Agent-based Intelligent KPIs Optimization of Public Transit Control System

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Abstract: Public transit has a wide variety of resources. There is an infrastructure including stations and routes with multiple trips provided by different modes of transportation (metro, subway, bus). These resources must be well exploited to ensure good quality of service to passengers and especially against perturbations that may occur during the day. The contribution of this work is to model and implement a transit control system that detects perturbations and finds, through optimization, the best regulation action while respecting the constraints of the traffic situation. This system combines various measures of Key Performance Indicators (KPIs) into a single performance value, covering several dimensions depending on the type of service quality to be guaranteed. To take into account the complex and dynamic nature of transportation systems, a multi-agent approach is adopted in the modelling of our system. The validation is based on real traffic data. The results show better performance of our system compared to the current resolution.

1 INTRODUCTION

Today, public transportation is one of the most important elements of the municipal plan. In densely populated urban areas, it carries a large number of people and becomes an indispensable service in daily life. In addition, public transport networks have been expanded. The number of vehicles, stations and itineraries continues to grow. This makes the management challenges even more complex.

With the emergence of many complex and random phenomena that disrupt transit traffic, it is becoming difficult to keep up with scheduled vehicle timetable in real time. For that reason, the quality of public transit service is deteriorating. Furthermore, the complexity of the road network means that several perturbations can occur at the same time and that one perturbation can generate others.

In addition, the transit system must be able to adapt to changing traffic conditions to ensure the required quality of service. They must therefore detect disturbances quickly and deal with new situations in order to improve the quality of service through performance measures. These measures are known as KPIs, which are quantitative measures or indices that numerically express a specific quality.

There is an extensive literature on various aspects of KPIs. (Mark Tromp et al., 2011) evaluates performance by EWT: Excess Wait Time, AWT: Actual Wait Time and SWT: Scheduled Wait Time. Moreover, in (M. Napiah et al., 2015) this performance is defined by the average waiting time expected by passengers. (Oded Cats et al., 2010) defines performance by the observed time interval deviations between trips of the same line compared to the regular frequency of vehicles during a given period.

(Neila Bhouri et al., 2016) describes the Gini index as another indicator in the form of a forward regularity index. (S. Carosi a, et al., 2015) describes regularity as an index on vehicle entries at stations.

Other projects define another indicator which is punctuality as a determining criterion in the final performance formulation. Punctuality is defined in (Noorfakhriah Y. and Madzlan N., 2011) as a comparison of actual departure times and scheduled departure times at the station. In (Xumei Chen et al.,

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2009) the authors distinguish three measures of punctuality. PIR: Punctuality Index based on Routes, DIS: Deviation Index based on Stops and EIS: Evenness Index based on Stops. However, in (Vaniyapurackal, 2015), the author converts the punctuality index to a percentage to define the proportion of the trip that was punctual.

In (Saberi, Meead, et al., 2012), three alternative performance measures are proposed: EI: Earliness Index, WI: Width Index and SSD: Second-Order Stochastic Dominance Index. These indices are used in two forms to measure the unreliability of a bus service: (i) the distribution of lane deviations for frequent services and (ii) the distribution of delays for infrequent services.

Other work, such as of (Ceder, 2007), add another indicator called transfer time that covers the time spent when the passenger is waiting for the vehicle while changing the line at a transfer station. (Zhenliang, 2013) details and explains the Input Buffer Time (IBT) formula that can be used to understand the additional unreliability caused by an incident. The authors of (Kenneth et al., 2004) and (Levinson, Herbert, 1983) discuss another indicator called "Dwell"" which is the parking time of vehicles. "Dwell" can also be used to hold for traffic to be restored (Vu The Tran et al., 2012) (Cats et al., 2011).

From this literature, we note that there is no standard meaning for specifying and formulating performance indicators. The challenge in defining key performance indicators is to select those that will sufficiently satisfy the overall performance of public transport.

In addition, the task of transit control is based on the optimization of the KPIs. The transit system must provide comparative information that allows the control system to identify performance gaps and set measures and targets to resolve them.

