

On the Issue of Modeling the Factor of Economic Development: Environmental Factor

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Abstract: The article is proposed a mathematical model of the ecological factor dynamics. Modeling of the environmental factor is carried out by types of costs aimed at environmental measures and activities leading to the rational using of natural resources. The work is considered investments, an attack on the protection of atmospheric air, water resources and land resources. The model of the ecological factor ensures the effectiveness of environmental protection measures. The calculation of the ecological factor is carried out on the basis of statistical data for the Udmurt Republic. Given the cumulative character of the negative impact of poor environmental conditions on human health, the long available period 1996-2019 is chosen. For the calculations statistical data on the annual indicators of pollution and purification of atmospheric air, water and land resources of the Udmurt Republic are used and information on current annual costs of environmental activities of the Udmurt Republic are reviewed. Calculations are shown that environmental pollution (air, water and land resources in the complex) in the region occurs at an average annual rate 0.48%. The estimated efficiency of environmental protection measures for atmospheric air is about 25.0%, for water resources is 47.5%, for land resources is 38.2%.

1 INTRODUCTION

Now a driving force of growth and a prerequisite for the sustainable development of regions is a set of qualitative population characteristics of the health, the intelligence, the education, the ability to master modern knowledge. Of course, the quantitative component of the population in regions is also important: the demographic one. These characteristics determine the quality of human capital, which in the context of rapidly changing technology, with the development of the components of an innovative economy, is necessary condition for ensuring social progress (Dubiei, 2021; Amanova, Turysbekova, Tazhibayeva, Izatullaeva and Kaltayeva, 2017; Ketova, Rusyak and Derendyaeva, 2013).

In terms of the influence degree on the economy, the most important components of human capital are health, education and culture. In this work, we study

the environmental factor that directly affects the health status of the population and on its demographic indicators such as life expectancy, fertility, mortality.

To improve the state of the environment, the government needs to direct financial resources and plan financial strategies (D'Orazio and Valente, 2019), since investments in the environment of health care reduce the levels of morbidity and mortality and extend the working life period (Yang, Zheng and Zhao, 2021; Seixas, Regier, Bryan and Mitton, 2021).

Good health of the population is the greatest value and benefit, it is the high importance for increasing the rate of socio-economic growth of society and the practical implementation of new innovative development paradigms (Tandon, Cain, Kurowski, Dozol and Postolovska, 2020; Lu, Chen, Hao, Wang, Song and Mok, 2017). The state of health determines the capabilities of the person during labor activity and the degree of his participation in it. A healthy person fully realizes himself, health problems limit him.

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Measures aimed at improving health status increase the potential of the labor force and reduce the economic losses from demographic losses (Ketova, 2007; Russyak and Ketova, 2008).

Recently, humanity is paid special attention to the state of the environment around it. Scientists around the world are actively engaged in the analysis of issues related to environmental problems and the search for their solutions. An overview of the progress of modern research by scientists around the world is presented, for example, in the work (Fan, He, Hou and Meng, 2020). Comparative analysis of the environmental pollution problem in the past, present, and forecasts for the future are assessed in the research (Huang, 2018).

The quality of the natural environment decreases as a result of active changes in people's lives: processes of industrialization and urbanization are taking place, traditional sources of energy and raw materials are being depleted, ecological balances in nature are disrupted, species of animals and plants are being destroyed, etc. Progress is far from the desired, the demographic "load" on nature is gradually increasing.

The direct objects of pollution are the atmosphere, water and soil. All living organisms are indirect objects of pollution. Failure to take measures to prevent environmental pollution can lead humanity to an environmental disaster. In this regard, the issues of constructing and implementing the concepts of environmentally oriented progress in order to preserve natural resources (McInnes and Roemer-Mahler, 2017) and the implementation of international cooperation on environmental problems (Fedulova, Korchagina, Vik and Martyanov, 2017; Gasanov, Kolotov and Kadnikova, 2017) are relevant.

