Comparing Classifiers' Performance under Differential Privacy

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Abstract: The application of differential privacy in privacy-preserving data analysis has gained momentum in recent years. In particular, it provides an effective solution for the construction of privacy-preserving classifiers, in which one party owns the data and another party is interested in obtaining a classifier model from this data. While several approaches have been proposed in the literature to employ differential privacy for the construction of classifiers, an understanding of the difference in performance of these classifiers is currently missing. This knowledge enables the data owner and the analyst to select the most appropriate classification algorithm and training parameters in order to guarantee high privacy requirements while minimizing the loss of accuracy. In this study, we investigate the impact of the use of differential privacy on three well-known classifiers, *i.e.*, Naïve Bayes, SVM, and Decision Tree classifiers. To this end, we show how these classifiers can be trained in a differential privacy setting and perform extensive experiments to evaluate the effect of this privacy enforcement on their performance.

1 INTRODUCTION

In the data-driven society of the 21st century, machine learning algorithms are largely employed to infer additional knowledge and intelligence from the increasing amounts of data available in the Internet (Marr, 2019). In particular, classifiers have been widely used in many real-world applications, such as face and speech recognition, text analysis, fraud and anomaly detection, recommendation system, weather forecasting, medical image analysis, and biometric identification. A classifier assigns labels (classes) to new instances based on its model trained on data whose classes are known beforehand.

Classifiers are often trained under the assumption that the underlying data is freely accessible. However, this assumption may not hold when training data contains sensitive information as its publication might raise the data owners' privacy concerns. Consider, for instance, a situation in which a hospital owns a dataset describing patient information, including age, address, gender, symptoms, and diseases. A classifier trained on this dataset might leak sensitive information about the individual patients.

To address this issue, a large body of work has been devoted to train a classifier on datasets containing sensitive information in such a way that the privacy of the individuals whose data is present in the dataset is guaranteed. Existing privacy-preserving solutions can be categorized into two main classes. One class comprises cryptographic-based approaches that securely train a classifier over protected data (Khodaparast et al., 2019). These approaches, however, are not scalable both in terms of execution runtime and bandwidth usage (Naehrig et al., 2011). The other class comprises solutions relying on data anonymization techniques, in which the data under analysis is perturbed before being released, *e.g.*, k-anonymity, ℓ-diversity, and t-closeness (Sheikhalishahi et al., 2021). These anonymization techniques have been criticized for not being rigorous enough in protecting the individuals' confidential information, and *differential privacy* is emerging as the de-facto privacy standard for data anonymization (Dwork et al., 2006). It addresses the weaknesses of other anonymization techniques by limiting the disclosure of private information of individual records when published data aggregates information in the dataset.

The rigorous privacy guarantees offered by differential privacy has led to its broad application in the field of privacy-preserving data analysis. In particular, differential privacy is used to introduce noise during the training of classification algorithms, where the noise is scaled according to the sensitivity of the training algorithm. One of the main scenarios in which differential privacy is applied is the training of classifiers, where

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one party owns the data (containing sensitive information) and the other party is interested in obtaining a classifier trained over that data.

In this setting, multiple classification algorithms have been extended to incorporate differential privacy, *e.g.*, Nearest Neighbor (Gursoy et al., 2017), Naïve Bayes (Vaidya et al., 2013), Support Vector Machine (SVM) (Chaudhuri et al., 2011), and Decision Tree (Jagannathan et al., 2009). However, there is still a lack of understanding on the impact of differential privacy on their classification accuracy. Such knowledge would enable the data owner and the analyst (model requester) to decide which classifier to be trained according to *(i)* dataset properties, such as dataset size, *(ii)* structural properties of the classification algorithm, and *(iii)* the amount of privacy required (ε in differential privacy). Accordingly, this study aims to answer the following research questions:

RQ1. Which dataset properties influence the accuracy of differentially private classifiers?
RQ2. How does the accuracy of different classification algorithms change when applied in a differential privacy setting?
RQ3. How is classifier accuracy affected by the privacy level enforced?