Multi-agent modelling can provide a solution fitted to the activities of public transport networks where autonomous entities (buses, stations, itineraries, etc.), called agents, dispersed in a dynamic environment which is the traffic of the transport network. They adapt their behaviours to the perturbation they perceive and interact with each other to perform optimal regulation actions.

Our objective is to model and implement an intelligent control system that manages perturbations. It detects and finds the best regulation action while respecting the constraints of the traffic situation. This system combines various KPI measures into a single performance value, covering several dimensions depending on the type of service quality to be guaranteed.

This paper is organized as follows. Section 2 defines the perturbation and describes the control method. Section 3 presents the performance measures. Section 4 details mathematical model by formulating the optimization problem. Section 5 presents the multi-agent model by describing the agents with their behaviours. Section 6 validates the control strategy of our system on a real network in the city of Portland in Oregon. In Section 7, we conclude and provide some perspectives.

2 PERTURBATION AND CONTROL METHOD

2.1 Perturbation

In general, a perturbation is the unexpected, sudden, or progressive appearance of events that can modify or cancel a program. In the field of public transport, a perturbation is an event that appears suddenly modifying the traffic state of the network in a situation that is generally unsatisfactory in terms of service quality. The perturbation then affects the normal operation of the network, which consists of keeping the scheduled timetable of vehicles.

Therefore, to improve service performance, it is necessary to optimize the adequacy between scheduled and actual operations as shown in Figure 1. In his study, (Van Oort, 2009) values the perturbation as guidelines to control the variation of scheduled operations from actual.

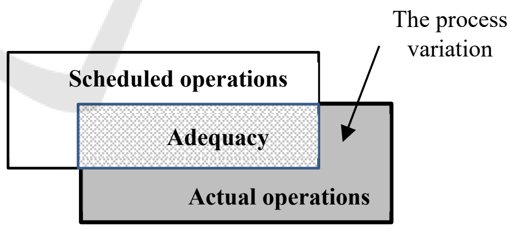

Figure 1: Inadequacy between scheduled and actual operations.

2.2 Control Method

(Ashby, W.R., 1956) presents three main methods of general control. These control methods are designed to mitigate the negative effects of perturbations on the process and allow to achieve the desired results.

- Use of a Buffer: consists in providing resources in the form of a measurement buffer allowing to detect the perturbation without generating repercussions on the desired result;

- Feedback: In case of unacceptable variation between scheduled and actual operations, a regulator intervenes to restore the desirable situation;
- Feed-forward: Instead of observing the trends of the variation, the regulator prepares the regulation action based on the simulation.

The three methods have specific advantages and disadvantages. In practice, they can also be used in combination. In our study, the control method is mainly used to evaluate current performance and to adjust operations in case of inadequate performance.

Therefore, it is essential to have the resources for detecting the perturbation and a regulator to reduce the variation as much as possible. Consequently, in our model it is necessary to have a buffer for the detection. While, after detection the selection of the control action is based on the calculation of the performance measurements of several regulation maneuvers. This is the feedback method.

As well, it is necessary to have scenarios of control maneuvers in memory to be able to compare the predicted results with the results of the scenarios. This is the feed-forward method.

Therefore, the three control methods that are mentioned above are used in combination in the modelling of our control system. We describe in the figure 2 the control method used in our modelling.

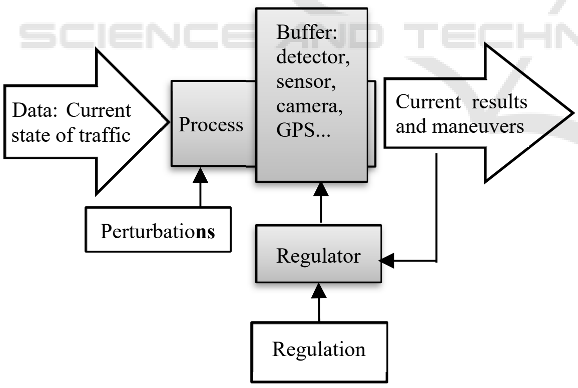

Figure 2: Control method of our system.

