Balancing Quality and Efficiency in Private Clustering with Affinity Propagation

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Abstract: In many machine learning applications, training data consists of sensitive information from multiple sources. Privacy-preserving machine learning using secure computation enables multiple parties to compute on their joint data without disclosing their inputs to each other. In this work, we focus on clustering, an unsupervised machine learning technique that partitions data into groups. Previous works on privacy-preserving clustering often leak information and focus on the k-means algorithm, which provides only limited clustering quality and flexibility. Additionally, the number of clusters k must be known in advance. We analyze several prominent clustering algorithms' capabilities and their compatibility with secure computation techniques to create an efficient, fully privacy-preserving clustering implementation superior to k-means. We find affinity propagation to be the most promising candidate and securely implement it using various multi-party computation techniques. Privacy-preserving affinity propagation does not require any input parameters and consists of operations that are relatively efficient with secure computation. We consider passive security as well as active security with an honest and dishonest majority. We offer the first comparison of privacy-preserving clustering between these scenarios, enabling an understanding of the exact trade-offs between them. Based on the clustering quality and the computational and communication costs, privacy-preserving affinity propagation offers a good trade-off between quality and efficiency for practical privacy-preserving clustering.

1 INTRODUCTION

The field of machine learning (ML) has received considerable attention in recent years thanks to its manifold applications. Furthermore, the increased storage capabilities and computational power of devices enable models to be trained on immense data pools. Training data is often aggregated from multiple sources to increase the utility of the resulting model, and cloud providers such as Amazon SageMaker and the Google AI Platform offer the necessary storage and computation as a service. However, as the availability of training data for such algorithms increases, the relevance of protecting its security and privacy also grows. Regulations such as GDPR restrict the use of personal information, and a need for privacy-preserving solutions arises.

For this reason, using secure multi-party computation (MPC) for privacy-preserving machine learning (PPML) has become a hot research topic (Juvekar et al., 2018; Mishra et al., 2020; Rathee et al., 2020; Patra et al., 2021). MPC uses cryptographic techniques to allow several parties to compute the output of a function without revealing the private input values to each other. Using MPC, multiple data owners can securely train an ML model without any information leakage to an (internal or external) adversary.

In this work, we focus on clustering, a form of unsupervised ML in which similar data points are grouped together. Clustering algorithms are useful, for example, to segment a market using consumer preferences (Chaturvedi et al., 1997) or group photos of diseased organs in medical imaging (Masulli and Schenone, 1999). A well-known and simple clustering algorithm is k-means (Steinhaus, 1956), which iteratively updates cluster centers and assignments. This algorithm has been the focus of much privacy-preserving clustering research (cf. §6); however, the quality of clustering results from this algorithm is limited (cf. §3). Furthermore, k-means requires the number of clusters k to be chosen in advance, which is a challenge if no party has access to the full pool of training data, as is the case in many privacy-preserving settings. In general, choosing parameter values for clustering algorithms becomes significantly less trivial when algorithms are executed in a privacy-preserving manner.

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Furthermore, privacy research has focused on private clustering in the passive security model (cf. §2.2), neglecting the stronger active security model needed to securely perform secure computation between mutually distrusting parties.

Contributions and Outline. After discussing the preliminaries in §2, we provide the following contributions:

- We give a comprehensive analysis of multiple clustering algorithms with respect to their potential for efficient and high-quality privacy-preserving clustering with MPC techniques (cf. §3).
- Based on our analysis, we identify affinity propagation as a promising candidate. Affinity propagation is used in a wide variety of privacy-critical medical applications, such as epilepsy (Leone et al., 2007) and cancer detection (John et al., 2016). It automatically determines the number of clusters and is tolerant to outliers (cf. §4). We provide the first fully privacy-preserving affinity propagation protocol with MPC usable in the passive and active security model.
- We provide an experimental evaluation of our protocol in multiple security models, i.e., considering an active/passive adversary and honest/dishonest majority (cf. §5). Our results enable the assessment of the overhead associated with stronger security, which is important for meaningfully balancing privacy and efficiency in privacy-preserving clustering applications. Our code is available at https://encrypto.de/code/ppAffinityPropagation.

We discuss related work on privacy-preserving clustering in §6 and conclude with §7.

2 PRELIMINARIES

This section introduces the preliminaries of clustering (cf. §2.1) and secure computation (cf. §2.2).

2.1 Clustering

Clustering is an unsupervised ML technique that groups data points into clusters. Since it is an unsupervised ML technique, learning occurs without knowledge of any true grouping of points or any data labels. The goal is to group similar records into the same cluster, while elements in different clusters should be maximally different (Jain et al., 1999).

Clustering Algorithm Types. We differentiate between partitioning-based, hierarchical, distribution-based, and density-based clustering algorithms. Partitioning-based algorithms separate the data set into several non-overlapping groups, whose center is considered the center of the data points in this group (Xu and Tian, 2015). Often these algorithms optimize an objective function in an iterative fashion (Xu et al., 1998). Examples are k-means (Steinhaus, 1956) and affinity propagation (AP) (Frey and Dueck, 2007), which we both examine closely in this work (cf. §3). This approach realizes a hard clustering, i.e., every input record is assigned to exactly one cluster, whereas soft clustering, e.g., based on distributions, may assign a point to several clusters with varying probabilities. Distribution-based clustering approximates the original distributions from which data points are assumed to have been drawn (Xu and Tian, 2015), as is the case for Gaussian mixture models clustering (GMM). Hierarchical clustering algorithms represent a data set as a binary tree of data points and iteratively merge or divide clusters based on the derived tree structure (Xu and Tian, 2015). The computational complexity of these algorithms is very high; already the first step of most hierarchical clustering algorithms, computing pair-wise distances and performing a sort, requires $\mathcal{O}(n^2 \log n)$ time complexity (Xu and Tian, 2015). Therefore, we do not consider them as candidates for our privacy-preserving algorithm. Density-based algorithms, such as Density-Based Spatial Clustering of Applications with Noise (DBSCAN) (Ester et al., 1996), cluster data points that lie closely together in a dense area.

