \((\text{male}/\text{female})\) and the work status \((\text{employee}, \text{unemployed})\). According to the above definitions, \((\text{gender}=\text{male}), (\text{work-status}=\text{unemployed})\) are items and the set \((\text{age}=3, \text{gender}=\text{female}, \text{work-status}=\text{employee})\) is un itemset. Hence, the conceptual links \(((\text{age}=3, \text{gender}=\text{female}, \text{work-status}=\text{employee}), (\text{age}=4, \text{gender}=\text{male}, \text{work-status}=\text{employee}))\) describes all the links between female workers aged between 30 and 39 years and male workers aged between 40 and 49 years old.

The number of these links constitutes the support of the conceptual link.

A conceptual link is said to be frequent if its support is greater than a predefined threshold \(\beta\).

\[
(m_1, m_2) \text{ frequent } \equiv \text{Support}((m_1, m_2)) \geq \beta
\]  

(3)

A conceptual link \((m_1, m_2)\) is included in another conceptual link \((m_1', m_2')\) if: \(m_1 \subseteq m_1'\) and \(m_2 \subseteq m_2'\), thus:

- \((m_1, m_2)\) is a sub-conceptual link of \((m_1', m_2')\)
- \((m_1', m_2')\) is a super-conceptual link of \((m_1, m_2)\).

From this definition, we can deduce two properties whose proofs are detailed in (Stattnner, 2012c):

- If a conceptual link is frequent, all its sub-conceptual links are frequent.
- If a conceptual link is infrequent, all its super-conceptual inks are infrequent.

Finally, a frequent conceptual link is said to be maximal if it is not included in any other frequent conceptual link.

3 THE PROPOSED SOLVING APPROACH, BIN-MFCLMIN

We propose in this work, a new solution for extracting maximum FCLs from a social network, namely the Bin-MFCL algorithm. It is based on a bottom-up search using the Apriori principle to gradually prune the search space. The main novelty of the proposed solution is in the data structure. Indeed, we use in this work a compressed binary presentation of the network (Leon, 2008) in order to decrease the number of processing operations during the extraction of FCLs. In this representation, the entire network (nodes and links) is represented by a matrix of integers of size \(m \times n\), where \(n\) is the number of all possible attribute values and \(m\) is the number of links in the original network divided by a compression factor, thus, reducing the complexity of the input data and reducing the search time. The next section gives more detail about the input structure construction process.

3.1 The Compressed Binary Representation

The frequent conceptual link extraction technique proceeds on an attributed social network which is depicted through two 2-dimensional array: the profile array lists all the nodes with their attributes, and the relation array lists all the existed links between nodes.

Figure 2, and table 1 give an example of an attributed social network. The nodes in this network are individuals described by the five attributes listed in the table 2.

![Figure 2: Example of a social network with 10 nodes related by 9 links.](image)

<table>
<thead>
<tr>
<th>Id user 1</th>
<th>Id user 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
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<td>1</td>
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<td>9</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 1: Link matrix of the social network example.
Table 2: Attribute matrix of the social network example.

<table>
<thead>
<tr>
<th>User id</th>
<th>Profile privacy</th>
<th>Completion rate of profile</th>
<th>gender</th>
<th>Registration year</th>
<th>Age class (Age/10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Public</td>
<td>&lt;30%</td>
<td>Male</td>
<td>&lt;2007</td>
<td>2</td>
</tr>
<tr>
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<td>Public</td>
<td>&gt;=60%</td>
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<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Private</td>
<td>&gt;=30% and &lt;60%</td>
<td>Male</td>
<td>&lt;=2007</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>Public</td>
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<td>2</td>
</tr>
<tr>
<td>5</td>
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<td>&gt;=30%</td>
<td>Male</td>
<td>&gt;=2007</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
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<td>&lt;30%</td>
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<td>&gt;=2007</td>
<td>2</td>
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<tr>
<td>7</td>
<td>Public</td>
<td>&lt;30%</td>
<td>Male</td>
<td>&lt;2007</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>Private</td>
<td>&lt;30%</td>
<td>Male</td>
<td>&lt;2007</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>Private</td>
<td>&lt;30%</td>
<td>Female</td>
<td>&lt;2007</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>Public</td>
<td>&lt;=60%</td>
<td>Female</td>
<td>&lt;=2007</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 3: Merge the attribute and link matrices.

