Modelling Social Protests in the Republic of Belarus in 2020 based on Diffusion Equations

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Abstract: In this article, we propose a model of social activity based on diffusion equations and a comparison of the modeling results with real data of protest activity (based on data in social networks) in Belarus in 2020. A model uses the diffusion Langevin equation. The model is based on the idea that individuals interact in society through a communicative field - h. Besides, the control is introduced into the system through the dissipation function. Protest data indicators were collected using the authors' content analysis of the main hashtags associated with Belarusian protests. Then the results of modeling were compared with the obtained data and analyzed. Based on the modeling we have revealed a general similarity in dynamics and characteristic patterns as well as have made a forecast for the development of the situation in 2021.

1 INTRODUCTION

Social conflict can be defined as a peak stage in the development of contradictions between individuals, groups of individuals, and society as a whole, is characterized by the existence of conflicting interests, goals, and views of the subjects of interaction. Conflicts may be hidden or explicit, but they are always based on the absence of compromise, and sometimes even arise from a dialog between two or more parties (Dollard et al. 1993). The development of general conflictology at the present stage was significantly influenced by the works of international scientists, who had laid the theoretical foundation for solving specific problems of complex interdisciplinary science. These are the classic works of L. Coser, R. Dahrendorf, J. Habermas, H. Becker, A. S. Akhiezer as well as other studies on social conflicts (Dahrendorf, 1965; Gurr and Harff, 1994; Galtung, 1969; Gurr, 1993; Greenfeld, 1992; Isajiw, 1974; Boulding, K, 1969; Krisberg, 1998), modeling of social processes (Castellano, Fortunato, and Loreto, 2009; Smith, 2003; Traud, 2011).

Mathematical modeling based on nonlinear dynamics is widely used in natural science, but it is still applied quite rarely in sociological research. In recent years, significant progress has been made in the development of models of social and political processes (Plotnitskiy, 2001).

As a rule, the modeling of the dynamics of the linear system in classical models is based on the use of multidimensional equations, difference equations, the mathematical apparatus of cellular automata, the mathematical apparatus of catastrophe theory, the mathematical apparatus of self-organized criticality theory, the stochastic differential Langevin equations and Itô-Stratonovich, the analysis of systems with chaos and reconstruction of stable states (attractors) by time series (Malkov, 2009; Romanovsky Stepanova and Chernavsky, 1984; Haken, 1985, Malinetskiy and Potapov, 2000).

A certain class of works was devoted to ethnic diversity and its impact on economic and sociocultural development, as well as other social processes associated with it (Shabrov, 1996; Ottaviano and Peri, 2005; Weber, Davydov and Dower, 2015). These are the interdisciplinary

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researches dealing with social problems and their correlation with the economy taking into account ethno-cultural factors, as well as their joint influence on potential and existing conflicts.

Forecasting and description of socio-political processes are carried out by many other methods (Mikhailov and Gorbatikov, 2012; Smith, 2003; Traud, 2011).

In fact, given the significant impact of such phenomena on the society and the processes associated with it, the methods and ways for describing and predicting ethno-social conflicts are extremely important.

In recent years, significant progress has been made in the development of models of social and political processes (Abzalilov, 2012). Mathematical modeling based on nonlinear dynamics is widely used in natural science, but it is still applied quite rarely in sociological research.

2 FUNDAMENTALS OF THE MODEL

Socio-political processes are subject to constant changes and deformations, therefore from the point of view of mathematical modeling they cannot be set with a high degree of precision. Here we can trace the analogy with the Brownian particle, i.e. a particle that seemingly moves along a rather defined trajectory, but under close examination, this trajectory turns out to be strongly tortuous, with many small fractures (Holyst, Kasperski and Schweitger, 2000; Petukhov et al. 2018; Gutz and Korobitsyn, 2000). These small changes (fluctuations) are explained by the chaotic motion of other molecules. In social processes, fluctuations can be interpreted as manifestations of the free will of its participants, as well as other random manifestations of the external environment (Gutz and Korobitsyn, 2000).

In physics, these processes are, as a rule, described by the Langevin equation of the stochastic diffusion, which has been relatively approved for modeling of some social processes as well. For example, the model of public opinion, developed by Holyst J.A., Kacperski K., Schweiter F. (Holyst, Kasperski and Schweitger, 2000), is based on the use of this equation.

This approach has several advantages:

1. As it has already been mentioned, the approach allows taking into account the manifestations of the free will of its individual participants, as well as other random manifestations of the external environment for the social system.

