Analysis of a Nonlinear Control Law with Cubic Nonlinearity

Melnikov Vitaly[1,2][a], Melnikov Gennady[2][b] and Dudarenko Natalia[2][c]

[1]*Department of Mechanics, Saint Petersburg Mining University, 2, 21st Line, Saint-Petersburg, 199106, Russia*

[2]*Department of Control Systems and Industrial Robotics, ITMO University, 49 Kronverksky Pr., Saint-Petersburg, 197101, Russia*

Keywords: Nonlinear Control, Cubic Nonlinearity, Polynomial Transformation.

Abstract: The paper is considered the problem of improving the stabilization of nonlinear controlled systems. It is proposed to solve the problem by introducing cubic components into the control law. Using the method of polynomial transformation, a comparative analysis of the influence of cubic components on the dynamics of a controlled systems is presented. As a result, some conclusions about the choice of the structure and parameters of the nonlinear control law are presented and recommendations are given.

1 INTRODUCTION

The presence of nonlinearity in the model description of any controlled rigid body can have a negative impact on the quality of control system processes. This problem is relevant for many controlled robotic systems. There are many approaches to reduce the influence of nonlinearity with correction devices and provide a control system with required dynamic quality (Popov, 1989; Frank et al., 2004; Ivanov et al., 2014). For example, correction methods for nonlinear systems can be realized with the changing of the structure and parameters of a linear part of the system, additional feedbacks or elements. There are modes of a control system when it is reasonable to do the correction of a controlled system with nonlinear control law (Fang et al., 2003; Ilyukhin et al., 2015; Reichensdorfer et al., 2018).

The choice of parameters of the nonlinear control law makes it possible to control the regulation time of the process and its oscillation, which improves the dynamic qualities of a wide class of systems, including aircraft stabilization systems, robotic and mechatronic complexes and its applications (Seraji, 1998; Ansarieshlaghi and Eberhard, 2020; Aleksandrov et al., 2018; Qi et al., 2018). The case of nonlinear control law with cubic nonlinearity is considered in the paper and the influence of the cubic components of the nonlinear control law on the system state variables is analysed. The results of the paper can be useful for the design of robotic applications with nonlinear control law including a cubic nonlinearity.

The paper is laid out as follows. Firstly, the problem of nonlinear systems correction with a nonlinear control law with cubic nonlinearity is discussed. Then, a methodology of reducing a first-order aperiodic controller to an ideal controller and integrating a simplified equation is proposed. Thereafter, expressions for the choice of characteristic coefficients and control parameters are presented. The analysis results are discussed and the paper is finished with some concluding remarks about the choice of the parameters for a cubic control law.

[a] https://orcid.org/0000-0002-2114-7891
[b] https://orcid.org/0000-0003-2606-7572
[c] https://orcid.org/0000-0002-3553-0584

2 PROBLEM DESCRIPTION

Let the motion of the controlled system be described by a system of differential equations of the third order.

$$\begin{aligned} \ddot{\theta} &= -a\alpha + c\theta, \\ \dot{\alpha} &= f(\sigma), \end{aligned} \tag{1}$$

where a, c are constants and $f(\sigma)$ is the known nonlinearity of the control device, represented in the form of the following odd cubic polynomial

$$f(\sigma) = k_*\sigma - k^*\sigma^3, \tag{2}$$

where k_*, k^* are constants σ is a control, a linear or non-linear function of $\alpha, \theta, \dot{\theta}$. The function $f(\sigma)$ can also be an odd piecewise linear function admitting a