<table>
<thead>
<tr>
<th>Profile privacy</th>
<th>Completion rate of profile</th>
<th>gender</th>
<th>Registration year</th>
<th>Age class (Age/10)</th>
<th>User id 1</th>
<th>User id 2</th>
<th>User id 3</th>
<th>User id 4</th>
<th>User id 5</th>
<th>User id 6</th>
<th>User id 7</th>
<th>User id 8</th>
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<th>User id 10</th>
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<td>9</td>
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<td>9</td>
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<td>public</td>
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<td>&lt;2007</td>
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<td>1</td>
<td>6</td>
<td>1</td>
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<td>6</td>
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</tr>
</tbody>
</table>

Table 4: The attribute link matrix in one-hot encoding and the extraction of the compressed representation.

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<th>Profile privacy</th>
<th>Completion rate of profile</th>
<th>gender</th>
<th>Registration year</th>
<th>Age class</th>
<th>User 1</th>
<th>User 2</th>
<th>User 3</th>
<th>User 4</th>
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</tr>
</tbody>
</table>

1023 0 1023 0 0 1023 0 1023 0 0 722 301 188 227 578 328 695 95 928 516 475 32
The transformation of this network into an integer matrix is described in the following steps:

1. Merge the profile and the link data in the same matrix [Table 3].

2. Transform the merged matrix into a binary array using the one-hot encoding (Potdar, 2017). This is made by transforming each attribute column with \( d \) possible values into \( d \) binary columns, the value of an attribute for each node is indicated by the presence (1) or absence (0) of the binary variable [Table 4]. For our social network example, the attribute profile privacy is divided into two columns, one indicates if the attribute value is public and the other indicates the private value of the attribute.

3. Finally, if \( F \) is the compression factor, convert for each column, \( F \) rows into an integer [Table 4]. For instance, if we take \( F = 9 \), the original link-profile matrix is transformed into a matrix with one row and 24 columns which represent the number of all possible attribute values.

In general, the result of the compression process is an \( MN \) matrix where \( N \) is the number of items and \( M \) is the number of links in the original network divided by the compression factor. It is important to note that this compression is done without any loss of data and it is possible to revert to the original network entirely. Moreover, the operation of searching links satisfying an itemset turns into an operation of logical ANDs between the columns corresponding to each item in the itemset in question.

At this stage, the compressed data structure can be exploited by the proposed algorithm, in order to optimize the MFCL extraction process in large networks. The pseudocode of the Bin-MFCLMin algorithm is given below [Algorithm 1]. As it is depicted in the listing code, the algorithm implements a bottom-up search by looking for the frequent conceptual links involving itemsets of size 1 at first (lines 4-15). After that, every frequent itemset will generate longer candidate itemsets by a join operation (lines 18-19).

Again, these itemsets constitute candidate conceptual links to be checked according to a predefined threshold (lines 20-31).

This process is repeated until no more candidate can be generated (while loop line 17)

Algorithm 1: The Bin-MFCLMin algorithm.

**Generation of the 1-frequent conceptual links**

1. \( \textbf{LeftFreqItemset} \leftarrow \text{list of 1-itemsets } m \text{ where } \text{CountItemSupport} (m) \geq \beta \ast S \)
2. \( \textbf{RightFreqItemset} \leftarrow \text{list of 1-itemsets } m \text{ where } \text{CountItemSupport} (m) \geq \beta \ast S \)
3. For each \( \textbf{leftItem} \) in \( \textbf{LeftFreqItemset} \)
4.   For each \( \textbf{rightItem} \) in \( \textbf{RightFreqItemset} \)
5.     \( \textbf{Support} = \text{countConceptualLinkSupport}((\text{leftItem}, \text{rightItem}), \text{data}) \)
6.     If(\( \textbf{Support} \geq \beta \ast S \))
7.       Add the conceptual link \((\text{leftItem}, \text{rightItem})\) to \( \text{listMFCL} \)
8.       Add \textbf{leftItem} to \( \textbf{t-leftItemSetCand} \)
9.       Add \textbf{rightItem} to \( \textbf{t-rightItemSetCand} \)
10. End if
11. End for
12. End for