Clustering Quality Scores. To measure clustering quality, internal or external clustering indices are used. External indices compare the clustering result to a known ground truth, which is a known true assignment of each point to a cluster. A ground truth is only available for benchmark data sets and not present for practical applications, since clustering is an unsupervised ML technique. In those cases, internal indices are used instead to assess the quality of a clustering result. These indices focus on internal characteristics, i.e., they measure the compactness of the elements assigned to one cluster and the separation between different clusters (Arbelaitz et al., 2013). To provide a comprehensive quality assessment in §3 and §5, we use an external and an internal clustering quality index:

The adjusted rand index (**ARI**) (Hubert and Arabie, 1985) is a widely used (Arbelaitz et al., 2013; Vinh et al., 2010) external index that assesses all inner-cluster and inter-cluster data point pairs, where the 2 points were assigned to the same cluster and to different clusters, respectively. Any pairs correctly identified as belonging to the same cluster or to different clusters increase the ARI value. The ARI lies in a range of $[-1, 1]$, and a value of 1 indicates that the clustering result is equal to the ground truth.

The silhouette index (**SI**) (Rousseeuw, 1987) is a

Table 1: MPC protocols for different security models, categorized by their security level and number of corruptions. Names are from MP-SPDZ (Keller, 2020).

	Passive	Active
Dishonest Majority	*Semi2k*	*SPDZ2k*
Honest Majority	*Replicated2k*	*PsReplicated2k*

well-known internal index (Arbelaitz et al., 2013) with range $[-1, 1]$ and quantifies the relation between inner-cluster and inter-cluster distance between points. A good result has an SI value close to 1, indicating a small inner-cluster and large inter-cluster distance.

2.2 Multi-Party Computation (MPC)

MPC protocols allow the realization of privacy-preserving clustering algorithms. They privately evaluate an algorithm under "encryption" s.t. the inputs and intermediate values remain hidden and only the output is revealed. The two most prominent cryptographic protocols for MPC are constant-round garbled circuits (GC) by (Yao, 1986) and the multi-round Arithmetic or Boolean Sharing protocol by (Goldreich et al., 1987).

Honest/Dishonest Majority. This terminology specifies the fraction of possible corruptions among the parties involved in MPC. In a *dishonest majority* setting, the adversary may corrupt all but one party. This setting is more complex and, thus, more expensive than an *honest majority* setting, where only a minority of parties are assumed to be corrupted.

Passive/Active Security. Adversarial behavior can be passive or active, which implies specific assumptions about the adversary's capabilities in an MPC protocol. *Passive* security protects against passive adversaries, who adhere to the protocol's specifications while attempting to learn as much as possible about that data of other parties (Goldreich, 2009). In the *active* security model, an active adversary can arbitrarily deviate from the protocol, i.e., this model provides stronger security guarantees. However, a trade-off between efficiency and security must often be made, since actively secure protocols incur significant overhead (Lindell and Pinkas, 2015).

Tab. 1 contains an overview of several MPC protocols used in our evaluation. We implement privacy-preserving affinity propagation (cf. §4) with the MP-SPDZ framework for MPC (Keller, 2020): *SPDZ2k* (Cramer et al., 2018) is the first protocol for the active, dishonest majority setting over $\mathbb{Z}_{2^k}$, and was implemented by (Damgård et al., 2019). *Semi2k* is a trimmed-down version of SPDZ2k for the passive, dishonest majority setting. *Replicated2k* is a passively secure protocol for 3 parties and honest majority based on (Araki et al., 2016). *PsReplicated2k* (Eerikson et al., 2019) extends (Lindell and Nof, 2017) to the ring setting and uses ideas from (Cramer et al., 2018), offering protection in the passive, honest majority setting.

3 ANALYSIS OF CLUSTERING ALGORITHMS

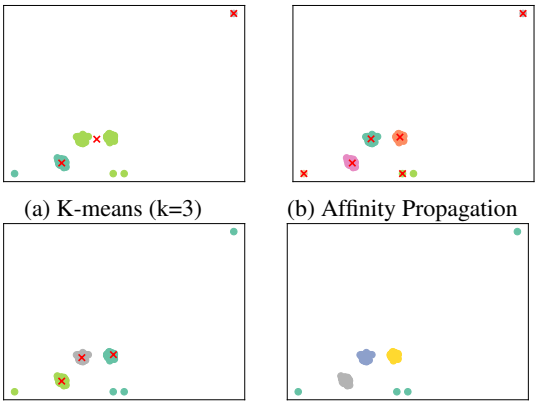
In order to determine promising algorithm candidates for privacy-preserving clustering, we compare the following prominent algorithms: k-means (Steinhaus, 1956), Gaussian mixture models (GMM) (Dempster et al., 1977), affinity propagation (AP) (Frey and Dueck, 2007), and Density-Based Spatial Clustering of Applications with Noise (DBSCAN) (Ester et al., 1996). These algorithms were selected based on (1) their widespread use (Bano and Khan, 2018; Xu and Tian, 2015; Xu and Wunsch, 2005) and (2) simplicity, which is crucial for efficiency with MPC. K-means and AP are partitioning-based algorithms (cf. §2.1). GMM is a distribution-based clustering algorithm, and DBSCAN is a density-based clustering algorithm.