Vitaly, M., Gennady, M. and Natalia, D.
Analysis of a Nonlinear Control Law with Cubic Nonlinearity.
DOI: 10.5220/0010604407010706
In *Proceedings of the 18th International Conference on Informatics in Control, Automation and Robotics (ICINCO 2021)*, pages 701-706
ISBN: 978-989-758-522-7

sufficiently accurate approximation of the form (2) using the Chebyshev polynomials (Melnikov, 2005; Melnikov, 2010; Mason and Handscomb, 2002). For example, the function $f(\cos\varphi)$ is approximated by two terms of the Fourier series, then it is assumed that $\cos\varphi = \sigma$

$$\begin{aligned} f(\sigma) = f(\cos\varphi) \approx A_1\cos\varphi + A_3\cos 3\varphi = \\ = (A_1 - 3A_3)\cos\varphi + 4A_3\cos^3\varphi = \\ = (A_1 - 3A_3)\sigma + 4A_3\sigma^3, \end{aligned} \quad (3)$$

where $A_k = \frac{2}{\pi}\int_0^\pi f(\cos\varphi)\cos k\varphi d\varphi$. Another example of $f(\sigma)$:

$$f(\sigma) = \left\{ \begin{array}{ll} \sigma\,, & 0 \le \sigma \le 0{,}4 \\ 0.4, & 0.4 \le \sigma \le 1 \end{array} \right\} \approx 0{,}90\sigma - 0{,}56\sigma^3. \quad (4)$$

Let $\theta, \dot\theta$ can take values from some elliptic neighborhood of zero values $\theta, \dot\theta$ determined on the phase plane by an inequality of the form

$$\theta^2 + a_1^2\dot\theta^2 \le \varepsilon_0^2, \quad (5)$$

where a_1^2 is a given coefficient, ε_0 is the main parameter characterizing the size of the area. This area will be called the "large ellipse ε_0", from which we will extract the "small ellipse" by the following condition

$$\theta^2 + a_1^2\dot\theta^2 \le (\frac{\varepsilon_0}{2})^2. \quad (6)$$

Definition 1. We will say that a controlled system has a performance defined as (h_1, h_2, h_3, h_4) if for any initial perturbations from the large ellipse ε_0 the perturbations remain in some "overshoot ellipse" h_1 and by the time $t = T$ they contract inside the ellipse of "residual perturbations" h_2 and if for any initial perturbations from the small perturbation ellipse $\frac{\varepsilon_0}{2}$, remaining in ellipse h_3 contract over time into an ellipse h_4, where $\varepsilon_0 \ge h_1 \ge h_2 \ge h_3 \ge h_4$, $\varepsilon_0 \gg h_2$, $\varepsilon_0 \gg h_4$.

Definition 2. Let us consider two systems with performance (h_1, h_2, h_3, h_4) and (h_1', h_2', h_3', h_4'), respectively. We will say that the second system has a higher performance than the first system if the following inequalities are satisfied:

$$h_1 \ge h_1',\ h_2 \ge h_2',\ h_3 \ge h_3',\ h_4 \ge h_4', \quad (7)$$

where at least one of the four inequalities is strict, for example $h_1 > h_1'$.

Let the linear control law have the form

$$\sigma \equiv \tilde\sigma \equiv \frac{1}{k_*}(-k\alpha + k_1\dot\theta + k_2\theta), \quad (8)$$

and it provides stabilization of the system with a certain performance (h_1, h_2, h_3, h_4) for given ε_0, T.

Let the parameters of the linear part of system (1), (2), (8) satisfy the condition that the characteristic equation has a pair of complex roots $\lambda_{1,2}$ with negative real parts and one real negative root λ_3 with large modulus:

$$\begin{aligned} &\lambda^3 + k\lambda^2 + (ak_1 - c)\lambda + ak_2 - ck = 0; \\ &\lambda_{1,2} = -\chi \pm \mu i; \quad \mu \ge \chi; \quad \lambda_3 = -p. \end{aligned} \quad (9)$$

Let the performance be good enough for small perturbations $\frac{\varepsilon_0}{2}$, but insufficient for large perturbations ε_0. That is, we believe that characteristics h_3, h_4 are satisfactory, and h_1, h_2 are unsatisfactory, and that the linear law (8) with the selected parameters is considered the best in the sense that no other linear law of the form (8) can significantly improve the performance of the system.