2. The behavior of a social system can be calculated, both for its entirety and separate individuals.

3. This approach allows identifying some distinctive stable modes of functioning of social systems, depending on various initial conditions.

4. Diffusion equations, as a mathematical apparatus, have been sufficiently validated and studied from the point of view of numerical simulation.

The model is based on the assumption that individuals interact in society through a communicative field - h (a similar concept was introduced in (Holyst, Kasperski and Schweitger, 2000), but with another parametrization and another type of initial equations).

The need to introduce this concept was caused by a number of factors:

1. Any alive cognitive system (including both society and a person) carries out the activity and functioning on the basis of the exchange of information in one form or another. It can be either information from electrical impulses in the brain or the cerebellum opioid system, or information flows on the Internet.

2. Therefore, modeling of individuals' behavior in society will be directly related to modeling their information exchange.

3. Accordingly, it is reasonable to introduce a function as the basis of the model that will simulate the information exchange between individuals.

This is the reason why the function h, in fact, the function of information exchange between individuals, was developed. Physically, it is a field created by every person in society, simulating information interaction. Besides, from the point of view of physical and mathematical description of this field, we should keep in mind that here we are talking about a society, which is difficult to refer to an object in classical physical spatial topology. Objectively, from the point of view of information transfer from an individual to an individual, space in society combines both classical spatial coordinates and additional specific parameters and features. This is associated with the fact that in the modern

information world there is no need to be close to the object of influence to transmit information to it.

Thus, the society is a multidimensional, social-physical space that reflects the ability of one individual to "reach" another individual with his communicative field, that is, to influence it, its parameters, and the ability to move in a given space. Accordingly, the position of the individual relative to other individuals in such a space, among other things, models the level of relationships between them and involvement in the information exchange. The proximity of individuals to each other in this model suggests that there is a regular exchange of information between them and a social connection has arisen. The conflict in such a statement of the problem should be regarded as a variant of the interaction of individuals, or groups of individuals, as a result of which the distance (i.e., social distance xi - xj, where xi and xj are the coordinates in social and physical space, i, j = (1, N), where N is the number of individuals or consolidated groups of individuals) between them is growing rapidly.

Conflict management or various options for conflict mediation (Dahrendorf, 1965; Gurr and Harff, 1994; Isajiw, 1974; Boulding, K, 1969), from the point of view of modeling, are an additional function that depends at least on the coordinates and affects the overall stability and structure of the social system. There are several physical analogies that similarly influence physical systems; for example, a dissipative function that can have different forms in different physical conditions (Holyst, Kasperski and Schweitger, 2000).

2.1 Mathematical Representation of the System

The communicative field, as in (Petukhov et al. 2018), is represented by a diffusion equation with a divergent type of diffusion:

$$
\begin{aligned}
\frac{\partial}{\partial t} & h(x_i, t) \\
&= \sum_{j=1}^{N} f(x_i, x_j) \vartheta(x_i, x_j) \bar{\delta}_{(k_s^j + k_c^j),(k_s^i + k_c^i)} \\
&\quad + D(h(x_i, t) - h(x_i, t_0)),
\end{aligned}
\tag{1}
$$

where f(x_i,x_j) is a function that describes the interaction between individuals, which is modeled by the classical Gaussian distribution:

This interaction is an information exchange between individuals, which is carried out through any communicative physical means.

$$
\vartheta(x_i, x_j) = \frac{1}{\varepsilon\sqrt{\pi}} e^{\frac{-(x_i - x_j)^2}{\varepsilon^2}},
$$

Function $\vartheta(x_i, x_j)$ is introduced instead of the delta-function to simplify the process of computer modeling;

$\bar{\delta}_{(k_s^j + k_c^j),(k_s^i + k_c^i)}$ – is the inverse Kronecker symbol;

D – is the diffusion coefficient describing the spread of the communicative field.

The movement of an individual in space is described by the Langevin equation:

$$
\begin{aligned}
\frac{dx_i}{dt} &= u(x_i) + k_c^i k_s^i \left(\sum_{j=1, j \neq i}^{N} \frac{\partial}{\partial x_j} h(x_j, t) \right) \\
&\quad + \sqrt{2D} \xi_i(t),
\end{aligned}
\tag{2}
$$

$u(x)$ – is the control function, which we set as:

$$
u(x) = -\frac{x_i}{\tau}
$$

where τ – is the time of relaxation in the society,

k_c^i – coefficient of social activity of the ith individual or a group of individuals,

k_s^i – coefficient of the scientific and technological progress of the ith individual or a group of individuals,

$\xi_i(t)$ – stochastic force.