3 THE PERFORMANCE MEASURES

3.1 Selected Traffic Performance Indicators

The list of selected KPIs that measure performance quality are based on the indices of the "operational efficiency" objective. According to literature reviews by (Cambridge Systematics Inc. 2005), this objective will be focused on three indicators: punctuality, regularity, and correspondence. The other indicators that are related to the costs of transport, such as oil consumption and the number of kilometers traveled, are not included in our study, because our control system is oriented towards the user of the transport system and not the operator, and they are irrelevant for the passengers.

The formulas for these measures are taken from (European Commission, 2011) and some other research works such as (Yan et al., 2009), (Noorfakhriah Y. and Madzlan N., 2011) and (Ceder, 2007). They represent a standard that can evaluate overall traffic performance in transportation engineering and Intelligent Transportation Systems.

3.2 Punctuality Index

Punctuality is defined in (Noorfakhriah Y. and Madzlan N, 2011) (Saberi, Meead, et al, 2013) as a comparison between the actual and the scheduled arrival times at the station. Its formula is:

$$I_{PUN} = \frac{S_1^2}{\overline{PUN}^2} \quad \text{avec} \quad S_1^2 = \frac{1}{n}\sum_{i=1}^{n}(t_i - t_t)^2 \qquad (1)$$

- n: the number of vehicles.
- $\overline{PUN}$: $\frac{1}{n-1}\sum_2^{n-1}(PUN_i - PUN_{i-1})$ the average punctuality for the n vehicles.
- $t_i = t_{arr\_i} + Dwel_i$: the actual departure time of the i-th vehicle, while $Dwel_i$ is the time spent board the passengers.
- $Dwel_i = t_{Mont} * N_{Mont} - t_{Desc} * N_{Desc}$, with t_{Mont} et t_{Desc} are the average time spent by the passenger to get on or off the vehicle and N_{Mont} et N_{Desc} are the number of passengers to be picked up and dropped off the vehicle.
- t_t: the scheduled departure time.

3.3 Regularity Index

The regularity index measures the differences in time intervals observed at the station between successive vehicles of the same line compared to the scheduled frequencies. The formula of the regularity index is:

$$I_{REG} = \frac{S_2^2}{\bar{h}^2} \quad \text{avec} \quad S_2^2 = \frac{1}{n-1}\sum_{i=2}^{n}(h_i - h_t)^2 \qquad (2)$$

- $\bar{h}$: $\frac{1}{n-1}\sum_2^{n-1}(t_i - t_{i-1})$ the average frequency for the n vehicles.
- h_i: $t_i - t_{i-1}$ (i=2,…I), the current time interval.
- h_t: the scheduled time interval.

3.4 Correspondence Index

The correspondence index represents the differences between the observed correspondence values and those of scheduled correspondence. Its formula is the following:

$$I_{COR} = \frac{S_3^2}{\bar{c}^2} \text{ avec } S_3^2 = \frac{1}{n}\sum_{i=1}^{n}(c_i - c_t)^2 \quad (3)$$

- c_i: the actual correspondence.
- c_t: the scheduled correspondence.
- $\bar{c}$: $\frac{1}{n-1}\sum_2^{n-1}(c_i - c_{i-1})$ the average correspondence for the n vehicles.

The 'c_i' or 'c_t' of the i-th vehicle is equal to:

$$C_i = \sum_{j=1}^{n} \Delta_{ij} \quad (4)$$

- Δ_{ij} : the remaining arrival time. It is equal to:

$$\Delta_{ij} = t_i - t_j + D_{ij} \quad (5)$$

- t_i: the actual arrival time of vehicle 'i'.
- t_j: the actual departure time of the vehicle in connection 'j'.
- D_{ij}: the walking time between the two connecting stops of the two vehicles 'i' and 'j'.