The input parameters of these algorithms have a direct impact on the clustering result and its quality. In the following, we discuss and visualize the influence of these parameters, as well as other characteristics such as the tolerance to outliers, flexibility for multiple cluster shapes, and other non-visual components on the clustering quality. Furthermore, we investigate the algorithms' suitability for a privacy-preserving realization via MPC.

3.1 Tolerance to Outliers

Data sets often contain outliers, i.e., data records that are very untypical compared to the majority of the set. Therefore, in the context of clustering, it is imperative that the clustering result, including identified cluster centers and cluster assignments, is not distorted or greatly affected by these data records. If a clustering algorithm either explicitly identifies outliers or if outliers do not distort the result, this algorithm can be considered as tolerant to outliers. Especially in a privacy-preserving context, data owners may not know whether their own data contains an outlier or not, since the other data owners' input data is unknown.

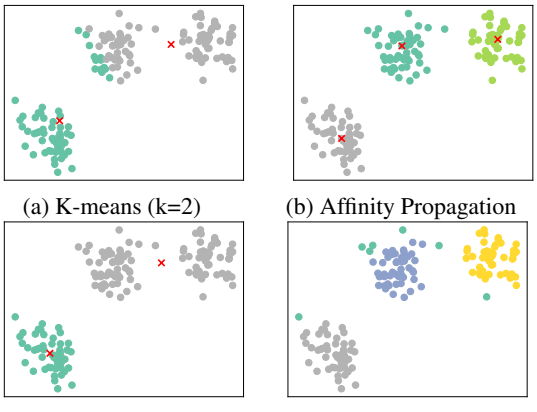
DBSCAN defines a special notion of *noise* that allows outliers to be flagged as such (Ester et al., 1996; Xu and Tian, 2015). Since AP does not require the number of clusters as input parameter, outliers are typically detected as small, separate clusters, which can be marked as outliers based on their size and which do not affect the attributes of other identified clusters (Xu and Tian, 2015). Therefore, both DBSCAN and AP



(a) K-means (k=3) (b) Affinity Propagation

(c) GMM (k=3) (d) DBSCAN

Figure 1: Example for outlier tolerance of four clustering algorithms. The red crosses mark the clusters' centers in the respective clustering algorithm's output.



(a) K-means (k=2) (b) Affinity Propagation

(c) GMM (k=2) (d) DBSCAN

Figure 2: Effect of setting the wrong numer of clusters k for clustering with k-means and GMM, whereas DBSCAN and AP determine the number of clusters automatically. The red crosses mark the clusters' centroids in the respective clustering algorithm's output.

are tolerant to outliers. In contrast, k-means and GMM are both sensitive to outliers. Specifically, k-means requires all data points to be assigned to one cluster, even if a point does not fit to any cluster. Since the algorithm specifies the cluster center as the mean of all points, outliers heavily distort these centers or form an own cluster, forcing other true clusters in the data to become merged (Xu and Wunsch, 2005). Similarly, the clusters resulting from GMM are assumed to have been drawn from a Gaussian distribution, so the center of all clusters is also the mean of data points, which is inherently sensitive to outliers (Xu and Tian, 2015).

To demonstrate these strengths and weaknesses of the four clustering algorithms with respect to outliers, we provide an example in Fig. 1. We show the clustering results for a data set with three circular clusters, each drawn from a 2-dimensional Gaussian distribution, and 4 manually added outliers. DBSCAN (Fig. 1d) does not include the concept of cluster centers. We provided both k-means and GMM with the correct number of clusters k=3. However both algorithms are sensitive to outliers. K-means (Fig. 1a) marks one outlier as a single cluster while it merges two true clusters to a combined one, thus, it returns a result that is far from optimal. GMM (Fig. 1c) groups the 3 clusters correctly; however, one centroid is affected by the outliers and, thus, not centered in the "core" cluster. AP and DBSCAN, on the other hand, perform well; AP (Fig. 1b) identifies the outliers as additional clusters that do not influence the 3 other clusters and can be identified as outliers based on their cluster size. DBSCAN inherently identifies all outliers as such.

3.2 Clustering Parameters

Another problem of k-means and GMM is the necessity to choose the number of clusters k in advance (Xu and Tian, 2015). Especially in a privacy-preserving setting, it may be difficult to set parameters like k in advance without knowledge of the entire data set. Fig. 2 shows that when the number of clusters k is set to the wrong value (k=2 instead of 3), the clustering result of k-means and GMM can be heavily distorted.

AP and DBSCAN do not require k to be chosen in advance and are initialized with other parameters. DBSCAN requires 2 parameters, *minPts* and ε, where *minPts* quantifies the minimal number of data points required to form a cluster, while ε is the maximum distance between 2 data points considered "neighbors." Especially setting ε can be challenging in a privacy-preserving clustering application, as each data owner only holds a subset of the input data. AP depends on the *preference* parameter, which can be tuned manually by testing several possible values, running the AP clustering with them, and then choosing the best one (using a clustering quality index). Alternatively, preference can be set to the minimum or median Euclidean squared distance between data points (Frey and Dueck, 2007). Therefore, AP is the most simple to initialize, which is important in a privacy-preserving setting.