Let us introduce into the control law (8) such nonlinear terms that, without significantly changing the performance at small perturbations, would improve the performance at large perturbations. We add to (8) a set of cubic terms with constant coefficients

$$\sigma = \tilde\sigma + \frac{1}{k_*}\sum_{\nu_1+\nu_2+\nu_3=3} p^{(\nu_1,\nu_2,\nu_3)}\alpha^{\nu_1}\dot\theta^{\nu_2}\theta^{\nu_3}, \quad (10)$$

where (ν_1, ν_2, ν_3) is a vector index. If the coefficients $p^{(\nu_1,\nu_2,\nu_3)}$ are small in comparison with the characteristic coefficients of the linear part of the system, then for small values of the parameters $\alpha, \dot\theta, \theta$ the cubic terms in (11) are negligible and we can assume that $\sigma \approx \tilde\sigma$. This means that at small perturbations the regulation is performed approximately as according to the linear law (8), therefore the parameters h_3', h_4' of the new system are approximately equal to the parameters h_3, h_4 of the system controlled according to the law (8). For large perturbations from the ellipse ε_0, cubic terms can be as large as linear ones, so their influence becomes significant, and the action of some terms is to some extent equivalent to a change in the coefficients k, k_1, k_2 in the linear law (8). For example, the presence of a term $p^{(0,0,3)}\theta^3$ is equivalent to a change in the coefficient at large values of θ, since in expression (11) two terms can be combined and for them approximately write

$$\begin{aligned} &(k_2 + p^{(0,0,3)}\theta^2)\theta \approx \\ &\approx \left\{ \begin{array}{ll} k_2\theta\,, & |\theta| \le \frac{\varepsilon_0}{2} \\ (k_2 + p^{(0,0,3)}\theta_{av}^2)\theta\,, & \frac{\varepsilon_0}{2} \le \theta \le \varepsilon_0. \end{array} \right. \end{aligned} \quad (11)$$

The main feature of the effect of cubic terms, in comparison with linear terms, is that they have almost no effect on the behaviour of the system if the system is near a stationary state $\theta = \dot\theta = 0$ (for example, it does not accelerate the system to the position $\theta = 0$, which would lead to subsequent overshoot in $\theta = 0$). Consequently, cubic terms can be introduced in order to improve the performance of the system at significant $(\theta, \dot\theta)$ without impairing the performance at small $\theta, \dot\theta$.

Let the control law be given in the form (10), where cubic terms are to be determined Let's find out the influence of each of the cubic terms on the stabilization of the system, determine which of the cubic terms improve the performance of the system, and which terms are inappropriate to leave in expression (10). Note that if in the process of synthesizing a linear system it is more convenient to specify the parameters χ, μ, p instead of specifying the coefficients k, k_1; k_2, then we will use the following formulas.

$$k = p + 2\chi, \; k_1 = \frac{1}{a}(c + \chi^2 + \mu^2 + 2\chi p), \quad k_2 = \frac{1}{a}(cp + 2c\chi + p(\chi^2 + \mu^2)). \tag{12}$$

3 REDUCING A FIRST-ORDER APERIODIC CONTROLLER TO AN IDEAL CONTROLLER

Let us approximately decrease the order of system (1) by one using the condition $p \gg \chi$. This can be interpreted as replacing the first order aperiodic controller with an ideal controller. Let us rewrite system (1) up to terms of the fifth order under conditions (2), (8), (10):

$$\dot{\alpha} = -k\alpha + k_1\omega + k_2\theta + \sum_{\nu_1+\nu_2+\nu_3=3} {p_1}^{(\nu_1,\nu_2,\nu_3)}\alpha^{\nu_1}\omega^{\nu_2}\theta^{\nu_3}, \quad \dot{\theta} = \omega, \quad \dot{\omega} = -a\alpha + c\theta \tag{13}$$

where cubic terms are defined by rearranging the expression

$$\sum_{\nu_1+\nu_2+\nu_3=3} p^{(\nu_1,\nu_2,\nu_3)}\alpha^{\nu_1}\omega^{\nu_2}\theta^{\nu_3} - \frac{k^*}{k_*^3}(-k\alpha + k_1\omega + k_2\theta)^3 \equiv \sum_{\nu_1+\nu_2+\nu_3=3} {p_1}^{(\nu_1,\nu_2,\nu_3)}\alpha^{\nu_1}\omega^{\nu_2}\theta^{\nu_3}. \tag{14}$$