To answer these questions, we investigate three well-known classification algorithms, namely the Naïve Bayes, SVM and Decision Tree classifiers in a differential privacy setting. We show how these classification algorithms can be adapted to train differentially private classifiers and apply them to several largely-used benchmark datasets. For each classification algorithm, we analyze the effect of dataset properties and privacy levels on the classifier accuracy. The experiment results show that in a differential privacy setting, no classification algorithm is a *one-size-fits-all* solution for all datasets and privacy levels. Nonetheless, under some conditions one might outperform the others. For example, our experiments show that a differentially private SVM classifier returns higher accuracy when the training dataset is large; on the other hand, the accuracy of a Decision Tree classifier mainly depends on the privacy level where the accuracy notably increases when the privacy level is relaxed.

The contribution of this work can be summarized as follows:

- We show how three well-known classification algorithms can be adapted to the differential private setting and prove that these adaptations satisfy the prescribed privacy level.
- We apply the revised algorithms to several benchmark datasets and empirically evaluate classifier accuracy with respect to the dataset properties and privacy level.

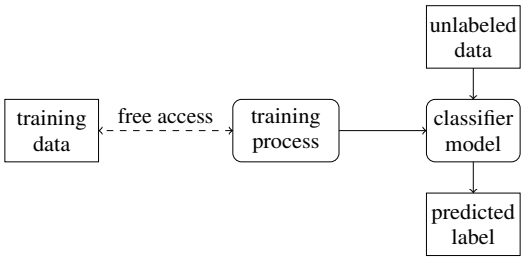


Figure 1: Classifier learning setting.

- Based on the experiment results, we draw recommendations to guide data owners and analysts in the selection of a classification algorithm for training a classifier and directions for future work to improve the state-of-the-art in differentially private classifier learning.

Outline. The remainder of the paper is organized as follows. The next section presents the classification algorithms studied in this work. Section 3 presents the differential privacy classifier learning setting and discusses the differentially private counterparts of the classification algorithms. Sections 4 and 5 describe our experimental setup and results, respectively. Section 6 discusses related work, and Section 7 concludes the paper and provides directions for future work.

2 CLASSIFICATION ALGORITHMS

In this work, we consider a classifier learning setting in which an analyst aims to train a classifier based on the data owned by another party. The setting is depicted in Figure 1. We assume the training data consists of a dataset D with n rows $\vec{x}$, which is described by a set of attributes $\mathcal{A}$ and a distinct *class* attribute C (hereafter we assume that C is a categorical attribute). Each row $\vec{x}$ is a vector in which each element x_A is a value of attribute $A \in \mathcal{A}$ and a class x_C. We write $\bar{x}$ for the unlabeled row, *i.e.*, for $\vec{x}$ with the class x_C removed. The analyst's goal is to create a *classifier*, which can be used to predict the class x'_C of a new unlabeled observation $\vec{x}'$. The analyst has free access to D, which is used to train the classifier. When the classifier is trained, it is published to the general public to be used in the classification of new unlabeled data.

Next, we describe the three classification algorithms studied in this work.

2.1 Naïve Bayes Classifier

Naïve Bayes classifier is a probabilistic classifier built based on Bayes' theorem and assumes the attributes describing the data to be mutually independent. For-

mally, for an unlabeled data point $\bar{x}$, the conditional probability of $\bar{x}$ being class c is denoted by $p(c|\bar{x})$ and (with the use of Bayes theorem) is computed as:

$$p(c|\bar{x}) = \frac{p(c)p(\bar{x}|c)}{p(\bar{x})} = \frac{p(c)\prod_{A\in\mathcal{A}} p(x_A|c)}{p(\bar{x})} \quad (1)$$