3.3 Other Attributes and Summary

The clustering quality scores ARI and SI (cf. §2.1) for the example data sets from §3.1 and §3.2 can be found in Tab. 2; the underlying distributions from which these data sets were drawn are considered as the

ground truth. The scores confirm the visual observations discussed in the previous subsections.

We summarize the strengths and weaknesses of all clustering algorithms in Tab. 3 including additional attributes of each algorithm, namely flexible choice of distance metrics, determinism, cluster shapes, computational complexity, and complex operations.

Since k-means averages input records to determine cluster centers and GMM uses the Gaussian distribution, they do not directly support nominal variables (Huang and Ng, 1999); AP and DBSCAN are more flexible, as they can use any distance function (Ester et al., 1996; Frey and Dueck, 2007). Furthermore, the original k-means and GMM algorithms are instances of the expectation-maximization (EM) algorithm, which is non-deterministic and often uses random initialization. This initialization strategy can lead to convergence at a local optimum (Peña et al., 1999). AP is deterministic (Frey and Dueck, 2007), and DBSCAN only varies in rare cases for elements that lie exactly in the neighborhood of elements from different clusters (Ester et al., 1996).

So far, we focused on circular or spherical clusters, which are non-convex. However, data can also come in the form of elongated or irregularly shaped clusters influenced, e.g., by the data's dimensions. Since some clustering algorithms, such as k-means and AP, depend on a dissimilarity measure like Euclidean distance and assign points to the cluster with the closest center, these algorithms do not perform as well on irregular data sets (Xu and Tian, 2015). GMM assumes that data is drawn from Gaussian distributions and is therefore more flexible, since oval-shaped distributions still fall within this assumption; however, GMM performs poorly on other irregular cluster shapes (Xu and Tian, 2015). Since DBSCAN is based on density of points rather than the distance to a cluster center, DBSCAN can detect arbitrarily shaped clusters (Ester et al., 1996). These observations are confirmed by our experiments reported in Tab. 2.

Secure computation protocols add significant overhead to their plaintext equivalents; therefore, we must balance clustering quality and complexity. We include the computational complexity of all algorithms in Tab. 3. The naive implementation of k-means and AP have a computational complexity of $\mathcal{O}(n^2)$ (Xu and Tian, 2015; Frey and Dueck, 2007; Ester et al., 1996), where n is the number of data points. DBSCAN and GMM, in contrast, have a complexity of $\mathcal{O}(n^3)$ when implemented using MPC. GMM requires an expensive matrix inversion operation, and DBSCAN uses a queue or stack for cluster expansion; to obliviously realize this queue, the computational complexity of the original baseline DBSCAN algorithm (Ester et al., 1996) increases from $\mathcal{O}(n^2)$ to $\mathcal{O}(n^3)$ (Bozdemir et al., 2021). Thus, privacy-preserving DBSCAN and GMM are significantly more costly than the other algorithms.

Table 2: Clustering quality measured with ARI and SI (cf. §2.1) of the four clustering algorithms k-means, AP, GMM, and DBSCAN for different data sets. Larger values (best in bold) indicate a better clustering result.

Attribute	Algorithm	ARI	SI
Outliers (Fig. 1, §3.1)	k-means (k=3)	0.55	0.67
	AP	0.96	**0.85**
	GMM (k=3)	**0.98**	0.79
	DBSCAN	0.96	0.83
# of Clusters (Fig. 2, §3.2)	k-means (k=3)	0.47	0.55
	AP	**1.00**	**0.73**
	GMM (k=3)	0.57	0.60
	DBSCAN	0.93	0.65
Shape (§3.3)	k-means (k=3)	0.52	**0.51**
	AP	0.61	0.50
	GMM (k=3)	**1.00**	0.45
	DBSCAN	0.95	0.45

Based on the results for the four clustering algorithms, we find AP to offer a good trade-off between clustering quality and complexity, taking its favorable handling of outliers and easily determinable clustering parameters into account.

4 PRIVACY-PRESERVING AFFINITY PROPAGATION

Affinity propagation (AP) is an iterative message-passing algorithm, where intuitively, each data point sends two messages to all other points in every iteration. A point's first message communicates the attractiveness of another point as its cluster center, or exemplar. This message is known as the **responsibility**. Based on all received responsibility values, each data point replies with a message quantifying its suitability as a cluster center for each other point, known as the **availability**. These messages are revised in each iteration until a consensus emerges, which identifies exemplars and point assignments to those exemplars. All message updates are based on the previous changes and the distance between data points in general. Computationally, message passing is implemented through iterative matrix updates of the availability and responsibility matrices, which store the pair-wise values.

Adapting AP for MPC (cf. §2.2) requires an in-depth analysis of the necessary operations of the algorithm. Fortunately, AP mainly consists of multiplication and addition in terms of arithmetic, which have efficient MPC instantiations based on Arithmetic

Table 3: Suitability of plaintext clustering algorithms for privacy-preserving clustering. ✓ indicates that the algorithm provides the property, while ✗ indicates that it does not.