We believe that the distinctive parameters of the system can take on values:

$$
0 < k_c, k_s, D < 1.
$$

In the general case, the following are chosen as the initial conditions for equations (1) and (2):

$$
\begin{aligned}
x_i|_{t=0} &= x_{0i}, \\
h(x_i, t = 0) &= h_{0i}.
\end{aligned}
$$

The field $h_k(x, t)$ affects an individual I in the following way. Being at the point x_i, the individual falls under the influence of the communication field of another individual (or several). Depending on the difference between its coefficients and the coefficients affecting individuals, it can react in the following ways:

1. Changes the value of its coefficients under the influence of other individuals

2. Moves in the direction of the area where the difference of the coefficients is relatively minimal at the moment

Let us consider $p_{ij}(k_i, k_j, t, x_i, x_j)$ as the probability of the impact of the communication field of an individual (or a cluster of individuals) j on the communication field of an individual i in a way to change its coefficients K_s and K_c (separately or together) at time t. In this case, the probability of movement of an individual i in the direction of the area where the difference of the coefficients is relatively minimal at present $-1 - p_{ij}(k_i, k_j, t, x_i, x_j)$.

The change in probability then:

$$\frac{d}{dt} p_{ij}(k_i, k_j, t, x_i, x_j)$$
$$= \sum_{k'_i} v(k_i|k'_i) \, p_{ij}(k'_i, k'_j, t, x_i, x_j) \vartheta(\Delta x_{ij} \Delta k_{ij}) -$$
$$-p_{ij}(k_i, k_j, t, x_i, x_j) \sum_{k'_i} v(k'_i|k_i) \, \vartheta(\Delta x_{ij}, \Delta k'_{ij}),$$
$$(3)$$

$\vartheta(\Delta x_{ij} \Delta k_{ij})$ – a parameter characterizing the induction effect of the communication field.

$v(k_i|k'_i)$ – abstract probabilities of changing the coefficients per unit of time:

$$v(k'_i|k_i) =$$
$$\begin{cases} k_i \neq k'_i \rightarrow \eta \, exp\left\{\left[h_{k_i'}(x_i, t) - h_{k_i}(x_i, t)\right]/Q\right\} \\ k_i = k'_i \rightarrow 0 \end{cases},$$
$$(4)$$

где Q – a parameter of social freedom characterizing the degree of freedom of movement of individuals in a given social system.

The next stage of the study must be focused on the data of protests in Belarus, for the reason that the results of the modeling are of special interest for us in this study, and then compare them with real data.

3 SOCIAL MEDIA AS A SUBSYSTEM OF THE MODERN POLITICAL AND COMMUNICATIVE SPACE

If we consider the general field of modern political communication as a separate system, so the political and communicative space of social media is its subsystem. Such features of communicative interactions as dynamism, interactivity, and connectivity have a special manifestation in this subsystem. This subsystem is closely connected with the other subsystems of political communication which often play the role of an external environment to it, sending its impulses that cause a certain reaction within this subsystem. The example of a similar reaction in the political and communicative space of social media can be noticed in the course of the protest events in the Republic of Belarus.

The choice of this social conflict is caused by its relevance, sufficiency of informational representation, and active presentation on social media. The experience of previous social conflicts with high integration of social media demonstrates that the activity of participants in the process on the internet can serve as a marker for determining their level of involvement (Erz, Marder and Osadchaya, 2018; Lidgren, 2019; Bonilla and Rosa, 2015).

To conduct our research, we carried out a quantitative content analysis of the subject of posts published by residents of the Republic of Belarus in the support system of modern social networks Facebook, dedicated to protesting actions that began immediately after summing up the results of the presidential elections in Belarus on 9 Aug. 2020. We have chosen a message (Facebook users' notes) as a unit of analysis and a word as a unit of calculating (keyword, hashtag).