where $p(c)$ is the probability of class c to occur in the dataset, $p(\bar{x}|c)$ is the conditional probability that $\bar{x}$ occurs given it is labeled c, and $p(\bar{x})$ is the probability of $\bar{x}$ to occur.[1] These probabilities are computed by observing frequencies in the dataset. The classifier assigns a class label $\hat{c}$ to given data with a maximum a posteriori probability as follows:

$$\hat{c} = \arg\max_c p(c) \prod_{A\in\mathcal{A}} p(x_A|c) \quad (2)$$

This equation is equivalent to Eq. 1, removing the constant value $p(\bar{x})$ in the denominator. From this formula, it can be inferred that for constructing a Naïve Bayes classifier, it is enough to compute the conditional probabilities and the probability of each class label. For a class c, a categorical attribute A, and a value $v \in A$, the conditional probability $p(x_A = v|c)$ and the probability $p(c)$ are computed as:

$$p(x_A = v|c) = \frac{n_{Avc}}{n_c} \quad , \quad p(c) = \frac{n_c}{n}, \quad (3)$$

where n_{Avc} is the number of rows $\vec{x}$ in D with $x_A = v$ and $x_C = c$ and n_c is the number of rows with $x_C = c$. For a numerical attribute A and $z \in \mathbb{R}$, the distribution of x_A given $x_C = c$ is assumed to be normal, and its probability density function is computed as:

$$p(x_A = z|c) = \frac{1}{\sqrt{2\pi}\sigma_{Ac}} \mathrm{e}^{\frac{(z-\mu_{Ac})^2}{2\sigma_{Ac}^2}}, \quad (4)$$

where μ_{Ac} is the mean value of x_A among the rows $\vec{x}$ of D with class c, and σ_{Ac} is the standard deviation of these values.

2.2 Support Vector Machine

The Support Vector Machine (SVM) algorithm is a classifier defined based on a statistical learning framework, in which the class attribute is binary, *i.e.*, the classes are ± 1, and each attribute A is numerical. As such we can represent each $\bar{x}$ as a point in $|\mathcal{A}|$-dimensional space. The aim of the SVM training algorithm is to find a hyperplane in this space that best separates the sets of points corresponding to the two labels. The degree to which a hyperplane, represented by a normal vector $\bar{w}$, fails at separating the sets of points is measured by

$$J(\bar{w}, D) = \frac{1}{n}\sum_{i=1}^{n} l_h(x_C^i(\bar{w}\cdot\bar{x}_i)) + \frac{\Lambda}{2}||\bar{w}||^2, \quad (5)$$

where $x_C^i \in \{\pm 1\}$ is the class of the i-th data point, $\bar{w}\cdot\bar{x}_i$ is the inner product of $\bar{w}$ with the unlabeled data point $\bar{x}_i$, and l_h is the Huber loss function given, for a fixed parameter $h > 0$, by

$$l_h(z) = \begin{cases} 0 & \text{if } z > 1+h, \\ \frac{1}{4h}(1+h-z)^2 & \text{if } |1-z| \le h, \\ 1-z & \text{if } z < 1-h. \end{cases} \quad (6)$$

The term $\frac{\Lambda}{2}||\bar{w}||^2$ in (5) is to prevent overfitting. Following (Chaudhuri et al., 2011), we take $h = 0.05$ and $\Lambda = 10^{-2.5}$. The SVM returns the $\bar{w}$ that minimizes (5), *i.e.*, $\hat{w} = \arg\min_{\bar{w}} J(\bar{w}, D)$.

2.3 Decision Tree Classifier

A Decision Tree classifier is a classifier that takes the form of a rooted tree, which is iteratively trained from the top (root) to down (leaves). Each leaf is labeled with a class and each internal node corresponds to an attribute in which the outgoing edges are the attribute-values. A new data point is classified by starting from the root and passing along its attribute-values until it reaches a leaf. The class label of the associated leaf is returned as the class label of the new instance.