Attributes		k-means (Steinhaus, 1956)	Affinity Propagation (Frey and Dueck, 2007)	GMM (Dempster et al., 1977)	DBSCAN (Ester et al., 1996)
Automatic choice of # clusters k		✗	✓	✗	✓
Tolerance to outliers		✗	✓	✗	✓
Non-spherical cluster support		✗	✗	✓	✓
Flexible choice of distance metrics		✗	✓	✗	✓
Determinism		✗	✓	✗	✓
MPC-friendly operations		✓	✓	✗	✗
Computational Complexity in MPC		$\mathcal{O}(n^2)$	$\mathcal{O}(n^2)$	$\mathcal{O}(n^3)$	$\mathcal{O}(n^3)$
Private Clustering Protocols (cf. §6)	Leakage of intermediate values	e.g., (Vaidya and Clifton, 2003), (Jagannathan and Wright, 2005)	(Zhu et al., 2012)	(Lin et al., 2005), (Hamidi et al., 2019)	e.g., (Liu et al., 2013), (Rahman et al., 2017)
	Fully private	(Bunn and Ostrovsky, 2007), (Patel et al., 2012), (Jäschke and Armknecht, 2019), (Mohassel et al., 2020)	**This work**		(Bozdemir et al., 2021)

sharing (cf. §2.2). Furthermore, the iterative matrix updates of AP often require maximum or minimum operations, which can be implemented with comparisons using special subprotocols for secure computation. In §4.1 to §4.3, we formalize protocols for the constituent parts of privacy-preserving AP, where all defined variables are secret shared among all participating parties, excluding the loop indices. This approach hides all intermediate results, making our private AP fully privacy-preserving, as discussed in §4.4. §4.1 discusses the necessary setup operations, including distance calculations and a private calculation for the preference parameter value. §4.2 and §4.3 describe how responsibility and availability matrices are updated, respectively. In practice, each update is damped with the previous update using a weighted average of the previous and current updates. The computation and communication costs reported in §5.2 to §5.4 omit the damping step to enable integer-only computation; however, including damping does not significantly change the runtime and communication costs, since its addition only incurs two arithmetic operations per iteration: multiplication by the damping coefficient and addition of the components.

4.1 Setup and Output

Alg. 1 specifies the modular steps of our privacy-preserving AP protocol for both the first distance calculation, solely with addition and multiplication, and the final choice and assignment of exemplars, mainly using comparison operations. Before the first iteration, the algorithm begins with the calculation of pair-wise squared Euclidean distances for all points in the data set in Step 5, whose additive inverses are stored in the similarity matrix S. Alternatively, other distance metrics could be used. Our protocol automatically selects the minimal negative difference between 2 data points as the preference value in Step 7, as recommended by (Frey and Dueck, 2007). This calculation easily integrates into the initial calculation of the distance between data points and is therefore not expensive.

Using the calculated distances, the availability and responsibility matrices are iteratively updated until convergence, when cluster centroids and assignments no longer change with additional iterations. In a final step, the state of availability and responsibility values determines which data points are chosen as exemplars, i.e., cluster centers.

4.2 Responsibility Update

Intuitively, the responsibility value between two data points quantifies how likely one point is to serve as an exemplar for the other point. The updates for responsibility r based on availaibility a and similarity s in AP are executed using the following rule:

$$r(i,k) \leftarrow s(i,k) - max_{k's.t.k' \neq k}\{a(i,k') + s(i,k')\},$$

where responsibility $r(i,k)$ quantifies the evidence for point k as an exemplar for point i. Expanding and modularizing this equation for use in secure computation yields Alg. 2.

4.3 Availability Update

The availability of one data point for another represents how likely one point is to choose the other point as its exemplar. Availability is quantified using two rules. The availability a of a point i to another point k based on responsibility r is:

$$a(i,k) \leftarrow min\left\{0, r(k,k) + \sum_{i's.t.i' \notin \{i,k\}} max\{0, r(i',k)\}\right\},$$

where availability $a(i,k)$ quantifies the evidence for point k to choose point i as an exemplar. The self-availability, or availability for a data point k to itself,

Algorithm 1: Privacy-Preserving Affinity Propagation.

Input: iterations iter, data set $\mathcal{D}$ with N points of dimension features
Output: E, exemplars of all points exEach

```
S = zeros((N,N))
for i in range(N) do
    for j in range(N) do
        for  k in range(features) do
            d=(D[k][i]-D[k][j])·(D[k][i]-D[k][j])
            S[i][j] = S[i][j] + d
            pref = pref>S[i][j]?pref:S[i][j]
        end for
    end for
end for
S = -S
for i in range(n) do
    S[i][i] = preference
end for
N=size(S)
A=zeros(N,N), R=zeros(N,N)
λ=0.5
for it in range(iter) do
    AS = A+S
    UpdateResponsibilityMatrix(AS,S,R)
    UpdateAvailabilityMatrix(AS,S,A,R)
end for
E = R+A
ex = zeros(N)
exEach = zeros(N)
for i in range(N) do
    ex[i] = (E[i][i]>0)?1:0
end for
for i in range(N) do
    for j in range(N) do
        exEach[i]=((ex[j]>0)∧
(exEach[i]≥0)∧(E[i][j]>E[i][exEach[i]])∨
(ex[j]>0)∧(exEach[i]<0))?j:exEach[i]
    end for
end for
return E, exEach
```

is calculated differently:

$$a(k,k) \leftarrow \sum_{i's.t.i' \neq k} max\{0, r(i',k)\}.$$

We combine, expand, and modularize these update rules for implementation with a secure computation protocol in Alg. 3.