Instead of α, we introduce a variable z in two stages. First we put

$$U = \alpha - A\omega - B\theta, \tag{15}$$

where A and B are determined from the condition that the new variable satisfies the diagonal equation

$$\dot{U} = -pu + \sum_{\nu_1+\nu_2+\nu_3=3} Q^{(\nu_1,\nu_2,\nu_3)}u^{\nu_1}\omega^{\nu_2}\theta^{\nu_3}. \tag{16}$$

We have

$$A = \frac{k-p}{a} = \frac{2\chi}{a}, \quad B = k_1 - \frac{2p\chi}{a}. \tag{17}$$

Then we perform a polynomial transformation

$$z = u - \sum_{\nu_1+\nu_2+\nu_3=3} A^{(\nu_2,\nu_3)}\omega^{\nu_2}\theta^{\nu_3}, \tag{18}$$

we determine the coefficients from the condition that z satisfies an equation of the form

$$\dot{z} = \left\{ -p + \sum_{\nu_1+\nu_2+\nu_3=3} a^{(\nu_1,\nu_2,\nu_3)}z^{\nu_1}\omega^{\nu_2}\theta^{\nu_3} \right\} z. \tag{19}$$

Substituting expressions (13), (15), (16), (18) into (19), then reducing each term in the resulting equality to the form of the $Q^{(\cdots)}u^{\nu_1}\omega^{\nu_2}\theta^{\nu_3}$, renaming the superscripts and equating the coefficients at the $u^{\nu_1}\omega^{\nu_2}\theta^{\nu_3}$, we obtain

$$[(\nu_2+1)p - \nu_2 k]A^{(\nu_2,\nu_3)} + (\nu_2+1)rA^{(\nu_2+1,\nu_3-1)} + (\nu_3+1)A^{(\nu_2-1,\nu_3+1)} = Q^{(0,\nu_2,\nu_3)}, \tag{20}$$

$$a^{(\nu_1-1,\nu_2,\nu_3)} = Q^{(\nu_1,\nu_2,\nu_3)} - (\nu_2+1)a_2A^{(\nu_2+1,\nu_3)}, \tag{21}$$

where $r = c - k_1 a - (p-k)p$, $\nu_2 = 0,1,2,3$, $\nu_3 = 3 - \nu_2$, $\nu_2 + \nu_3 = 2 - \nu_1$, $\nu_1 \geq 1$.

From here we can find all the coefficients, and they are small for a sufficiently large ρ. Let equation (19) be defined. Then it follows from it that z tends to zero approximately according to the law of $z = z_0\exp(-pt)$, since ρ is relatively large, and after a short period of time $[0, t_0]$ we can assume that for $t \geq t_0$ we have

$$z \approx 0, \quad \alpha \approx A\omega + BQ + \sum_{\nu_1+\nu_2=3} A^{(\nu_1,\nu_2)}\dot{\theta}^{\nu_1}\theta^{\nu_2}. \tag{22}$$

We replace the last equation of system (1) with relation (22). This relation defines an ideal regulator, in a sense equivalent to the original regulator.