Among the existing Decision Tree classifiers, we consider the Classification And Regression Tree (CART) algorithm (Breiman et al., 1984) for its simple training process. CART is a binary decision tree, where each attribute $A \in \mathcal{A}$ only takes values $\{0,1\}$ and the class attribute C can take more than two values. The tree is built recursively from the root. At every node the attribute that gives the best splitup is selected. Formally, it is defined as follows.

The purity of node $\mathcal{N}$ (*i.e.*, its homogeneity in terms of class labels) is measured with the *Gini index*, denoted by $G(\mathcal{N})$, and it is computed as:

$$G(\mathcal{N}) = 1 - \sum_{c\in C} p_{\mathcal{N}}(x_C = c)^2, \quad (7)$$

where $p_{\mathcal{N}}$ is the probability among all dataset rows that end up in node $\mathcal{N}$ when walking from the root. Note that $G(\mathcal{N}) = 0$ iff all rows in $D_{\mathcal{N}}$ have the same class. The CART classifier selects the attribute whose split creates the children with the least average Gini index (the purest children), *i.e.*,

[1] The second equivalence of (1) is derived by assuming that attributes are independent.

$$A_{\text{best}} = \arg\min_{A \in \mathcal{A}} \sum_{v \in \{0,1\}} p_{\mathcal{N}}(x_A = v) G(\mathcal{N}_{A,v}), \quad (8)$$

where $\mathcal{N}_{A,v}$ is the child of $\mathcal{N}$ with value v in the splitup along with A. More concretely, for a given A and $v \in \{0,1\}$ and class c, and for a fixed $\mathcal{N}$, let m_{Avc} be the number of rows $\vec{x}$ in D that end up at node $\mathcal{N}$ and that satisfy $x_A = v$, $x_C = c$. The best attribute is selected as:

$$A_{\text{best}} = \arg\min_{A \in \mathcal{A}} \sum_{v \in \{0,1\}} \frac{(\sum_c m_{Avc})^2 - \sum_c m_{Avc}^2}{(\sum_c m_{Avc})\left(\sum_{v',c} m_{Av'c}\right)}. \quad (9)$$

If the stopping condition is reached, then $\mathcal{N}$ is a leaf, and we assign to $\mathcal{N}$ the class that is most prevalent among the training items that end up in $\mathcal{N}$, *i.e.*, $\hat{c} = \arg\max_c m_c$,, where m_c is the number of rows $\vec{x}$ in D that end up at node $\mathcal{N}$ and that satisfy $x_C = c$.

While theoretically one can continue splitting up nodes until all attributes have appeared in the tree structure, this generally results in overfitting. Therefore, the algorithm is stopped when a maximum depth d is reached. We take $d = \lceil \sqrt{m} \rceil$, which satisfies the best trade-off between the underfitting and overfitting of the Decision Tree model (for the selected dataset) (Mantovani et al., 2018).

3 DIFFERENTIAL PRIVACY CLASSIFIER LEARNING

To measure information leakage when classifiers are trained over sensitive data, we use the *de facto* standard metric named Differential Privacy (DP) defined as follows (Dwork et al., 2006).

Definition 3.1. *Two datasets are called* adjacent *if they differ in at most one row. Let $\varepsilon \in \mathbb{R}_{\geq 0}$, and let f be an algorithm operating on datasets. We say that f satisfies ε-Differential Privacy if for all adjacent datasets D, D' and all sets of possible outputs S:*

$$\mathbb{P}(f(D) \in S) \leq e^{\varepsilon} \mathbb{P}(f(D') \in S). \quad (10)$$

By ensuring that the probability distributions on the output space originating from two input datasets cannot differ too much, ε-DP provides plausible deniability about any row's true value, even if all other rows are compromised. The lower ε, the stronger privacy ε-DP guarantees. To ensure privacy in the classifier learning setting, we demand that a classifier training algorithm satisfies ε-DP. Thus, we aim to solve the following problem:

