Determination of the Risk of Deviation of an Event from the Mathematical Expectation in the Management of Regional Development

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Abstract: The paper analyses various time segments of a random process and shows how the covariance of the values of random functions for different times is displayed. It is accepted that any type of management involves the optimal use of factors, avoiding the uncertainties that are inevitable with a statistical approach to the sequencing of scientific research. The model includes the risk minimization of the real process deviation from the anticipated one. The concept of the quasi-ergodic idea of two functions, which characterize this concept with the corresponding form of integrals, is important for a further understanding of the presented theory. Various time segment of a random process are analyzed during the scientific research, and it is shown how the covariance of random functions’ values for times is manifested. The suggested model allows us to expand the boundaries of research automation of management process in space and time, taking into account the risk associated with the use of factors of the digital model.

1 INTRODUCTION

It was established that it is possible to build the sequence of management strategy of scientific search on the basis of a digital model, which creates the prerequisites for the automation of research activities, a significant increase in the effectiveness of scientific and technological development. A digital model is formed from a combination of statistical information that reflects the retrospective results of the process under analysis, the interdependence of targets and factors.

Visualization of the digital model allows us to illustrate the construction of various variants of management strategy for the solution of the problem and determine the best option which is considered to be as the shortest distance between the isoquants of the surface described by the digital model (Prokhorov and Kulikovskikh, 2016), (Prokhorov et al., 1995), (Mikheeva, 2007), (Borovik et al., 2014), (Borovik, V.S and Borovik, V.V., 2016).

Regardless of the causes of the risk, deviations of the characteristics of research process from the given parameters, each agent seeks to reduce the possible losses associated with the implementation of this risk by making managerial decisions. The risk prediction which is based on its acceptable values for making unambiguous management decisions concerning the process under analysis in space and time is a very urgent task (Borovik et al., 2014), (Borovik, V.S and Borovik, V.V., 2016), (Borovik et al., 2018), (Borovik, V.S and Borovik, V.V., 2020).

The risk management is not an instantaneous act, but it should be included into the general managerial decision-making process. When setting optimization problems, along with the criteria, the restrictions are expected to be set on the parameters and process variables, i.e. the permissible variations that determine the functioning of the process (Borovik, V.
and Borovik, A., 2019). The achievement of objectives obtained by means of functions’ performance. In the management of research the computing equipment has the tasks of accounting and monitoring the achievement of the given parameters, starting and stopping the solution of the problem, maintaining the given operating mode of the equipment and stabilizing the given parameters. The main difference of the approach under consideration is that the digital model based on the statistical data of the process under analysis is the basis of risk management.

The problem is that the existing management system of the research process does not meet the requirements and accuracy opportunities that allow the process to be carried out automatically.

The aim of the study is to substantiate the determination of the risk of deviation of an event from the mathematical expectation when managing the research process in accordance with the regional development strategy.

Research objectives:
1. To establish a control criterion for an automated scientific search system.
2. To highlight the quasi-ergodicity of the process through the manifestation of the covariance of the values of random functions for different time segments.

2 RESEARCH METHODOLOGY

The largest companies in the world including NASA use the 4-D System. It allows measuring the team capabilities and people behavior that affect team performance and risk management. The 4-D system shows how to improve the performance of business and projects, and it is especially in demand in multi-level large-scale projects and assignments, as well as in complex or critical situations. An automated research management system is a person-machine control system that provides automated collection and processing of information necessary for the optimization of the management of a production facility in accordance with the accepted criterion.

The ratio that characterizes the quality of the managed object as a whole and takes specific numerical values depending on the managerial actions which are used is taken as the control criterion of an automated system of scientific search. It is proposed to conduct the planning on the basis of the theory of random processes, which allows the use of representative statistics that reflect the vast majority of external and internal processes taking into account the object of risk management. The design calculation of reliability is part of the mandatory work ensuring the reliability of any automated system and is based on the requirements of regulatory and technical documentation (Prokhorov, 2007).

The assumption is made that for a given sample size, the risk cannot be less than some $R_0$ (for example, 5%). It is necessary to achieve a narrowing of the control area (relative to the mathematical expectation of a random process) to the interval $[-\delta; \delta]$, which greatly facilitates the management, helps avoiding unforeseen risks. For the control in space and time, time segments and control parameters are indicated, which allows automating significantly the control process.