As a result, instead of the original problem, we can consider the problem of the angular motion of an object controlled by an ideal controller, which is described by a second-order nonlinear differential equation

$$\ddot{\theta} + 2\chi\dot{\theta} + (\chi^2 + \mu^2)\theta + \sum_{\nu_1+\nu_2=3} A^{(\nu_1,\nu_2)}\dot{\theta}^{\nu_1}\theta^{\nu_2} = 0. \tag{23}$$

Equation (19) shows that the controlled aircraft rotates according to the law of rotation around the fixed axis of a rigid body with a unit moment of inertia under the action of the following damping and restoring moments:

$$M_1 = -(2\chi + aA^{(3,0)}\dot{\theta}^2 + A^{(1,2)}\theta^2)\dot{\theta}, \quad M_2 = -(\chi^2 + \mu^2 + aA^{(2,1)}\dot{\theta}^2 + aA^{(0,3)}\theta^2)\theta. \tag{24}$$

As a result, we can also conclude that by substituting relation (18) into the cubic terms of expression (11), cubic regulation with respect to $\alpha, \dot{\theta}, \theta$ can be reduced to regulation with respect to the $\dot{\theta}, \theta$ and therefore, it makes no sense to introduce cubic terms containing α

in ((11)), it is enough to introduce regulation according to the following law:

$$\sigma = \frac{1}{k_*}(-k\alpha + k_1\dot{\theta} + k_2\theta) + \frac{1}{k_*}(\sum_{\nu_1+\nu_3=3} p^{(\nu_1,\nu_2)}\dot{\theta}^{\nu_1}\theta^{\nu_2}). \quad (25)$$

4 INTEGRATING A SIMPLIFIED EQUATION

We assume that the imaginary part of the roots exceeds the real part in absolute value $\mu > k$. We introduce a variable

$$\xi = \Theta + (\chi + \mu i)\Theta, \quad (26)$$

satisfying the diagonal equation

$$\dot{\xi} = \lambda_1\xi + \sum_{\nu_1+\nu_2=3} Q^{(\nu_1,\nu_2)}\xi^{\nu_1}\xi^{-\nu_2}, \quad \lambda_{(1,2)} = -\chi + \mu i. \quad (27)$$

By a polynomial substitution

$$\xi = z - \sum_{\nu_1+\nu_2=3} B^{(\nu_1,\nu_2)} z^{\nu_1} z^{-\nu_2} \text{ for } B^{(2,1)} = 0, \quad (28)$$

$$B^{(\nu_1,\nu_2)} = \frac{1}{(1-\nu_1)\lambda_1 - \nu_2\lambda_2} Q^{(\nu_1,\nu_2)}, \quad \nu_1 = 0,1,3;\ \nu_2 = 3-\nu, \quad (29)$$

we reduce equation (27) to the form

$$\dot{z} = (\lambda_1 + gr^2)z \text{ for } g = Q^{(2,1)} \equiv g_1 + ig_2,\ r = |z|. \quad (30)$$

Equation (30) is equivalent to two equations

$$\dot{r} = \chi r + g_1 r^3, \dot{\varphi} = \mu + g_2 r^2, \text{ where } z = re^{i\varphi}. \quad (31)$$

Integrating the last equations, we find

$$r = r_0[(-1 - \frac{g_1 r_0^2}{\chi})e^{-2\chi t} + \frac{g_1 r_0^2}{\chi}]^{-\frac{1}{2}}, \quad \varphi = (\mu + \frac{\chi g_2}{g_1})t + \frac{g_2}{g_1}\ln(\frac{r}{r_0}). \quad (32)$$

It follows from equations (31) or from solution (32) that the constants chi, g characterize the rate of decrease in r, i.e. the rate of damping of the disturbed motion, while mu, g_2 determine the oscillation of the motion. The influence of nonlinear terms in the expression for the control function (25) on the stabilization of the object is mainly characterized by a complex coefficient $Q^{(2,1)} = g_1 + ig_2$ having two parameters g_1, g_2. A decrease in g_1 and g_2 in the negative direction, respectively, increases the rate of decay of the process and reduces the oscillation of the process.