When writing texts of messages on social media, several specific hashtags are often used ("a keyword or phrase indicated by a hash sign that turns this word/phrase into a hyperlink" and reflects the subject and content of the message (Erz, Marder and Osadchaya, 2018, p. 50). During the reflection of the protest events in Belarus in the Facebook information field, a number of certain hashtags became integral elements of the symbolic semantic core of the communication activity of the republic's residents on Facebook. Among the designated hashtags are the following: "#Belarus2020", "#Беларусь2020", "#ЖывеБеларусь" ("#Long live Belarus"), "#ВерымМожамПераможам" ("#believecanwin"). As it is known, hashtags are not just words or expressions corresponding to the used sign, but, "tools of activating certain interpretive frames", according to S. Lindgren (Lidgren, 2019, p. 421). They play a semiotic role pointing out the intended meaning of the utterance, allowing users to state in the message that semantic meaning, which, otherwise, might not be so obvious (Bonilla, Y., and J. Rosa, 2015, p.5). Thus, in some cases, hashtags mark the corresponding user posts with a certain value, fixing a meaningful message to the audience in the post itself, or with a certain meaning, stating a meaningful message to the audience in the post.

The time interval for content analysis is from 11 Aug. to 2 Oct. During the indicated period, 604 messages were selected including the hashtags

mentioned above as a component element. It is necessary to emphasize that we have analyzed only messages published by residents of the Republic of Belarus based on open data on Facebook.

3.1 Results of Content Analysis

The dynamics of changes in the frequency of posting messages with the indicated hashtags in general by days is presented in Figure 1.

The dynamics of changes in the frequency of posting messages with the indicated hashtags separately by days is presented in Figure 2.

Number of messages with hashtags #Belarus2020, #ЖывеБеларусь, #Беларусь2020, #ВерымМожамПераможам в Facebook
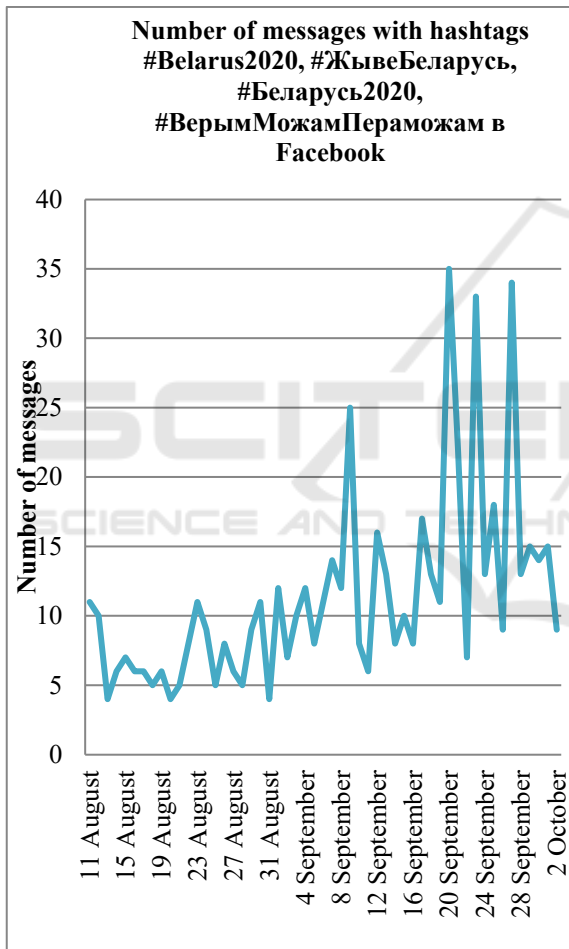
Figure 1.

In messages dedicated to protest activity in Belarus, it was more often used (in comparison with others) «#ВерымМожамПераможам» (213 messages) and «#Belarus2020» (202 messages) by residents of the Republic. The hashtag «#ЖывеБеларусь» was used in 164 in posts and «#Беларусь2020 – 25 in messages.
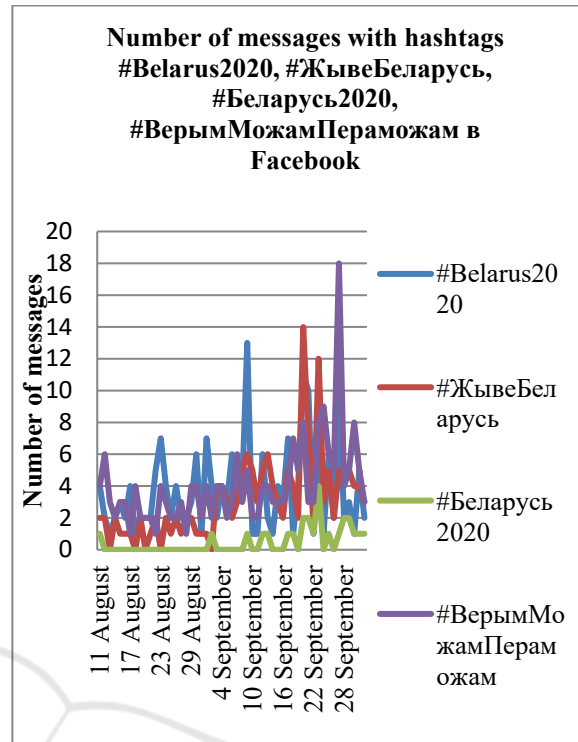
Number of messages with hashtags #Belarus2020, #ЖывеБеларусь, #Беларусь2020, #ВерымМожамПераможам в Facebook