4 THE MATHEMATICAL MODEL

4.1 Formulation of the Optimization Function

The performance measures used in our optimization problem are based on the indices mentioned above. The objectif function of our problem, which is the performance value "F", is formulated as follows:

$$F = w_{I_{PUN}}.I_{PUN} + w_{I_{REG}}.I_{REG} + w_{I_{COR}}.I_{COR} \quad (6)$$

It is necessary that the sum of the weights is equal to 1 ($w_{I_{PUN}} + w_{I_{REG}} + w_{I_{COR}} = 1$). These weights indicate the importance of the indicators in the control process.

Such "objectif" function can be optimized by a combinatorial method. Combinatorial optimization is a subject that consists in finding an optimal object from a finite set of objects. It operates in optimization problems where the set of feasible solutions is discrete or can be reduced to discrete. In our application, the estimation of the objectif function F is performed by simultaneous simulations of each maneuver to choose the best one that minimizes F.

4.2 Formulation of Constraints

The following constraints are based on the work of (Ceder, 2007). We Consider the following notations to model the problem constraints:

- H_{min_i} *and* H_{max_i} : is the minimum and the maximum time interval of two consecutives vehicles in station 'i'.
- t_{ij} : $t_j - t_i$ is the elapsed time between the departure time t_j of station 'j' and the departure time t_i of station 'i'. 'i' and 'j' represent the two successive stations of the link l_{ij} respectively.
- T_{ce_i}: is the estimated total travel time of trip 'i'.
- T_{ct_i}: is the scheduled total travel time of trip 'i'.
- N_i: is the number of trips conducted at station i
- I_{PUN_i}: is the punctuality index at station 'i'.
- $I_{PUN_{max}}$: is the max punctuality index allowed at station 'i'.
- I_{REG_i}: is the regularity index at station 'i'.
- $I_{REG_{max}}$: is the max regularity index allowed at station 'i'.

The problem is infeasible unless the following constraints are satisfied for each trip:

$$I_{REG_i} \leq I_{REG_{max}} \quad (7)$$

$$I_{PON_i} \leq \min(I_{PON_{max}}, I_{REG_{max}}) \quad (8)$$

$$t_i \leq N_i.H_{max_i} \quad (9)$$

$$t_i \geq (N_i - 1).H_{min_i} \quad (10)$$

$$T_{ce\_i} \leq T_{cmax}, T_{cmax} = T_{ce\_t} + (n * I_{REG_{max}}) \quad (11)$$

$$t_i \in [0, I_{REG_{max}}] \quad (12)$$

These constraints are mandatory to verify the following situations:

- Not to exceed the maximum regularity limit allowed (eq. 7)
- Not to catch up with the regulated trip (eq. 8)
- Not to exceed the maximum time allowed during a regulation. (eq. 9)
- Respect the minimum regularity between vehicles of the same line (eq 10)
- Not to exceed the maximum time allowed for a given trip (eq. 11)
- Not to have a conjunction of two consecutive trips in the starting station (eq. 12)

5 THE MULTI-AGENT MODEL

The system must detect and resolve traffic perturbations. It is composed of a society of agents. These agents communicate with each other via messages. To guarantee our goal, the system must detect and manage perturbations by providing a good coordination between the agents. Each agent has a specific role in its environment.

The agents in our model are described as follows:

- VEHICLE: The vehicle agent memorizes all the data that characterizes it. It collects the data related to the current link and the values of the KPIs. It calculates the overall performance to find the value of the performance variation to detect perturbation. In case of disturbance, it transmits a call to the regulator to trigger the decision-making step. Also, each vehicle agent transmits, regularly, its properties with those of the current link to the arrival station agent to estimate the remaining time.

- LINK: It represents the transition between two successive stations. It should be linked to at least one line. It stores two types of information (i) Static properties: distance, maximum speed allowed and maximum density. (ii) Dynamic properties: average speed, current density. This data is sent to the vehicle agent. The link agent used to analyse and detect link congestion by calculating the speed performance index as an indicator to evaluate the traffic condition of the connection. This indicator is passed to the KPI agent to calculate its value.