5 CHARACTERISTIC COEFFICIENT AND CONTROL PARAMETERS

The coefficient $Q^{(2,1)}$ can be expressed directly through the coefficients $p^{(\nu_1,\nu_2)}$. Differentiating the first of equations (1) and subtracting from it the second equation multiplied by a, we obtain one third-order equation equivalent to system (1)

$$\theta^{(3)} + (2\chi + p)\ddot{\theta} + (\chi^2 + \mu^2 + 2\chi p)\dot{\theta} + + (\chi^2 + \mu^2)p\theta = -a \sum_{\nu_1+\nu_2=3} \tilde{p}^{(\nu_1,\nu_2)}\dot{\theta}^{\nu_1}\theta^{\nu_2} \quad (33)$$

From (2), (8), (10), (25) we obtain the coefficients of the cubic terms

$$\tilde{p}^{(\nu_1,\nu_2)} = p^{(\nu_1,\nu_2)} + \frac{k^*}{k_*^3} C_3^{\nu_2} N_1^{\nu_1} N_2^{\nu_2}, \quad (34)$$

where $N_1 = kA - k_1$, $N_2 = kB - k_2$, $C_3^{\nu_2}$ - number of combinations from 3 to v_2. By grouping the linear terms of equation (33), we represent it in three forms

$$\dot{z}_s - \lambda_s z_s + a \sum_{\nu_1+\nu_2=3} \tilde{p}^{(\nu_1,\nu_2)}\dot{\theta}^{\nu_1}\theta^{\nu_2} = 0,\ (s = 1,2,3), \quad (35)$$

where

$$z_{1,2} = \ddot{\theta} + (p + \chi \pm \mu i)\dot{\theta} + p(\chi \pm \mu i)\theta; \quad z_3 = \ddot{\theta} + 2\chi\dot{\theta} + (\chi^2 \pm \mu^2)\theta. \quad (36)$$

We will express the cubic terms in equations (35) in terms of z_1, z_2, z_3. Substituting into the first of equations (1) and comparing the obtained expression with (18), (15), we conclude that $z_3 = -\frac{1}{a}z + O^{(3)}(\dot{\theta},\theta)$ and since $z \approx 0$ at $t \geq t_0$, then $z_3 = O^{(3)}(\dot{\theta},\theta)$ at $t \geq t_0$. Therefore, when substituted into cubic terms, we can assume $z_3 \approx 0$ and from system (36) we obtain

$$\theta = \frac{1}{2\mu i\Delta}(K_1 z_1 + K_2 z_2), \quad \dot{\theta} = \frac{1}{2\mu i\Delta}(K_1\lambda_1 z_1 + K_2\lambda_2 z_2), \quad (37)$$

where $\Delta = (p-\chi)^2 + \mu^2$, $K_1 = p - \chi - \mu i$, $K_2 = -[(p-\chi) + \mu i]$, $\lambda_{1,2} = -\chi \pm \mu i$.

Substituting (37) into (35), we obtain a system of two equations in the canonical form

$$\dot{z}_s - \lambda_s z_s + \frac{a_i}{(2\mu\Delta)^3} S = 0, \quad (38)$$

where

$$S = \sum_{v_1+v_2} \tilde{p}^{(v_1,v_2)}(K_1\lambda_1 z_1 + K_2\lambda_2 z_2)^{v_1}(K_1 z_1 + K_2 z_2)^{v_2}.$$

The real and imaginary parts of the coefficient at the term $z_1^2 z_2$ are determined by the formulas

$$Q^{(2,1)} = -\frac{\partial \tilde{f}}{\partial z_2}\Big|_{\substack{z_2=0\\ z_1=1}} = = \frac{a_i}{8\mu^3\Delta^2} \sum_{v_1+v_2=3} \tilde{p}^{(v_1,v_2)}(p + \lambda_2)(v_1\lambda_2 + v_2\lambda_1){\lambda_1}^{v_1-1}, \quad (39)$$

where $\tilde{f}$ denotes the entire cubic term of equations (38)

$$g_s = L^2 \sum_{v_1+v_2=3} L_3{}^{(v_1,v_2)} \tilde{p}^{(v_1,v_2)},\ (s=1,2), \quad (40)$$