3 RESEARCH RESULTS

We introduce the concept of quasi - ergodicity of two functions, which is important for a further understanding of the presented theory, characterizing this concept only by the form of integral Figure 1.

$$\frac{1}{b-a} \int_{a}^{b} f(t) dt$$

means that for any two random processes $y = f_i(t)$ and $y = f_j(t)$ the condition is fulfilled

$$\frac{1}{2t} \int_{t}^{b} f_i(t) dt = \frac{1}{2t} \int_{t}^{b} f_j(t) dt = 1,$$ (1)

with the ratio of the mathematical expectations of any two random processes on the same interval $[-t, t]$ are equal to one another.

Obviously, the formula (1) is also valid for the interval $[0; t]$, i.e.

$$\frac{1}{t} \int_{0}^{t} f_i(t) dt = \frac{1}{t} \int_{0}^{t} f_j(t) dt = 1,$$

Since mathematical expectations are equal for an ergodic process at all equal intervals, for example, for functions $y = -t^2 + \frac{2}{3}$ and $y = t^2$, then the ratio of their mathematical expectations is:
Figure 1 shows an illustration of quasi-ergodicity only from integrals of the type \( \frac{1}{b-a} \int_a^b f(t) dt \).

For the functions of a random type, the integrals of the mean values of the functions on the same interval are equal, i.e. the mathematical expectations are equal.

\[ \frac{1}{1} \left( -t^2 + \frac{2}{3} \right) \int_0^1 dt = -\frac{2}{3} + \frac{2}{3} = 1. \]

4 DISCUSSION OF THE RESULTS

It should be noted that all possible implementations of the random process under consideration are described by a function, for example, of type \( y_i = C_i \sqrt{t}, \quad i = \frac{1}{n} \). We introduce this function into the function (1) on the intervals \([0, t_1]\) and \([0, t_2]\).

\[
\frac{1}{t_2} \int_0^{t_2} C_i \sqrt{t} dt = \frac{1}{t_2} \int_0^{t_2} C_{MO} \sqrt{t} dt = \frac{1}{t_1} \int_0^{t_1} C_i \sqrt{t} dt = \frac{1}{t_1} \int_0^{t_1} C_{MO} \sqrt{t} dt
\]

Where, \( C_{MO} \) - management coefficient \( y = C_{MO} \sqrt{t} \), which describes the mathematical expectation of the curve of a random process.

The validity of formula (2) becomes obvious if we reduce the constants \( C_i \) and \( C_{MO} \) and the same multipliers in the left and right sides of formula (2).

By way of analogy with formula (1), we call a random such a process which has quasi-ergodic characteristics (Fig. 2). It should be noted that an additional justification for this term is given by the fact that formula (2) implies formula (1) accurate to a numerical factor.

Let’s take \( y_i = f_i(t) \) and \( y_j = f_j(t) \) as two random processes. Then for any time segments \( t_k \) and \( t_{k+1} \) we have:

\[
\frac{y_j(t_{k+1})}{y_j(t_k)} = \frac{y_i(t_{k+1})}{y_i(t_k)}, \quad \text{at } i \neq j. \tag{3}
\]

Since we will analyze various further time segments of a random process, we will show how the covariance of the values of random functions for different times is shown. In the scientific research [7], we obtained a function \( X_1(t) = 886.7 \sqrt{t} \), that describes the intensity of factor \( X_1 \) consumption over time. In all calculations, the time associated with \( X_1 \), is analyzed, i.e. \( t \).

Let’s find the integral,

\[
\int_0^6 886.7 \sqrt{t} dt = \left[ \frac{2}{3} \cdot 2 \right] = 886.7 \cdot \frac{2}{3} = 8689, \quad \text{and}
\]

it coincides with the mathematical expectation of a random variable given by the first column of the table given below (segment \( t=6 \)). The time in months is 6 and it is a planned deadline for the work. We similarly define the integral \( \int_0^6 886.7 \sqrt{t} dt = 6608.7; \)

\[
\int_0^4 886.7 \sqrt{t} dt = 4728.8; \quad \int_0^3 886.7 \sqrt{t} dt = 3071.4
\]