- STATION: The station agent is linked to one or more lines. Each agent memorizes the passenger arrival and departure flows, as well as the scheduled and actual passage times of the vehicles. It receives the necessary vehicle properties to calculate the remaining arrival time. Then it gives the necessary data to the KPI agents (scheduled arrival time, remaining time) so that they can measure the performance of the vehicle.

- KPI: It calculates the value of the key performance indicator and transmits it to the concerned vehicle agent. In our system the KPIs are classified by objectives.

- REGULATOR: Each "regulator" agent is responsible for a geographical area of the network. It receives the KPIs of each vehicle in perturbation. Then it performs an optimization to find the best regulation action. It should be noted that after each regulation action, each agent must update his knowledge to be coherent to the new current traffic situation.

6 EXPERIMENTATION AND RESULTS

6.1 Description of the Simulation Model

Our control system includes a graphical interface that visualizes the inputs and outputs of the simulation. The network infrastructure and stations are displayed graphically, and the vehicle movements are animated. The simulation provides the numerical data result in sheet and chart resolution.

We chose AnyLogic as a modelling tool (https://www.anylogic.com/). AnyLogic is a mesoscopic simulation tool that integrates transportation system-specific libraries to simulate transit scenarios and animate system behaviour in a single package. In AnyLogic, a model is built with one or more active agent classes. These agents can be controlled from (i) an individual point of view by its distinctive behaviour in its environment and (ii) a global point of view by the emergence of the whole system phenomena. In addition, AnyLogic provides a Java application programming interface (API) that guides the use of state diagrams, variables, functions, and other various tools.

6.2 Description of the Transit System: Portland's Real-World Traffic

In this experiment, we test the control strategy of our system on a real network in the city of Portland in Oregon. The data was collected from the general transit department of the District of Oregon's "Tri-County Metropolitan Transportation" (TriMet) and imported into AnyLogic as GTFS files to model the map data of the TriMet network. We test our control system model on the "2 Division" line connecting Portland City Center and Gresham Transit Center (round trip). This line has eight stations with 86 outbound trips from 5:26 AM to 1:41 AM of the next day and 87 return trips from 4:09 AM to 12:42 AM of the next day, as well as connections to several lines (https://ride.trimet.org/).

6.3 Description of the Scenario and Results

The scenario presents traffic congestion observed on the "2-Division" line at the September 20, 2019 due to bad weather conditions caused by fog. It occurred in the morning on the 10th trip at stop #1375 (SE Division & 12th). The solution indicates that the service in the station is temporarily disrupted and passengers are advised to go to the nearby station at address 2314. There is no action applied to the vehicle.

We present, in Figure 3, the delays observed in each station for all trips of the entire journey on the "2-Division" line after TriMet regulation. While Figure 4 presents the delays obtained without any regulation. It shows the contribution of the current TriMet control. In fact, the delays are considerably reduced (the highest value has become 15 minutes instead of 45 minutes) and the regulation has become faster.

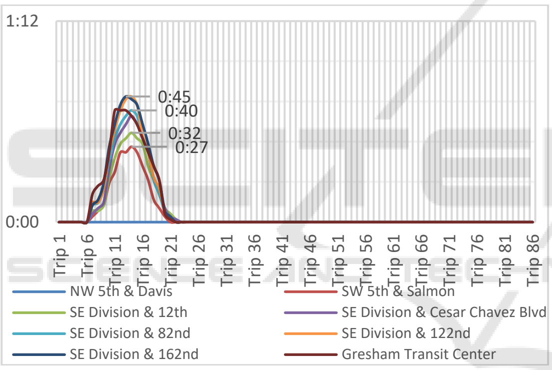

Figure 3: Observed delays using TriMet regulation.