where

$$\begin{aligned} &L_1^{(0,3)} = 3\mu, \quad L_2^{(0,3)} = 3(p-\chi), \quad L^2 = \tfrac{a}{8\mu^3\Delta^2} > 0,\\ &L_1^{(1,2)} = (p+2\chi)\mu, \quad L_2^{(1,2)} = \mu^2 - 3\chi(p-\chi),\\ &L_1^{(2,1)} = (\chi^2+\mu^2+2\chi p)\mu,\\ &L_2^{(2,1)} = 3\chi^2(p-\chi)+\mu^2(p-3\chi),\\ &L_1^{(3,0)} = -3(\chi^2+\mu^2)p\mu,\\ &L_2^{(3,0)} = -3(\chi^2+\mu^2)[-\mu^2+(p-\chi)]. \end{aligned} \quad (41)$$

Note that according to (34) the coefficients at $\sigma^{(v_1,v_2)}$ differ from $p^{(v_1,v_2)}$ only by additional terms; therefore, g_1, g_2 depends linearly on $p^{(v_1,v_2)}$, and the coefficients at $p^{(v_1,v_2)}$ are determined by formulas (41).

6 RECOMMENDATIONS ABOUT THE CHOICE OF PARAMETERS FOR THE CUBIC CONTROL LAW

Now, some recommendations about the choice of parameters for the cubic control law are given. As noted in (4), the rate of damping of the disturbed motion will increase if g_1 is reduced to the negative side by choosing the parameters $p^{(\nu_1,\nu_2)}$; fluctuation decreases with decreasing g_2. Each of $p^{(\nu_1,\nu_2)}$ affects the characteristics of g_1 and g_2, namely, from expressions (40), (41) it follows that:

1) The inclusion of a term $p^{(0,3)}\theta^3$ with a negative coefficient $p^{(0,3)}$ in the control function (25) helps to weaken the disturbed motion (since $L^2L_1^{(0,3)} > 0$), and also helps to reduce the oscillation of the movement (more precisely, to a decrease of g_2).

2) The inclusion of $p^{(1,2)} < 0$ contributes to the damping of motion in about the same way as the inclusion of the coefficient $p^{(0,3)}$. The coefficient $p^{(0,3)}$, is related to $p^{(1,2)}$ by the formula related to $p^{(1,2)}$ according to the formula

$$p^{(0,3)} = \frac{1}{3}(p+2\chi)p^{(1,2)} < 0. \quad (42)$$

At the same time, $p^{(1,2)}$ decreases the vibrational value less than $p^{(0,3)}$ (42) if the following condition is satisfied

$$\begin{aligned} &L_2^{(1,2)}(L_1^{(1,2)})^{-1} < L_2^{(0,3)}(L_1^{(0,3)})^{-1} \Rightarrow\\ &\Rightarrow \tfrac{\mu^2}{p-\chi} < p+2\chi. \end{aligned} \quad (43)$$

In the opposite case, the inclusion of $p^{(1,2)}$ is preferable.

3) The coefficient $p^{(2,1)} < 0$ contributes to the damping of motion as $p^{(0,3)}$ since

$$p^{(0,3)} = \frac{1}{3}(\chi^2+\mu^2+2\chi p)p^{(2,1)} < 0. \quad (44)$$

But the inclusion of the term $p^{(0,3)}$ is preferable, since it provides less oscillation due to the fulfillment of the condition

$$\chi - \frac{\mu^2}{p-\chi} < p. \quad (45)$$

4) The coefficient $p^{(3,0)} < 0$ contributes to the damping of motion as $p^{(0,3)}$ since

$$p^{(0,3)} = -p(\chi^2+\mu^2)p^{(3,0)}. \quad (46)$$

But taking into account condition (45), the coefficient $p^{(0,3)}$ is preferable.