\[
\int_0^2 886.7 \sqrt{t} dt = 1671.9; \quad \int_0^1 886.7 \sqrt{t} dt = 591.1.
\]
In all calculations, the time associated with $X_1$, is analyzed, i.e. $t_1$. Then we define the coefficients of quasi-ergodicity, i.e. the ratio of the time segments of the mathematical expectation of the random process of consumption of factor $X_1$ under consideration. The meaning of these coefficients is seen from their definition:

$$K_{m,n} = \frac{\int_0^m \bar{X}_1(t) \, dt}{\int_0^n X_1(t) \, dt}. \tag{4}$$

Let us show how this formula is used. We found all the necessary integrals. So:

$$K_{6,5} = \frac{\int_0^5 \bar{X}_1(t) \, dt}{\int_0^5 X_1(t) \, dt} = \frac{6689}{60897} = 1.314;$$

$$K_{5,4} = \frac{\int_0^4 \bar{X}_1(t) \, dt}{\int_0^4 X_1(t) \, dt} = \frac{60897}{47288} = 1.397;$$

$$K_{4,3} = \frac{\int_0^3 \bar{X}_1(t) \, dt}{\int_0^3 X_1(t) \, dt} = \frac{47288}{30714} = 1.54;$$

$$K_{3,2} = \frac{\int_0^2 \bar{X}_1(t) \, dt}{\int_0^2 X_1(t) \, dt} = \frac{30714}{16719} = 1.837;$$

$$K_{2,1} = \frac{\int_0^1 \bar{X}_1(t) \, dt}{\int_0^1 X_1(t) \, dt} = \frac{16719}{5911} = 2.828.$$

It should be noted that $K_{m,n}$ depends on the distance (time) between the segments $t_m$ and $t_n$, which can be seen from the following diagram (Fig. 3).

In accordance with the principle of quasi-ergodicity, the same $K_{m,n}$ are valid for any implementation of the random process under consideration. The value of the random variable $X_1$ for the time segment $t = 6$ was taken from the experimental data. From the formula for $K_{m,n}$, it follows that in order to find the values of a random variable in the time segment $t = 5$, it is necessary to divide the values of the random variable at $t = 6$ by $K_{6,5} = 1.314$, for the time segment $t = 5$ we get:

$$\frac{6691314}{8792}; \frac{6672314}{8767} = 6691 \text{ and so on till the 30th table row.}$$

For the time segment $t = 4$, we do the same, recalculate the sample for $t = 5$ with the quasi-ergodicity coefficient $K_{5,4} = 1.397$.

$$\frac{8767}{1314}; \frac{8792}{1314} = 6691 \text{ and so on.}$$

Any control involves the optimal use of factors, avoiding the uncertainties that are inevitable when the statistical approach to the planning of scientific research is used [4, 12]. It is necessary to minimize the risk of deviation of the real process from the planned one. Having received the data, we can calculate the optimal management. At the same time we assume that:

- In accordance with the Khinchin – Kolmogorov theorem, the spectral power density of a stationary, in the broad sense, random process is the Fourier transform of the corresponding...
The estimated minimum risk should be taken as \( R = 5\% \), i.e. the probability of the problem’s solution is 95\%, which corresponds to the conditions of the sample size \( n = 30 \).

We give a detailed calculation of the interval \( 2\delta \), in which the values of \( X_i \) for the time segment \( t_k \) fall with a risk of 5\%. Since the calculations for the remaining segments \( t = 5, 4, 3, 2, 1 \) are the same, they will be presented more briefly.

For the time period \( t = 6 \). Calculation method is \([\delta, \delta]\). The maximum and minimum values of \( X_i \) are in bold: \( X_{\text{min}} = 7537; \ X_{\text{max}} = 9432 \). We divide the interval \([7537; 9432]\) into three partial intervals in width \( h = \frac{X_{\text{max}} - X_{\text{min}}}{3} = \frac{9432 - 7537}{3} = 631.7 \).

The number of partial intervals is selected under the condition of obtaining in each interval 8-10 options.