Problem 1. *Given a privacy level ε and a dataset D, determine the ε-DP classifier training algorithm Q that maximizes the accuracy of the classifier $Q(D)$.*

Many classifiers are trained by retrieving information from the dataset through numerical queries. In this case, one can ensure ε-DP by making sure the responses to queries satisfy differential privacy. DP on a single query can be incorporated as follows. Let φ be a numerical function on datasets, and let $s := \max |\varphi(D) - \varphi(D')|$, where the maximum is taken over all adjacent D, D'. Suppose a query asks for $\varphi(D)$. Then the response

$$L(\varphi, \varepsilon) = \varphi(D) + \text{Lap}(0, s/\varepsilon), \quad (11)$$

where $\text{Lap}(0, s/\varepsilon)$ is a Laplace random variable with mean 0 and scale parameter s/ε, is ε-DP. Occasionally we will need responses that are positive, in which case we will use $L^+(\varphi, \varepsilon) = \max\{L(\varphi, \varepsilon), \alpha\}$, where α is a small positive number that should be substantially smaller than $\varphi(D)$. Since most of our queries are counts and therefore integers, we use $\alpha = 10^{-5}$ throughout. The response $L^+(\varphi, \varepsilon)$ is ε-DP as well. The following Theorem, which follows from standard properties of differential privacy (Dwork et al., 2006; Nguyen et al., 2013), shows that such DP responses can be used to construct DP classifier training algorithms:

Theorem 3.1. *Let Q be a classifier training algorithm, accessing the database via queries. Suppose that each row of the dataset is accessed through at most m queries and that the response to each query is $\frac{\varepsilon}{m}$-DP. Then, Q is ε-DP.*

3.1 ε-DP Naïve Bayes Classifier

To train the Naïve Bayes classifier in the ε-DP setting, we mainly follow the work presented in (Vaidya et al., 2013) with few adjustments. Specifically, 1) we have modified the definition of standard deviation sensitivity coming from using a different definition of adjacent datasets, and 2) we consider the effect of multiple queries by applying the lower value of ε per query (Theorem 3.1) such that the final algorithm satisfies ε-DP. Algorithm 1 details the process including our contribution and it can be summarized as follows.

Naïve Bayes relies on the dataset via the queries n_{Avc}, n_c, μ_{Ac} and σ_{Ac} for all $A_c \in \mathcal{A}$ (cf. Section 2.1). To insert differential privacy, we instead use the noisy versions of these values as:

$$L^+(n_{Avc}, \varepsilon'), L^+(n_c, \varepsilon'), L^+(\sigma_{Ac}, \varepsilon'), L(\mu_{Ac}, \varepsilon'), \quad (12)$$

where ε' is chosen such that the collection of noisy answers as a whole satisfies ε-DP. More concretely,

$$\varepsilon' = \frac{\varepsilon}{1 + \#\{\text{categorical } A\} + 2\#\{\text{numerical } A\}}. \quad (13)$$

Algorithm 1: Construction of ε-DP Naïve Bayes classifier.

Data: Privacy parameter ε.
Result: Prior probabilities $p(c)$; Conditional probabilities $p(x_A = v|c)$ for each class, categorical attribute, and value of that attribute; Obfuscated mean $\tilde{\mu}_{Ac}$ and standard deviation $\tilde{\sigma}_{Ac}$ for each class and numerical attribute.