Thus, the inclusion of a term $p^{(0,3)}\theta^3$ with a negative coefficient into the control function does not allow the system to be accelerated unnecessarily to a stationary state at relatively large values of θ.

If $L_2^{(1,2)} > L_2^{(0,3)}$, then it is preferable to include in the control the term $p^{(1,2)}\dot{\theta}\theta^2$ in addition to $p^{(0,3)}\theta^3$. This term at relatively small χ (more precisely, at $L_2^{(1,2)} > 0$) does not lead to an increase in the oscillation of the system.

If $L_1^{(2,1)}$ or $L_1^{(3,0)}$ are much larger than $L_1^{(0,3)}$, $L_1^{(1,2)}$, then regulation with the help of $\dot{\theta}^2\theta$ or $\dot{\theta}^3$ can also be introduced. Note that these terms can adversely affect the oscillation of the movement.

7 CONCLUSIONS

The paper presents the analysis results of the influence of the cubic components of the nonlinear control law on a system state variables. The choice of parameters of the nonlinear control law makes it possible to control the regulation time of the process and its oscillations. Expressions for the choice of characteristic coefficients and control parameters are obtained in the paper. The relationship between the parameters of the nonlinear control law with the cubic components and the quality of the system is described.

ACKNOWLEDGEMENTS

This work was financially supported by Government of Russian Federation, Grant 08-08.

REFERENCES

Aleksandrov, A., Aleksandrova, E., and Tikhonov, A. (2018). Monoaxial attitude stabilization of a rigid body under vanishing restoring torque. *Nonlinear Dynamics and Systems Theory*, 18(1):12–21. cited By 0.

Ansarieshlaghi, F. and Eberhard, P. (2020). Disturbance compensator for a very flexible parallel lambda robot in trajectory tracking. *Proceedings of the 17th International Conference on Informatics in Control, Automation and Robotics - Volume 1: ICINCO*, pages 394–401.

Fang, Y., Dixon, W., Dawson, D., and Zergeroglu, E. (2003). Nonlinear coupling control laws for an underactuated overhead crane system. *ASME Transactions on mechatronics*, 8(3):418–423.

Frank, L., Darren, M., and Chaouki, T. A. (2004). *Robot Manipulator Control: Theory and Practice*. Marcel Dekker.

Ilyukhin, Y., Poduraev, Y., and Tatarintseva, A. (2015). Nonlinear adaptive correction of continuous path speed of the tool for high efficiency robotic machining. *Procedia Engineering of 25th DAAAM International Symposium on Intelligent Manufacturing and Automation*, (100):994–1002.

Ivanov, S., Melnikov, G., and Melnikov, V. (2014). The modified poincare-dulac method in analysis of autooscillations of nonlinear mechanical systems. *Journal of Physics: Conference Series*, 570. cited By 13.

Mason, J. and Handscomb, D. (2002). *Chebyshev Polynomials*. CRC Press.

Melnikov, V. G. (2005). Chebyshev economization in poincare-dulac transformations of nonlinear systems. *Nonlinear Analysis, Theory, Methods and Applications*, 63(5-7):e1351–e1355.

Melnikov, V. G. (2010). Chebyshev economization in transformations of nonlinear systems with polynomial structure. In *International Conference on Systems - Proceedings*, volume 1, pages 301–303.

Popov, E. (1989). *Theory of nonlinear automatic regulation systems and control*. Nauka, Moscow.

Qi, L., Cai, J., Han, A., Wan, J., Mei, C., and Luo, Y. (2018). A novel nonlinear control technique with its application to magnetic levitated systems. *IEEE Access*, 6:78659–78665.

Reichensdorfer, E., Odenthal, D., and Wollherr, D. (2018). Nonlinear control structure design using grammatical evolution and lyapunov equation based optimization. *Proceedings of the 15th International Conference on Informatics in Control, Automation and Robotics*, 1:55–65.

Seraji, H. (1998). New class of nonlinear pid controllers with robotic applications. *Journal of Robotic Systems*, 3(15):161–181.