**Generation of the t-frequent conceptual links**

1. \( t = 2 \)
2. While (\( \textbf{t-leftItemSetCand} \neq \emptyset \) OR \( \textbf{t-rightItemSetCand} \neq \emptyset \))
3.   \( \textbf{t-leftItemSetCand} \leftarrow \text{list of t-itemsets constructed from (t-1)-frequent left itemsets by a join operation} \)
4.   \( \textbf{t-rightItemSetCand} \leftarrow \text{list of t-itemsets constructed from (t-1)-frequent right itemsets by a join operation} \)
5.   for each \( \textbf{leftItemSet} \) in \( \textbf{t-leftItemSetCand} \)
6.     for each \( \textbf{rightItemSet} \) in \( \textbf{t-rightItemSetCand} \)
7.       \( \textbf{support} = \text{countConceptualLinkSupport}((\text{leftItemSet}, \text{rightItemSet}), \text{data}) \)
8.       If(\( \textbf{support} \geq \beta \ast S \))
9.         Add the conceptual link \((\text{leftItemSet}, \text{rightItemSet})\) to \( \text{listMFCL} \)
10.        Add \textbf{leftItem} to \( \textbf{t-leftItemSetCand} \)
11.        Add \textbf{rightItem} to \( \textbf{t-rightItemSetCand} \)
12.        Remove all sub-conceptual links of the newly added frequent conceptual link
13.     End if
14.   End if
15. End for
16. End for
17. \( t = t+1 \)
18. End while
19. Return \( \text{listMFCL} \)
The main benefit of the compressed input structure is in the support counting implemented in the three following algorithms [Algorithm 2, Algorithm 3, Algorithm 4].

Algorithm 2 is the first called by the principal algorithm for the sake of computing the support of each single item. This allows us to discard the non-frequent items as early as possible and use solely the frequent items for generating longer candidate itemset in the subsequent iterations. Since each item is represented by a column in the compressed input data (line 2), the support counting is done by scanning all the rows of this column (number of rows = M = number of Links in the original network / compression factor) and adding the number of bits set to 1 in the binary representation of each row data (line 3-4). The support of every couple 1-itemset is then, counted using the algorithm 3 in order to determine the list of one frequent conceptual link.

Algorithm 2: CountItemSupport.
Input: item, network data in binary compressed representation
Output: support of item
1. Support = 0
2. itemCountColumn \( \leftarrow \) the integer array corresponding to the item in the input data
3. for each row in itemCountColumn
4.     support = support + number of 1 in the binary representation of row
5. end
6. return support

The support counting of a one conceptual link, involves two columns of the input data, the one which corresponds to the left itemset and the one which corresponds to the right itemset (lines 2-5). The algorithm 3 performs an and operation between these two columns in order to retain only the links where the two itemsets are satisfied (line 7), before counting the number of bits set to 1 in the binary representation just obtained (line 8).

Furthermore, in the case of a of a t-conceptual link (t>1), the left and right itemsets involved in the conceptual link correspond to more than column in the compressed input data. Thus, before counting the number of links where the left and the right itemset are satisfied, the algorithm 4 allows us to obtain the column associated to each itemset. In the lines 3-6, the algorithm constructs a matrix where each column correspond to an item of the itemset. Then, it performs an AND operation within this matrix to obtain the column representation of the whole itemset (line 7-11). Finally the support counting is made with the columns associated to the left and right itemset similarly to the one conceptual link support counting.