$\varepsilon' \leftarrow \frac{\varepsilon}{1+\#\{\text{categorical } A\}+2\#\{\text{numerical } A\}}$;
for *each class* c **do**
  $\tilde{n}_c \leftarrow L^+(n_c, \varepsilon')$;
  $p(c) \leftarrow \frac{\tilde{n}_c}{n}$;
  for *each categorical* A*, each value* $v \in A$ **do**
    $\tilde{n}_{Avc} \leftarrow L^+(n_{Avc}, \varepsilon')$;
    $p(x_A = v|c) = \frac{\tilde{n}_{Avc}}{\tilde{n}_c}$;
  end
  for *each numerical* A **do**
    $\tilde{\mu}_{Ac} \leftarrow L(\mu_{Ac}, \varepsilon')$;
    $\tilde{\sigma}_{Ac} \leftarrow L^+(\sigma_{Ac}, \varepsilon')$;
  end
end

Note that in (12) we use L^+ for the counts and the standard deviation because they are assumed to be positive, and L for the mean because it has no such restriction.

To calculate the expressions in relations (12), we need to know their sensitivities. The sensitivities of the counts n_{Avc} and n_c satisfy $s = 1$. We assume that for each numerical attribute A, a lower bound l_A and upper bound u_A are public knowledge. Then, the sensitivity of μ_{Ac} and σ_{Ac}, respectively, are given as

$$s_{\mu_{Ac}} = \frac{u_A - l_A}{n_c} \quad , \quad s_{\sigma_{Ac}} = \frac{u_A - l_A}{\sqrt{n_c}}. \tag{14}$$

Theorem 3.2. *Algorithm 1 satisfies* ε*-DP.*

Proof. Every row in dataset is queried in one n_c, in one $n_{A_v c}$ for each categorical attribute A, and in one μ_{Ac} and one σ_{Ac} for each numerical attribute A, so the total amount of times each row is queried is

$$1 + \#\{\text{categorical } A\} + 2\#\{\text{numerical } A\}. \tag{15}$$

The result now follows from Theorem 3.1. □

Compared to (Vaidya et al., 2013), we work with ε' rather than ε, we have a different formula for $s_{\sigma_{Ac}}$ in (14), and we use L^+ to round up certain negative responses, rather than resampling until a positive response appears. These changes are necessary to ensure ε-DP.

Algorithm 2: Construction of ε-DP SVM classifier.

Data: Privacy parameter ε; Huber parameter h; overfitting parameter Λ.
Result: Separating hyperplane $\bar{w}_{\text{priv}}$.

$\varepsilon' \leftarrow \varepsilon - \log\left(1 + \frac{1}{nh\Lambda} + \frac{1}{4n^2h^2\Lambda^2}\right)$;
if $\varepsilon' > 0$ **then**
  $\varepsilon'' \leftarrow \varepsilon'$;
  $\Lambda' \leftarrow \Lambda$;
else
  $\varepsilon'' \leftarrow \frac{\varepsilon}{2}$;
  $\Lambda' \leftarrow \frac{1}{2nh(e^{\varepsilon/4}-1)}$;
end
draw $\bar{b}$ according to $p(\bar{b} = \bar{z}) \propto e^{-\frac{\varepsilon''}{2}||\bar{z}||}$;
$\bar{w}_{\text{priv}} \leftarrow \arg\max_{\bar{w}} \frac{1}{n}\sum_{i=1}^{n} l_{\text{Huber}}(\bar{w} \cdot \bar{x}^i, x_C^i) + \frac{\Lambda'}{2}||\bar{w}||^2 + \frac{1}{n}\bar{b} \cdot \bar{w}$;

3.2 ε-DP SVM Classifier

We adopt the ε-DP implementation of SVM introduced in (Chaudhuri et al., 2011), which is detailed in Algorithm 2 and works as follows. In SVM, the resulting hyperplane $\bar{w}$ can leak information about D, since it minimizes an objective function J depending on D. To avoid this, the objective function is perturbed so that it does not rely on any row in D significantly. More precisely, instead of the objective function J from Section 5, we use

$$J_{\text{priv}}(\bar{w}, D) = \frac{1}{n}\sum_{i=1}^{n} l_h(\bar{w} \cdot \bar{x}^i, x_C^i) + \frac{\Lambda'}{2}||\bar{w}||^2 + \frac{1}{n}\bar{b} \cdot \bar{w} \tag{16}$$