In this situation, the environmental factor is the most important condition for economic development. Thus, investments in environmental protection are becoming priority sources of positive economic dynamics in the regions (Danilova, Podoprigora and Ufimtseva, 2020; Maher, Fenichel, Schmitz and Adamowicz, 2020). In this regard, the urgency of studying and modeling the environmental factor, as well as evaluating the effectiveness of environmental protection measures, is obvious.

Since the ecological factor in the deteriorating environment is an one of factors of the successful development of socio-economic systems, therefore we need to consider ecological factor when constructing strategies for optimal management of these systems (see, for example, (Belenky and Ketova, 2006; Ketova, Russyak and Derendyaeva, 2013; Ketova, Russyak, Saburova and Vavilova, 2020). The process of constructing strategies for

optimal management of territories involves the using mathematical and computer modeling tools (Gertseva and Gertseva, 2004; Bravo de la Parra and Poggiale, 2005). In this regard, it becomes necessary to build mathematical models to study the environmental factor in order to be able to further take it into account in models of economic dynamics. As a rule, when describing the processes of the ecological systems dynamics, differential equations are used (Vincenot, Giannino, Rietkerk, Moriya and Mazzoleni, 2011).

This research is devoted to the mathematical model construction of the ecological factor dynamics, which allows to estimate the magnitude and compare the change in time of this factor for three basic components of the environment: atmospheric air, water resources and land resources.

2 RESEARCH MATERIALS AND METHODS

Let's define the ecological factor as the percentage of the volume of the environment that does not need cleaning to its total volume, and denote it Ω . The measure of environmental pollution determines the degree of deterioration of the environmental factor.

We divide the ecological factor according to the types of costs aimed at environmental protection and rational using natural resources: investments directed to the protection of atmospheric air, water resources and land resources. The total value of the environmental factor is determined by linear combination:

$$\Omega(t) = \alpha_1\Omega_1(t) + \alpha_2\Omega_2(t) + \alpha_3\Omega_3(t) \quad (1)$$

where α_i – the corresponding terms proportion in the total volume of the factor;

$$\alpha_i \in (0, 1); \sum_{i=1}^3 \alpha_i = 1 \quad (2)$$

values $\Omega_i = \Omega_i(t)$ are measured as a percentage, calculated as part of an unpolluted natural resource in its total volume; index $i=1$ corresponds to the component of atmospheric air, $i=2$ – component of water resources, $i=3$ – component of land resources of the natural environment.

To describe the dynamics of the i – component of the ecological factor, we use an equation of the form:

$$\dot{\Omega}_i = Z_i k_i - \eta_i(t) \Omega_i \quad (3)$$

where Z_i – operating costs for measures taken to environmental protection and activities for rational using of natural resources; k_i – coefficient of conversion of the value, expressed in monetary units, into the amount of refined resources; $\eta_i(t)$ – intensity of pollution of the i – component of the environmental factor.

Note that dynamic equation of the form (3) is easy to integrate into models of economic dynamics. This is due to the internal logic of building these models. The functioning of regional economic systems is a dynamic system of financial, material and information flows, within the framework of which the produced final product is formed. Expressed in monetary terms, the final product is divided into parts: investments in production activities, in the social sphere, in improving the environment, in the field of increasing production efficiency, etc. This distribution should be carried out in an optimal way, based on their criteria for increasing the efficiency of the functioning of the regional economy. An optimal strategy can be constructed using a mathematical apparatus that includes the L.S. Pontryagin and R. Bellman’s optimality principle (Pontryagin, 1961; Belenky, 2007; Ioffe, 2020). The possibility of using this mathematical apparatus presupposes the presence of dynamic equations for the development factors of the economic system of the form (3).

The initial condition at $t = t_0$ looks like:

$$\Omega_i(t_0) = Z_i(t_0)k_i \quad (4)$$

where $Z_i(t_0)$ – a known value from statistical data.

The dynamics equation (3) with initial data (4) is the Cauchy problem. The solution can be carried out by the numerical three-stage Runge-Kutta method of the 3rd order.