Then we find the middle of partial intervals:
\[
\frac{7537 + 8168.7}{2} = 7852.8; \\
\frac{8168.7 + 8800.4}{2} = 8484.5; \\
\frac{8800.4 + 9432}{2} = 9116.2.
\]

For further calculations, we need to determine the sample mean and variance, which we will calculate by means of the production method, which is used for the samples with equally spaced options: \( \bar{X}_n = M \bar{h} + C; \ D_n = [M^2 - (M_1^2)]h^2 \), where \( C \) – false zero (option located in the middle of the sample). We have \( C = 8484.5, h \) – partial interval width – 631.7. Let us calculate: \( u_i = \frac{x_i - C}{h} \) – conditional option; \( M_1^* = \frac{\sum u_i}{n} \) - conditional option of the first order; \( M_2^* = \frac{\sum (u_i^2)}{n} \) - conditional option of the second order.

Let us carry out some preliminary calculations:
\[
t = \sqrt{n} \frac{\delta}{\sigma} , \text{ where } n = 30 \text{ – sample size, } \delta = 181.74 \text{ – evaluation precision, } \sigma = 508.12 \text{ – mean square deviation, } t \text{ - Laplace function argument – } \Phi(t). \]

\[
\sqrt{\frac{30}{508.12}} 181.74 = 1.96 . \text{ Then from the table of values on the basis of the Laplace function is } \Phi (1.96) = 0.475. \text{ Calculation reliability: } 95\%.
\]

The values of the mathematical expectation in the time segments \( t = 1, 2, 3, 4, 5, 6 \) are obtained, as shown earlier, using the integral \( \int \frac{886.7}{\sqrt{t}} dt \), where we successively take \( t_k = 1, 2, 3, 4, 5, 6 \).

In order to determine the calculated values of factor \( X_i \) for various time fragments, it is necessary to protect the process from external risks associated with the influence of other factors.

Therefore, in order to ensure a reliable operation of the system, it is necessary to have the corresponding value, for example, of the elements \( K \), which is determined for the time segment \([t_k, t_{k+1}]\) using the formula:
\[
\frac{X_i(t_{k+1}) - X_i(t_k)}{Q} = K, \quad (5)
\]

Where \( Q \) – intensity of impact of elements \( K \).

It can be interesting to compare the result obtained by the calculations by means of Lorentz transformation for \( X_i \). Let us assume that
\[
X_i^* = X_i - nt, \quad (6)
\]

where \( c \) – light speed in the special theory of relativity. Let us take \( c = 1 \), because the speed of light \( c \) in Lorentz transformations is often written equal to unity (Borovik, V.S and Borovik, V.V., 2016), (Carmeli, 1977), (Einstein,1955), (Ko, 2010).

In our example, \( c \) makes sense having the maximum performance (or normative one). Then \( q \) is the intensity of additional elements for a particular system in fractions of \( c \).

Let us insert into the formula
\[
8689 = \frac{6608.7 - 6q}{\sqrt{1 - q^2}}.
\]

Further \( 75498721(1 - q^2) = (6608.7 - 6q)^2 = 43674915.7 - 79304 + 36q^2 \).

From here it follows that:
\[
q^2 - 0.00/q - 0.42 = 0 \Rightarrow q = 0.0005 + 0.648 = 0.6485 \text{ (out of 1)}. \]

From here the intensity \( q \) is determined, calculated in space and time, as a percentage of the maximum (or standard). From here it is easy to find the required number of elements.
5 CONCLUSION

In the article the concept of quasi ergodicity of two functions was introduced and these functions are presented by corresponding forms of integrals. These notions are important for a further understanding of the theory. The paper analyses various time segments of a random process and it is shown how the covariance of the values of random functions for different time periods is manifested. It was established that when planning a process it is extremely important to determine the risk of random situations with unforeseen consequences. The need is shown to have a functional relationship, which would allow calculating the risk of unforeseen consequences, as a random event that develops over time. When using the statistical characteristics, it is advisable to proceed from a 5% error (risk). For individual calculation points, in fact, the width of the 2σ interval should be calculated, which includes the influence factor with unforeseen consequences with the probability of 0.95. This allows us to obtain the tabular dependence δ = δ(t), (t), and then move on to its analytical expression.

So that the system could function in a given way at the right time the necessary parameters of the factors are provided in an amount determined by the corresponding time interval. For the calculated parameters of the factors used at various time periods, the reliability of the system's work from external risks is ensured. The given model allows us to expand the boundaries of automation of the research management process in space and time, taking into account the risk associated with the use of factors of the digital model.

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