where $\bar{b}$ is a random vector, whose probability distribution is defined below, and Λ' depends on the choice of Λ and the privacy parameter ε. More concretely, given ε, Λ and the Huber parameter h, we define

$$\varepsilon' = \varepsilon - \log\left(1 + \frac{1}{nh\Lambda} + \frac{1}{4n^2h^2\Lambda^2}\right), \tag{17}$$

$$\varepsilon'' = \begin{cases} \varepsilon', & \text{if } \varepsilon' > 0 \\ \frac{\varepsilon}{2}, & \text{otherwise,} \end{cases} \tag{18}$$

$$\Lambda' = \begin{cases} \Lambda, & \text{if } \varepsilon' > 0 \\ \frac{1}{2nh(e^{\varepsilon/4}-1)}, & \text{otherwise,} \end{cases} \tag{19}$$

and $\bar{b}$ is drawn according to $\mathbb{P}(\bar{b} = \bar{z}) \propto e^{-\frac{\varepsilon''}{2}||\bar{z}||}$. The algorithm then outputs the hyperplane

$$\bar{w}_{\text{priv}} = \arg\min_{\bar{w}} J_{\text{priv}}(\bar{w}, D).$$

Theorem 3.3 (Theorem 9 of (Chaudhuri et al., 2011))**.** *Algorithm 2 satisfies* ε*-DP.*

Algorithm 3: Construction of ε-DP Decision Tree classifier.

Data: Privacy parameter ε; Maximum depth d.
Result: Rooted tree T with labeled edges and leaves.

$\varepsilon' \leftarrow \frac{\varepsilon}{|\mathcal{A}|(d+1)}$;
create root $\mathcal{R}$;
$\mathcal{R}$.expandable $\leftarrow$ True;
while #{*expandable nodes*} > 0 **do**
  choose expandable node $\mathcal{N}$;
  if $\mathcal{N}.depth = d$ **then**
    for *each class c* **do**
      $\tilde{m}_c \leftarrow L(m_c, \varepsilon')$;
    end
    $\mathcal{N}$.label $\leftarrow \arg\max_c \tilde{m}_c$;
  else
    for *each attribute A not set on path* $\mathcal{R} \to \mathcal{N}$ **do**
      for $v, c \in \{0,1\}$ **do**
        $\tilde{m}_{vc} \leftarrow L^+(m_{Avc}, \varepsilon')$;
      end
      $G(A) \leftarrow \sum_v \frac{(\sum_c \tilde{m}_{vc})^2 - \sum_c \tilde{m}_{vc}^2}{(\sum_c \tilde{m}_{vc})(\sum_{v',c} \tilde{m}_{v'c})}$;
    end
    $A \leftarrow \arg\min_A G(A)$;
    create nodes $\mathcal{N}_0, \mathcal{N}_1$;
    add labeled edges $\mathcal{N} \xrightarrow{A=0} \mathcal{N}_0, \mathcal{N} \xrightarrow{A=1} \mathcal{N}_1$;
    $\mathcal{N}_0$.expandable, $\mathcal{N}_1$.expandable $\leftarrow$ True;
  end
  $\mathcal{N}$.expandable $\leftarrow$ False;
end

3.3 ε-DP Decision Tree Classifier

An overview of differentially private Decision Tree classifiers is given in (Fletcher and Islam, 2019). We follow its general framework, adapted to the CART classifier as presented in Algorithm 3. It works by replacing the counts m_{Avc} and m_c with differential privacy equivalents. More precisely, instead of m_c we use the noisy version $L(m_c, \varepsilon')$, where $\varepsilon' = \frac{\varepsilon}{|\mathcal{A}|(d+1)}$, in which d is the depth of the tree. The Gini impurity needs positive counts as inputs, so we use $L^+(m_{Avc}, \varepsilon')$. Both these noisy counts have sensitivity $s = 1$.