Let’s introduce a variable t grid with a step h . We consider the set of points $t_n = nh, n = 0,1,2,\dots$ For calculating the formulas are:

$$\Omega_i^{n+1} = \Omega_i^n + \frac{h}{6} (F_i^1 + 4F_i^2 + F_i^3), \quad (5)$$

$$\begin{cases} F_i^1 = Z(t_n)_i k_i(t_n) - \eta_i(t_n) \Omega_i^n \\ F_i^2 = Z(t_n)_i k_i(t_n) - \eta_i(t_n) (\Omega_i^n + \frac{h}{2} F_i^1) \\ F_i^3 = Z_i(t_n)k_i(t_n) - \eta_i(t_n) (\Omega_i^n - hF_i^1 + 2hF_i^2) \end{cases} \quad (6)$$

3 RESEARCH RESULTS

We consider the regional socio-economic system of the Udmurt Republic. Let’s analyse the development of the ecological situation using its example. To do this, we use the proposed mathematical model of the form (1)-(4) and the solution methods (5) and (6). We fill the mathematical model with the statistical data necessary to solve the problem. Since the environmental factor is characterized by a long-term, deferred effect of exposure, we use a long time interval for analysis. We have the opportunity to analyze the indicator under study for the period 1996-2019.

The dynamics of the ecological factor of the UR, which characterizes atmospheric air, water resources and land resources, for the period 1996-2019, is shown in Figure 1. In this paper, the environment is the totality of atmospheric air, water and land resources. In this regard, in terms of the entire environment as a combination of these three components, we get the graph shown in Figure 2. It displays the degree of environmental pollution in dynamics over the years for the period under study.

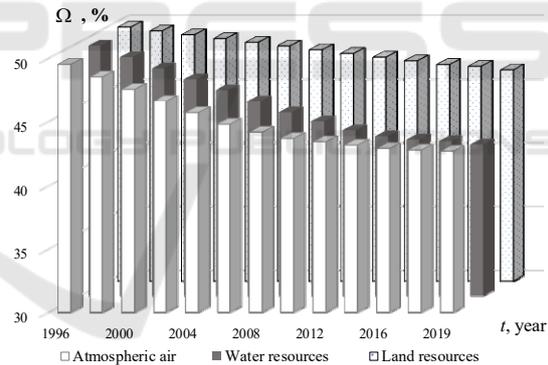


Figure 1: Ecological factor dynamics of the UR, characterizing atmospheric air, water resources and land resources.

It is obtained that the average rate of pollution in the region of atmospheric air was 0.64%, water – 0.73%, land resources – 0.31% (Figure 1). The share of the environment, which requires the using of cleaning measures, on average for the period 1996-2019 in UR was 53.9%. The change over the 23-year period under study is from 50.3% to 56.4% (Figure 2). The most intense was pollution of the atmosphere and water resources. The soil is polluted less intensively, but, nevertheless, production and economic activities cause significant damage to the nature of this cumulative nature of this phenomenon.

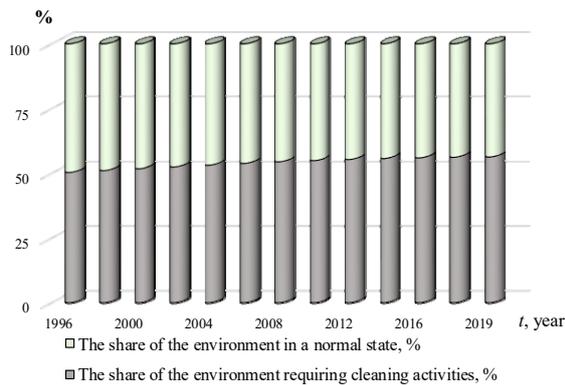


Figure 2: Change in the percentage of polluted environment of the UR.

The calculated dynamics of the environmental factor are shown that the rate of pollution has decreased. This is due to the strengthening of measures aimed at combating environmental pollution in the region in the last decade. This government policy is carried out within the framework of the state program of the Russian Federation “Environmental Protection”. In the future, the policy of respect for the environment should be continued.