Algorithm 3: CountConceptualLinkSupport.
Input: conceptualLink, network data in binary compressed representation
Output: support of the conceptual link
1. Support = 0
2. leftItemSet \( \leftarrow \) left ItemSet of the conceptual link
3. rightItemSet \( \leftarrow \) right ItemSet of the conceptual link
4. LeftItemSetColumn = CountItemSetSupport(leftItemSet)
5. rightItemSetColumn = CountItemSetSupport(rightItemSet)
6. for each row i in LeftItemSetColumn, rightItemSetColumn
7.     binaryRepresentationRow = LeftItemSetColumn[i] AND rightItemSetColumn[i]
8.     support = support + number of 1 in the binaryRepresentationRow
9. End for
10. return support

Algorithm 4: CountItemSetSupport.
Input: itemset, network data in binary compressed representation
Output: support of itemset, itemSetColumn
1. itemSetMatrix \( \leftarrow \) []
2. support = 0
3. for each item in itemset
4.     itemCount \( \leftarrow \) the integer array corresponding to the item in the input data
5.     add the itemCount to itemSetMatrix
6. End for
7. for each row in itemSetMatrix
8.     for each column N in row
9.         binaryRepresentationRow = binaryRepresentationRow AND itemSetMatrix [row][N]
10.     add the current binaryRepresentationRow to the itemSetColumn
11. End for
12. support = support + number of 1 in the binaryRepresentationRow
13. End for
14. return support, itemSetColumn
4 EXPERIMENTAL RESULTS AND DISCUSSION

In this section, we present the experiments performed to validate our proposed new solution for solving the FCL problem. We will first justify the choice of MFCLMin for the comparison, then, we will present the dataset used in the tests. Finally, the results will be compared and discussed in term of the execution time for both the algorithms.

The table 5 resumes the main characteristics of the previous implementations and compares them with our proposed solution. From this table, we can notice that the MFCLMin and D-MFCL-Min algorithms have the same properties of our solution and are consequently the most suitable for the comparison. Nevertheless, the experiments accomplished with the D-MFCLMin algorithm on the pokec network, stated that the execution time increases up to 15% and this lost in the search process performances was justified by the absence of dependency in the assessed network. Hence, for a fair comparison, the MFCLMin algorithm will be used in the experiments with the same data and the same min sup threshold.

The input dataset is a Slovak online social network (Tabatabaee, 2017) which includes one million nodes and 31 million links, the nodes are described by a set of 51 attributes which are not all filled in. A task of cleaning and preparation is therefore necessary and consists essentially of discarding null values, transforming the values of some attributes in the suitable classes (see below the age, comp and the reg attributes) and preparing the input data structure.

The number of attributes considered is equal to 5, with a total valence of 13. Table 6 below describes the attributes and their possible values.

All tests are carried out on several instances by varying each time the number of nodes and links and calculating the execution time taken by the two algorithms. This is done with four possible support thresholds.

Figures 6 and 7 show respectively the runtime of both the MFCLMin and Bin-MFCLMin algorithms and the gain achieved by the proposed solution. The results were obtained by a compression factor of 63 while varying support threshold between: 0.1, 0.15, 0.2 and 0.25. One can notice here that the smaller the threshold, the more tedious the task of detecting frequent links is, and therefore more time consuming since the number of frequent links become large.

Firstly, we can observe that the gain on runtime is at least of 43% which confirms the positive impact of the input structure on the search process performance. The second remark that can be done is that while the performances of Bin-MFCLMin remains relatively stable for bigger instances, the MFCLMin algorithm runtime increases considerably, thus proving the superiority of our data structure compared to the one used in MFCLMin algorithm. Indeed, the later uses a vertical layout representation for the network where it associates to each node, input and output links, then it calculates the itemset supports by intersecting the links list associated with each itemset (Stattner, 2012b), whereas, our solution looks for all the maximum FCLs, by scanning a reduced input data to a compression factor of 63 in the assessed instance and uses the AND logic operation to count the support of each itemset. Hence, this reduces the support counting process complexity.