4.4 Security Discussion

The security of our privacy-preserving AP protocol follows from the security of the employed MPC techniques. These techniques guarantee that a passive or active adversary learns nothing beyond what can be learned from the output. Initially, all data owners secret-share their clustering input among themselves or among several non-colluding parties in an outsourcing scenario (Kamara and Raykova, 2011). As these parties have access only to secret shares, no private information can be extracted. The distance calculations for the similarity matrix and for the preference value are realized with Arithmetic or Boolean sharing (Yao, 1986; Goldreich et al., 1987; Beaver et al., 1990). MP-SPDZ also includes conversions between these realizations, which are also provably secure (Rotaru and Wood, 2019). The same techniques are used for all other calculations in the protocol. All data but the output of the clustering remains secret-shared during the entire protocol, so our privacy-preserving AP protocol is fully privacy-preserving.

Algorithm 2: Update Responsibility Matrix.

Input: AS, S, oldR
Output: R

```
R = oldR
row = zeros(N)
idx = zeros(N)
Y = zeros(N)
for r in range(N) do
    for p in range(N) do
        index = idx[r]
        max = row[r]
        idx[r] = AS[r][p]>max?AS[r][p]:index
        row[r] = AS[r][p]>max?AS[r][p]:max
    end for
end for
R = S-repeat(row, columnRep=N, rowRep=1)
for index in idx do
    AS[index] = -inf
end for
for r in range(N) do
    for p in range(N) do
        Y[r] = AS[r][p]>Y[r]?AS[r][p]:Y[r]
    end for
end for
for i in range(N) do
    R[i,idx[i]] = S[i, idx[i]] - Y[i]
end for
return R = (1-λ)·R + λ·oldR
```

5 EVALUATION

In this section, we benchmark our privacy-preserving AP implementation (cf. §4) w.r.t. the achieved clustering quality, its efficiency in comparison to related

Algorithm 3: Update Availability Matrix.

```
Input: AS, S, oldA, R
Output: A
RP = zeros(N)
for r in range(N) do
    for p in range(N) do
        RP[r] = R[r][p]>0?R[r][p]:0
    end for
end for
for i in range(N) do
    RP(i,i) = R(i,i)
end for
sum = sum(R, dimension=1)
A = repeat(sum, columnRep=1, rowRep=N) - RP
AP = zeros(N)
for r in range(N) do
    for p in range(N) do
        AP[r] = A[r][p]>0?0:A[r][p]
    end for
end for
for i in range(N) do
    AP(i,i) = A(i,i)
end for
A = AP
return A = (1-λ)·A+λ·oldA
```

Table 4: Data sets and parameter values.

Data Set	Size	# Clusters	Iterations	Preference
LSUN	400	3	127	24
Blobs	100	2	28	6
	200	2	61	8
	300	2	104	20
	400	2	95	23
	500	2	126	18.8
	100	6	23	3
	100	10	23	40

works, as well as its scalability.

MP-SPDZ. We implement our protocol using the MP-SPDZ framework for MPC (Keller, 2020). It realizes secure multi-party computation (MPC) in the active and passive security models with honest and dishonest majorities. The MPC protocols of MP-SPDZ specified in Tab. 1 run over a ring $\mathbb{Z}_{2^k}$ with $k = 64$ bits. All experiments are run on a 16-core machine for each party, with a 2.8 GHz Intel Core i9-7960X processor and 128GB RAM, running Linux. We evaluated a LAN setting with bandwidth 10Gbps and RTT 0.2ms.

Data Sets. Several data sets were used in our evaluation. First, we chose the LSUN cluster benchmarking data set (Ultsch, 2005) for a comparison of privacy-preserving AP's efficiency with other works on privacy-preserving clustering (Mohassel et al., 2020; Jäschke and Armknecht, 2019). It contains 400 2-dimensional data points and 3 rectangular clusters, but no outliers. Furthermore, we created artificial "blob" data sets to benchmark privacy-preserving AP's scalability with respect to the data set's size. For simplicity, all clusters are spherically shaped and have a standard deviation of 0.3, as we only use them for demonstration purposes on how to choose a preference value, as well as runtime and communication cost benchmarks with varying data set sizes. The artificial data sets are also 2-dimensional, and their sizes range from 100 to 500 elements.

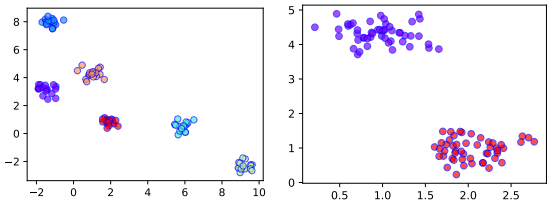
5.1 Clustering Quality & Input Parameters

We first discuss the effect of input parameters on the clustering quality. We measure the quality of a clustering output with the adjusted rand index (ARI) and silhouette index (SI), cf. §2.1. Here we do not focus on other aspects, e.g., the tolerance to outliers, the handling of nominal variables, or determinism as discussed in §3, but rather on the question of how to determine AP's input parameter, i.e., the preference, in a privacy-preserving setting. This is a crucial question for practical applications of privacy-preserving clustering, but was completely neglected so far by previous work.

Tab. 4 lists the manually tuned values for preference and the number of iterations until convergence for all data sets that we used in this work. For the artificial "blob" data sets, AP yields a perfectly clustered result that reflects exactly the ground truth. The result of clustering LSUN with AP corresponds to an ARI score (cf. §2.1) of 0.53 and a SI of 0.54. In comparison, k-means (with k set to the true number of clusters as shown in Tab. 4) results in a lower and hence worse ARI of 0.44 and SI of 0.50.