Legend: #Belarus2020, #ЖывеБеларусь, #Беларусь2020, #ВерымМожамПераможам

Figure 2.

4 RESULTS OF THE MODELING

The modeling was carried out in the MatLab 2013b environment. Two variants with different initial conditions were served as a basis for research.

The first variant (Figure 3) represents the modeling of social indignation without any external influence (without the control function), in order to
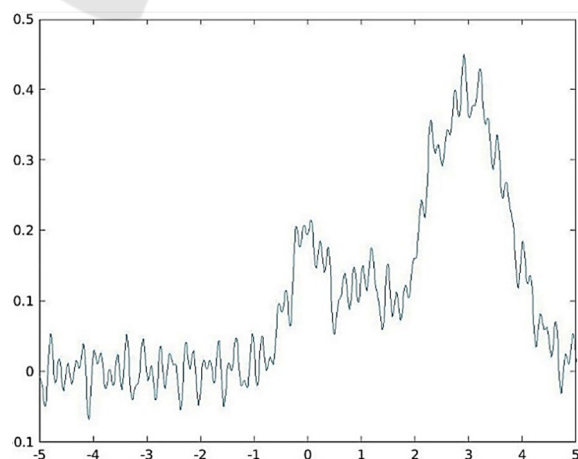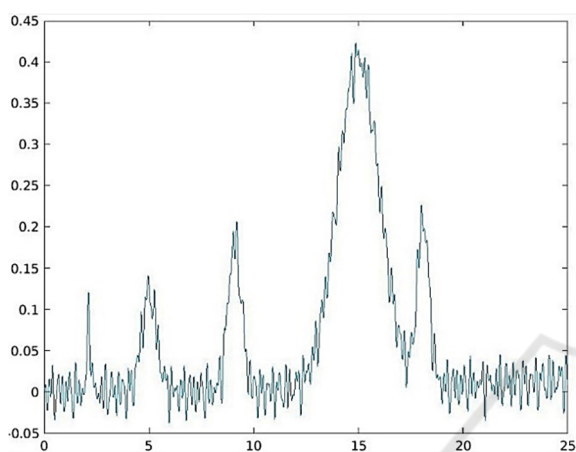
Along axes: axis y – change in social activity Δkc, axis x – timing t.

Figure 3.

analyze the variant of a closed system and to make a conclusion on how the internal social conflict should proceed in this case. The modeling results are similar to earlier works on the dynamics of social activity in conflict conditions (Petukhov 2020).

The second situation refers to a variant of social disturbance with external influence/control creating a sequential series of "disturbances" in the communication field of the social system (Figure 4).

Along axes: axis y – change in social activity Δkc, axis x – timing t.

Figure 4.

4.1 Data Analysis and Comparison

We have analyzed the intensity of use of the designated hashtags on Facebook (for the period from 11 Aug. to 2 Oct.) and identified a number of days when some of these hashtags were used especially intensively: 9, 20, 21, 23, and 27 September. Only 20 and 27 Sept. were days off, which is important to emphasize because the main rally activity took place in Belarus exactly on weekends. Therefore, the active use of the hashtags mentioned above could become a reaction to events that happened in the external environment (towards the political and communicative field of Belarus).

The reason for the active use of thematic hashtags on 9 Sept. could become a speech of one of the leaders

of the Belarusian opposition Svetlana Tsikhanouskaya at the Parliamentary Assembly of the Council of Europe (PACE) on 8 Sept. During her speech, Svetlana Tsikhanouskaya made an appeal to PACE for imposing sanctions against the leadership of Belarus[1].