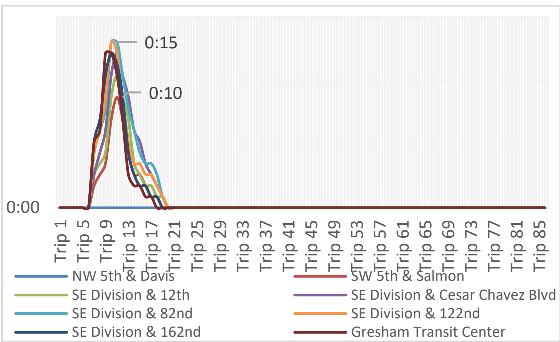

Figure 4: Observed delays with no controls.

Now we integrate our control system into the simulator and discuss the results. After simulation, the system detects a perturbation in the morning at 8:40 am on the 7th trip at the stop n° 1375 (SE Division & 12th). The performance variation "F" becomes 0.1701 which is higher than the critical value 0.15 (This value is supposed to be fixed by the traffic experts). We note that our system detects the disturbance three trips earlier than TriMet (7th trip instead of 10th trip).

After optimization, the regulator chooses "the deviation maneuver" for all vehicles in the disturbed area whose lowest value "F" is equal to 0.105. the list of the regulation actions is already defined and classified by experts (Van Oort, 2011). We note that the same value of F was estimated by the simulator to be 0.068 before the perturbation. We remark that the traffic performance variation is improved by a considerable decrease of the "F" value.

In Figure 5, we show the evolution of the observed performance variation "F" for each trip of the traffic with our control. The results obtained show an improvement in the quality of service by minimizing the values of the "F" variation during the perturbation. The area between the two curves represents the gain in performance variation when using our model.

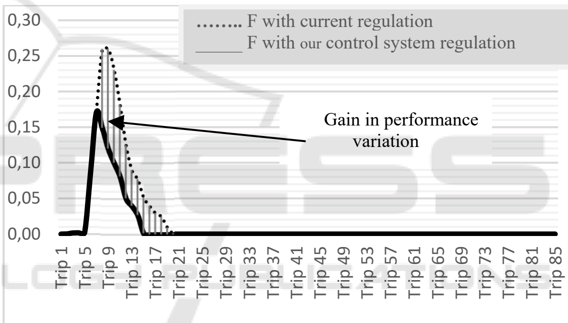

Figure 5: Performance variation with TriMet and with our control system for the 2-Division line.

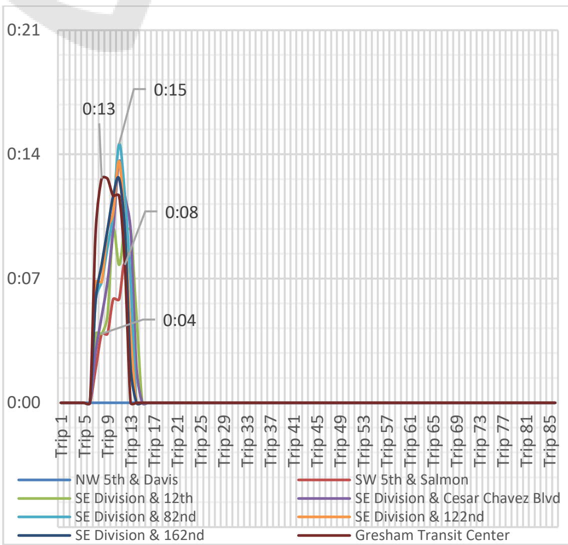

Figure 6: Observed delays using our control system.

Figure 6 shows the contribution of our control system by the considerable decrease of the delays of the disrupted buses. It indicates that the resolution period becomes faster. In fact, with our control system the perturbation is completely solved at the 15th trip instead of the 20th trip.

In the following, we present in the figures below (Fig. 7-8-9) the percentage increase of waiting passengers per station (PI) on the disrupted trips compared to the normal traffic without perturbation.