To support this result, we also considered the results of a well-known algorithm in the field of frequent pattern mining, namely, ECLAT algorithm (Zaki, 2000). It should be noted that the data structure employed in the MFCLMin algorithm is similar to that used by ECLAT while looking for frequent itemsets in the context of association rules mining. In fact, the author of this work implements a vertical layout representation of the input data where it associates to each item, the list of transactions where it appeared then, the support of an itemset is defined by the size of the intersection list of all the lists associated with each item constituting the itemset.

Table 5: Characteristics of FCLS extraction algorithms.

<table>
<thead>
<tr>
<th>Solution space</th>
<th>Type of implementation</th>
<th>Type of search</th>
</tr>
</thead>
<tbody>
<tr>
<td>FLMin</td>
<td>FCLs</td>
<td>Exhaustive</td>
</tr>
<tr>
<td>MFCLMin</td>
<td>Maximum FCLs</td>
<td>Sequential</td>
</tr>
<tr>
<td>H-MFCLMin</td>
<td>Maximum FCLs</td>
<td>Exhaustive</td>
</tr>
<tr>
<td>D-MFCLMin</td>
<td>Maximum FCLs</td>
<td>Sequential</td>
</tr>
<tr>
<td>PALM</td>
<td>Maximum FCLs</td>
<td>Exhaustive</td>
</tr>
<tr>
<td>Bin-MFCLMin</td>
<td>Maximum FCLs</td>
<td>Sequential</td>
</tr>
</tbody>
</table>

3 This is the same network used in our experiments
Table 6: Pokee social network attributes.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Description</th>
<th>Possible values</th>
<th>Valence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public</td>
<td>Indicates if the user profile is public or private</td>
<td>Public / private</td>
<td>2</td>
</tr>
<tr>
<td>Comp</td>
<td>The completion rate of the user profile</td>
<td>&lt;30%</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt;=30% and &lt;60%</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt;=60%</td>
<td></td>
</tr>
<tr>
<td>Gender</td>
<td>Indicates the user gender</td>
<td>Male / female</td>
<td>2</td>
</tr>
<tr>
<td>Reg</td>
<td>Indicates the registration year in the network</td>
<td>&lt;2007</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt;=2007</td>
<td></td>
</tr>
<tr>
<td>Age class</td>
<td>The class age of the user</td>
<td>&lt;10</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt;=11 and &lt;20</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt;=20 and &lt;30</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>&gt;=30</td>
<td></td>
</tr>
</tbody>
</table>

Figure 3: Runtime of the MFCLMin and Bin-MFCLMin algorithms.

Figure 4: Gain on the runtime obtained by Bin-MFCLMin algorithm.
We note that although ECLAT is more efficient than many leading algorithms in the area, it is known to be unsuitable for items with large lists of transactions to be intersected (Luna, 2019). This proves the superiority of our proposed data structure especially in that case, which is emphasized by the gain in runtime, which reaches 91% for the last network.

5 CONCLUSION

With the proliferation of social networks in recent years, data mining techniques have become essential for the knowledge extraction process. The pattern presented in this paper plays a dual role to accomplishing this task, on the one hand, it exploits the structural and attributed information of the network in order to extract a more relevant information which seems, hence, to be very promising in the analysis of real world application like the spread of diseases or the recommendation systems. On the other hand, the conceptual view provided by this approach summarizes the extracted knowledge in one simple and content-rich visualisation.

We proposed in this work a new algorithm for extracting frequent conceptual links from large networks, by adopting a new binary compressed structure for the network, hence reducing the input data complexity. The proposed approach allows extracting the FCLs in a shorter time comparing to MFCLMin for all the used instances. The improvement reaches up to 90% of gain in the execution time for items with large lists of transactions to cross-reference.

Nevertheless, some network instances are so large that makes approximated methods inevitable for dealing with the computational challenges that this problem reveals. Thus, the data structure presented in this work, may be used within a heuristic solution as an alternative to tackle this problem in the future.

REFERENCES

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