An important external factor that contributed to the increase in communication activity on Facebook (using the considered hashtags) in the period from 20 to 23 Sept. could be the resolution of the United Nations Human Rights Council, adopted at the meeting held on 18 Sept. The resolution condemns violations of human rights in Belarus, calling on the country's authorities to take measures to resolve the situation[2]. A statement by a representative of the US State Department was released directly on 23 Sept. According to this statement, the US did not officially recognize A.G. Lukashenko as the legitimate president of the Republic of Belarus[3]. On the same day the official representative of the German government S. Seibert, the head of the Czech Ministry of Foreign Affairs T. Petršicek, and the head of the Danish Foreign Ministry J. Kofod also declared that their states did not recognize the legality of A.G. Lukashenko for the presidency of the Republic of Belarus[4].

On 27 Sept. French President E. Macron declared in an interview with a weekly Journal du Dimanche that, in his opinion, the Belarusian leader A.G. Lukashenka must resign[5]. It provoked a response from the leadership of Belarus and can also be viewed as an informational influence of the external environment on the political and communicative space of Belarus.

When comparing curve trajectory in Figure 1 and Figure 2 with Figure 3 and Figure 4, it is necessary to emphasize the repeated cyclicity in Figure 1, Figure 2 and cyclicity of the same nature in Fig. which, as noted, demonstrates the reaction of the communicative field (subsystem) to external information impulses. They cause an increase in the intensity of social activity and contribute to the activation of communicative actions of social media users with the use of certain content-semantic

[1] Nikolaev P. Sanctions against the authorities: Tsikhanouskaya spoke at the PACE // Internet edition Gazeta.ru. URL: https://www.gazeta.ru/politics/2020/09/08_a_13241324.shtml

[2] Krayushkins M United Nations Human Rights Council condemned what is happening in Belarus // Internet edition Gazeta.ru. URL: https://www.gazeta.ru/social/news/2020/09/18/n_14958775.shtml

[3] Kazantseva K. The US does not officially recognize Lukashenko as the legitimate president of the Republic of

Belarus // Internet edition Gazeta.ru. URL: https://www.gazeta.ru/politics/news/2020/09/23/n_14981911.shtml

[4] Fakhrutdinov P. "What a farce": Europe condemned Lukashenka's inauguration // Интернет- Internet edition Gazeta.ru. URL: https://www.gazeta.ru/politics/2020/09/23_a_13263973.shtml

[5] Ermolov A. "Lukashenko must leave": Macron accused Minsk of authoritarianism // Internet edition Gazeta.ru. URL: https://www.gazeta.ru/politics/2020/09/27_a_13270117.shtml

(thematic) verbal tools (hashtags).

Also, a comparison of the results and general characteristic patterns indicates the similarity of real data (from the point of view of repetition of growing peaks of activity) with the simulation results in Figure 4, which from a perspective of the presented concept of the approach implies an increasing cyclical external interference in the social conflict in Belarus.

Based on the results of modeling, we should predict (at the time of the survey in Nov. 2020) that the main peak of social disturbance is likely still ahead. Of course, various additional factors, such as the coronavirus epidemic, can interfere in the process. However, from the point of view of the model, the process has not yet reached the main maximum and its appearance can be assumed at the beginning of 2021 if external influence and main internal trends persist.

5 CONCLUSIONS

This article proposed a model based on diffusion equations (Langevin equations, in particular) that models information and communication interactions of individuals in society in various conditions. One of the main parameters of the model is the coefficient of social activity, and the change of this parameter can be indicative in the context of ongoing social conflicts from the point of view of analyzing the state of society.

In order to test the model, we have chosen a specific real situation - public protests in Belarus in 2020.

It also should be pointed out that informational influences from the external environment can make an impact on the political and communicative field of the political system, causing a response from the communication participant. In particular, one of the consequences of such an impact is an increase of user activity in the context of modern social media support platforms, which use special symbolic elements, certain hashtags, in the process of text communication acts. According to the conducted analysis of the communicative actions of the inhabitants of Belarus on Facebook certain events (of a certain nature and political orientation) that took place in the information field of foreign states and international organizations, often preceded (and/or accompanied) the active use of the appropriate hashtags.

The results of the conducted content analysis partially correspond to the results of modeling, especially the general pattern in Figure 4, which

makes it possible to assume that there is an external influence/interference in conflict processes in the Republic of Belarus.

We have also made a forecast for the development of the situation and the possibility of a larger disturbance in the social system when/while maintaining the current trends and the level of external influence.

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