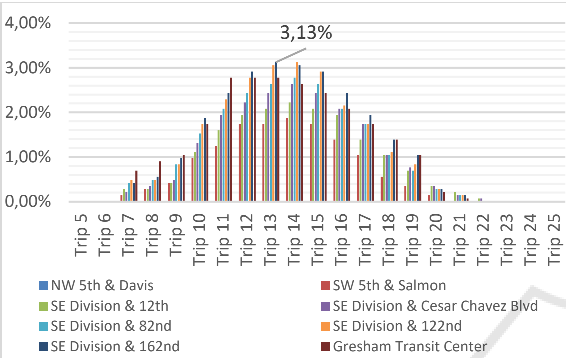

Figure 7: PI per station on disturbed trips with no control.

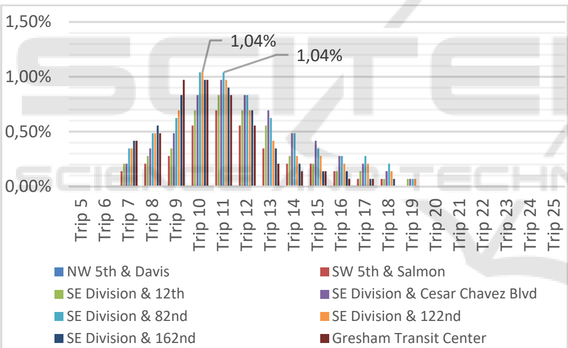

Figure 8: PI per station on disturbed trips with TriMed control.

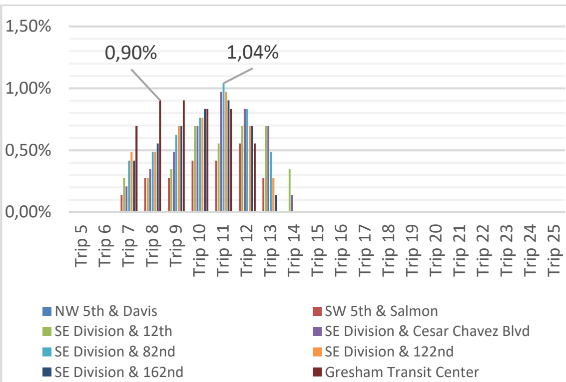

Figure 9: PI per station on disturbed trips with our control system.

We note that this percentage is relatively proportional to the bus delays. The simulation with our control system shows a clear improvement of the service quality by minimizing the PI value on the stations of the disrupted trips. In fact, the number of passengers waiting on each disrupted trip is significantly reduced with our control system.

7 CONCLUSION AND PERSPECTIVES

The main objective of this study was to model and develop a transit control system. This system simulates and controls the operational environment of a transit network. It detects in real time the traffic disturbances of the itineraries and generates the most appropriate regulation action. The modelling of the system based on a multi-agent approach dealing with an optimization problem. The optimization resolution includes mathematical model that describes the traffic dynamics, and a set of constraints represents the current traffic state. The main contribution of our system is the multi-agent modelling of control system using a mathematical model that treats all key performance indicators (KPIs) as variable elements with different weightings. To identify the variable elements, a detailed study is conducted on the literature of transit traffic performance measures.

To validate our control system, a simulation model reflecting real transit dynamics was built. The development is done with AnyLogic which is an agent-based modelling simulator. Our model gives visual and mathematical results justifying the choice of the control action. The results show that the proposed model is able to (i) evaluate the impact of the disturbance on the transit performance and (ii) regulate the disturbance with a better performance than the real one.

Finally, in a perspective, we mention two tracks: (i) to be able to manage disturbances in unfamiliar situations (unknown disturbance, new traffic parameter, etc.), we need to improve the behaviour of the system by providing an evolutionary approach in its resolution. This approach consists of making sure that, thanks to the regulator agent, the system can remember the results for these types of situations. The model should then suggest a fast neighborhood solution as a future action with new experiences and update the regulator's knowledge base by inserting these new rules to cope with future situations. (ii) to orient the control system towards the operator, we need to change the goal and include other performance measures related to the costs of transport. This track consists of adapting the optimization method to resolve a problem with antagonistic variables. Variables directed towards the user's view and variables directed towards the operator's view.

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