As the data set size increases, the number of iterations until convergence also increases. Furthermore, more complex data sets like LSUN (i.e., not simply spherical shaped clusters) require additional iterations to converge. However, increasing the number of clusters within a data set in general affects only the optimal preference value, but not the number of iterations. Often, the optimal preference value also tends to grow with the data set size and the number of clusters.

In a privacy-preserving setting, participants are likely not able to tune the preference value before the clustering given that they only have access to a subset of the input data. In such cases, we recommend following the suggestion of the original work that introduced AP (Frey and Dueck, 2007) and set the preference to a privately calculated median or minimum sum of



(a) Median (b) Minimum sum

Figure 3: Example results when setting the preference value to the median or minimum sum of squared distances between the input records.

Table 5: Effect of the preference on number of iterations for the "Blob" data sets with 100 elements and 2 or 6 clusters.

# Clusters	Preference		Iterations	ARI	SI
2	tuned:	−6	28	1.0	0.85
2	min:	−23.65	52	1.0	0.85
2	med:	−4.10	27	0.50	0.32
6	tuned:	−3	23	1.0	0.85
6	min:	−242.78	280	0.54	0.62
6	med:	−22.11	51	1.0	0.85

squared distances, which we simply call the distance between data points. To do so, e.g., (Makri et al., 2021; Aggarwal et al., 2010) propose efficient MPC-protocols for computing minimum and median. This approach often yields a good clustering result (Frey and Dueck, 2007). For example, we give the clustering results for our 2 artificial 100-element "Blobs" data sets with 2 and 6 clusters in Fig. 3. Setting the preference value to the minimum distance yields a perfect clustering result when the data set comprises 2 clusters in Fig. 3b. Similarly, the 6-cluster data set can be clustered perfectly when the preference value is set to the median distance as shown in Fig. 3a.

However, the preference value affects the clustering result and the number of iterations until convergence. We show this effect in Tab. 5. After preference tuning, AP converges on the 2-cluster "Blobs" data set after 28 iterations, while it takes 52 iterations when the preference is set to the minimum distance (yielding the same clustering result). Similarly, although choosing the median distance results in a good clustering result for the 6-cluster data set, the algorithm converges after 51 iterations, rather than 23 with tuning.

5.2 MPC Performance

We evaluate privacy-preserving AP on a wide range of MPC protocols and give an intuition for the efficiency-security trade-off in different security models for privacy-preserving AP. Therefore, we benchmark the costs associated with our privacy-preserving AP implementation for all settings described in Tab. 1 on the public data set LSUN (Ultsch, 2005). Furthermore,

Table 6: Computation and communication costs for clustering LSUN in the passive (P)/active (A) security model, as well as for honest (H)/dishonest (D) majority.

		# Parties	Runtime (hr)	Communication (GBytes)
P	H	3	22.67	103
A	H	3	33.01	550
P	D	3	107.18	42,658
A	D	3	660.05	312,127
P	D	2	54.92	17,471
A	D	2	500.19	156,079

we carried out experiments for 3 parties in an honest majority setting and for both 2 and 3 parties for a dishonest majority of parties. We executed each experiment 5 times and report the average cost per iteration (see Tab. 4 and Tab. 5 for examples how many iterations are needed). All resulting computational and communication costs for clustering LSUN in these settings are given in Tab. 6. As expected, the honest-majority protocols (H) are significantly more efficient than dishonest-majority protocols (D). Compared to passive security (P), the active security model (A) increases computational costs by a factor of about 9× for the dishonest-majority (2 parties) and about 1.5× for the honest-majority setting (3 parties). For the same scenarios, communication costs grow by a factor of approximately 9× with dishonest-majority and 5× with honest majority protocols. To choose the most efficient protocol, it is important to realistically assess the behavior and capabilities of a potential adversary w.r.t. the specific application (cf. §2.2).

5.3 Comparison to Related Work

We use the LSUN benchmark data set to compare the efficiency of privacy-preserving AP with state-of-the-art privacy-preserving clustering algorithms and gauge costs for the standardized data set LSUN. Tab. 7 lists the runtime necessary to cluster LSUN using three state-of-the-art efficient privacy-preserving k-means and DBSCAN protocols from (Jäschke and Armknecht, 2019; Mohassel et al., 2020; Bozdemir et al., 2021) compared to our work. Although (Mohassel et al., 2020) is by a factor of 3,674× faster than ours, we discussed in §3 that AP yields a better clustering result for many data sets (including LSUN), making the additional costs acceptable. Similarly, the parallel and independent work by (Bozdemir et al., 2021) on private DBSCAN is about 194× faster than ours. However, they use an optimized hybrid combination of secure two-party computation techniques (Demmler et al., 2015) which is not possible with MP-SPDZ. Additionally, (Bozdemir et al., 2021) reduce the complexity of DBSCAN on LSUN to close to $\mathcal{O}(n^2)$ by fixing a low number of neighborhood expansions which will likely not be possible for all kinds of datasets.