The value of the environmental factor directly depends on the effectiveness of environmental protection measures. Let’s analyze this indicator. We designate the effectiveness of environmental protection measures ϕ ; it is the reciprocal of the intensity of environmental pollution $\eta = \eta(t)$:

$$\phi_i = \frac{a_i}{\eta_i} \tag{7}$$

Based on (3), we determine the intensity of environmental pollution by the formula:

$$\eta_i = \frac{k_i Z_i - \Omega_i}{\Omega_i} \tag{8}$$

Let’s determine the value of conversion coefficient k , expressed in monetary units, into the amount of refined resources. To do this, we calculate the ratio of the cleaned volume resource U of the to the current costs of protecting this resource P . Next, we use information on the annual indicators of pollution and purification of atmospheric air, water and land resources of the UR for the period 1996-2019 and information on the current costs of

environmental protection of the UR for the period 1996-2019 (in the calculations, the current financial costs for the possibility of comparing the obtained results are adjusted to 2019 prices):

$$\zeta_i^j(t) = \frac{U_i^j(t)}{P_i^j(t)} \tag{9}$$

$$i = \overline{1,3}, j = \overline{1996, 2019}$$

Average value of the coefficient for each resource: $\bar{\zeta}_1 = 0.144$, $\bar{\zeta}_2 = 0.021$, $\bar{\zeta}_3 = 0.014$. Further, we calculate the values of the coefficients for converting the components of the environmental factor into monetary terms, as a ratio $\bar{\zeta}_i(t)$ to the known values of the total volume of these resources. Then $k_1 = 3.5 \cdot 10^{-6}$, $k_2 = 3.4 \cdot 10^{-6}$, $k_3 = 3.3 \cdot 10^{-6}$. As a result of calculations for the studied period 1996-2019 for UR, the following values of pollution intensity are obtained, in accordance with formula (8): for atmospheric air – $\eta_1 = 0.01 \pm 0.00023$, for water resources – $\eta_2 = 0.003 \pm 0.00006$, for land resources – $\eta_3 = 0.01 \pm 0.00057$.

Based on the values of the annual pollution indicators and purification of atmospheric air, water and land resources of the UR for the period 1996-2019 and the estimated rate of change in the percentage of environmental pollution, the values of the proportionality coefficients from formula (7) for the three resources (air, water, land): $a_1 = 0.003$, $a_2 = 0.001$, $a_3 = 0.004$.

The efficiency of environmental protection measures, determined by formula (7), for atmospheric air is about 25.0%, for water resources – 47.5%, for land resources – 38.2%.

4 CONCLUSIONS

The authors have proposed a mathematical model of the environmental factor in this research. Modeling of the environmental factor is carried out by types of costs aimed at measures taken to environmental protection and activities for rational using of natural resources. In the model of the environment, three components appear, which are presented in combination: water, air and land resources.

The presented in the article mathematical model allows solving the problem of calculating the

environmental factor dynamics, taking into account the effectiveness of environmental protection measures. The solution to the problem of the environmental factor modeling is found using a combination of analytical and numerical methods.

The mathematical model of the ecological factor dynamics is built in the form of a differential equation of this type, which allows taking into account the ecological factor in the models of economic dynamics, as well as when constructing strategies for optimal control of socio-economic systems using the L.S. Pontryagin and R. Bellman's optimality principle.

The calculation of the environmental factor is carried out using the example of the Udmurt Republic. For this, statistical data on the annual indicators of pollution and purification of atmospheric air, water and land resources of the UR and information on the current annual costs for measures to protect the environment of the UR are used. Since the environmental factor is characterized by a delayed impact, the long available period 1996-2019 was chosen.

It has been established that environmental pollution in the region occurs at an average annual rate of 0.48%. However, over the past decade, there is a decrease in the rate of environmental pollution. The estimated efficiency of environmental protection measures for atmospheric air is about 25.0%, for water resources is 47.5%, for land resources is 38.2%. This state of affairs leads us to the need to strengthen measures aimed at improving the state of the environment in the region.

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