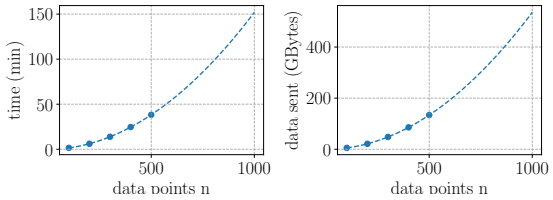
Table 7: Runtime comparison for privately clustering the LSUN data set with 2 parties and passive security.

k-means		DBSCAN	Affinity Propagation
(Jäschke and Armknecht, 2019)	(Mohassel et al., 2020)	(Bozdemir et al., 2021)	This work
25.79 days	22.21 seconds	420.72 seconds	22.67 hours

5.4 Scalability

We evaluate the runtime and computation costs of privacy-preserving AP on the "Blobs" data sets (cf. Tab. 4) with varying sizes (100, 200, 300, 400, and 500 data points) to assess the scalability of privacy-preserving AP. This evaluation was carried out for 2 parties and the passive security model. Note that all other MPC instantiations from Tab. 1 scale similarly with n and the results are, thus, transferable to the other security settings. Since the evaluated data sets have different optimal preference values and converge after a varying number of iterations, which would influence the scaling of cost values, we restrict our analysis to the computational and communication costs per iteration. We listed the required number of iterations for the data sets in Tab. 4 such that the runtimes of the full clustering can also be derived. Fig. 4 presents our benchmark results. As expected based on AP's complexity ($O(n^2)$), the costs grow quadratically with the number of data points n.We extrapolate the fitted polynomial to a maximum data set size of $n = 1000$. The fitted polynomials are $0.0001505n^2 + 0.001638n - 0.1583$ minutes for runtime and $0.0005357n^2 + 0.000001406n - 0.0001$ GBytes for communication.

We note that the cost per iteration is not dependent on the number of clusters in the original data set, since the calculations executed will be the same. The optimal choice of preference value and the number of iterations required for convergence may be affected by the true number of clusters, as discussed in §5.1. Therefore, our previous observations and benchmarking results hold not only for the 2-cluster data sets evaluated, but for data sets with *arbitrary* numbers of clusters. This characteristic is a clear advantage of AP over the k-means clustering algorithm, which becomes increasingly expensive as the number of clusters k in the data set grows.



(a) Runtime (b) Communication

Figure 4: Efficiency benchmark per clustering iteration for privacy-preserving AP with 2 parties and passive security.

6 RELATED WORK

Privacy-preserving machine learning (PPML) is an active research field, with much focus placed on cryptographic methods for privacy-preserving supervised and deep learning (Juvekar et al., 2018; Mishra et al., 2020; Rathee et al., 2020; Boemer et al., 2020; Patra et al., 2021). As training data, often without labels, is increasingly available, the relevance of privacy in unsupervised learning, e.g., clustering, has also grown. Some approaches to privacy-preserving clustering add noise to the clustering process, providing provable differential privacy (DP) guarantees (Su et al., 2017; Balcan et al., 2017; Cai et al., 2020). DP is a privacy-utility trade-off. Instead, we analyze cryptographic approaches for achieving privacy, which yield full accuracy but are more complex, so they are not directly comparable to approaches based on DP.

Existing research on private clustering has mainly proposed protocols for the k-means algorithm. Today's most efficient protocol is (Mohassel et al., 2020). Their work is based on MPC and provides a privacy-preserving k-means implementation that fully preserves privacy and does not incur significant computational overhead. For this reason, we compare our privacy-preserving implementation of AP to their work using the standard benchmark data set LSUN (cf. §5.3). Other work on k-means provides privacy as well; however, some protocols use homomorphic encryption and hence typically are significantly slower, e.g., (Bunn and Ostrovsky, 2007; Jäschke and Armknecht, 2019) or are applicable only in certain settings, requiring more than 2 non-colluding servers (Patel et al., 2012) or horizontally partitioned data (Gheid and Challal, 2016). Some privacy-preserving k-means implementations have fundamental issues, leaking intermediate cluster centers (Vaidya and Clifton, 2003; Jagannathan and Wright, 2005; Jha et al., 2005) or information about cluster sizes (Wu et al., 2020). In general, k-means is a simple clustering algorithm, which explains its popularity in privacy-preserving clustering. Unfortunately, its clustering capabilities and quality are limited, as discussed in §3.

A few passively secure privacy-preserving variants of clustering algorithms more advanced than k-means

have been proposed. (Bozdemir et al., 2021) design a fully privacy-preserving DBSCAN clustering protocol. Unfortunately, most other protocols do not preserve full privacy. For example, (Jagannathan et al., 2010) tackle privacy-preserving hierarchical clustering, but their protocol reveals merging patterns. The private DBSCAN protocols by (Jiang et al., 2008; Liu et al., 2013; Rahman et al., 2017) leak information such as parameter values, distances, cluster sizes, and neighborhoods. (Zhu et al., 2012) propose a privacy-preserving AP protocol for vertically partitioned data in the passive security model using additively homomorphic encryption. Their protocol leaks the permuted sensitivity values of all data records to one data owner.

7 CONCLUSION

In this work, we explored the suitability of k-means, AP, GMM, and DBSCAN for efficient crypto-oriented clustering. While the clustering quality of k-means is limited for many data sets, GMM and DBSCAN are expensive to implement using MPC techniques. AP provides more flexibility in terms of all attributes, so we design the first fully privacy-preserving AP with MPC techniques and implement it with the MP-SPDZ framework. We evaluated the resulting cluster quality and scalability, as well as the computational and communication costs for all combinations of passive/active security and honest/dishonest majority. With this, we are the first who evaluate the performance of a private clustering algorithm on all 4 scenarios.

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