Theorem 3.4. *Algorithm 3 satisfies ε-DP.*

Proof. At each level of the tree, each row of the training dataset is present in at most 1 node $\mathcal{N}$. At $\mathcal{N}$, it is present in exactly one of the m_c if $\mathcal{N}$ is a leaf, and in exactly one m_{Avc} for each attribute A if $\mathcal{N}$ is an interior node. Hence each row is queried at most $|\mathcal{A}|(d+1)$ times, and by Theorem 3.1, Algorithm 3 satisfies ε-DP. □

Table 1: Dataset statistics.

Name	Type	#Attributes	#Instances
Adult	Mix	14	48 842
Mushroom	Categorical	22	8 000
Nursery	Categorical	8	12 960
Congressional Voting	Binary	16	435
SPECT Heart	Binary	22	267
Skin Segmentation	Numerical	3	245 057

4 EXPERIMENTAL ANALYSIS

The experimental analysis aims to assess and compare the classifiers' performance when they are trained in an ε-DP setting w.r.t. dataset properties (**RQ1**), classification algorithm (**RQ2**) and privacy level (**RQ3**). Next, we present the experimental setup, the datasets used for the experiments and the evaluation approach.

Experiment Setup. We implemented the classification algorithms both in a non-private (Section 2) and an ε-DP setting (Algorithms 1, 2, and 3) in Python.[2]. The privacy levels ε used to train the classifiers in the ε-DP setting are taken from the set $\mathcal{E} = \{10^{-11}, 0.001, 0.005, 0.01, 0.05, 0.1, 0.25, 0.5, 0.75, 1\}$.

Datasets. For our experiments we selected six datasets from the UCI repository[3]. Table 1 summarizes the statistics of the selected datasets.

Adult:[4] The dataset describes 48842 individuals using 14 attributes such as age, occupation, education. The class attribute represents their income, which has two possible values: '$> 50K$' and '$< 50K$'. The attributes are both numerical and categorical.

Mushroom:[5] This dataset describes 8000 hypothetical samples of mushrooms, characterized using 22 categorical attributes, such as cap shape. The samples are classified into two classes: *edible* and *poisonous*.

Nursery:[6] This dataset has originally been developed to rank applications for nursery schools. The dataset includes 12960 instances, described with 8 categorical features such as health situation. The records are classified into five classes, each representing a level of being recommended for the position.

Congressional Voting:[7] This dataset includes votes for each of the U.S. House of Representatives Congressmen on the 16 key votes identified by the CQA. The dataset includes 435 records, described with binary

[2]The codes of our experiments are available in https://github.com/jeroenkivits/seminar
[3]https://archive.ics.uci.edu/ml/datasets/
[4]https://archive.ics.uci.edu/ml/datasets/adult
[5]https://archive.ics.uci.edu/ml/datasets/Mushroom
[6]https://archive.ics.uci.edu/ml/datasets/nursery
[7]https://archive.ics.uci.edu/ml/datasets/Congressional+Voting+Records

attributes, such as immigration, where the records are labeled either *democrat* or *republican*.

SPECT Heart:[8] The dataset describes diagnosing of cardiac Single Proton Emission Computed Tomography (SPECT) images. Each of the patients is classified into two categories: *normal* and *abnormal*. It contains 267 instances that are described by 23 binary attributes.

Skin Segmentation:[9] This dataset comprises 245057 samples of face images of people. Each sample is described by its RGB value (3 numerical attributes) and is classified into two classes: *skin* and *non-skin*.

Given that the selected SVM and Decision Tree algorithms are respectively applicable on numerical and binary attributes, we convert the attributes of the selected datasets in such a way that they respect the requirements of these algorithms. For SVM, integer numbers are randomly assigned to the distinct values of categorical attributes. For Decision Tree, each attribute-value of a categorical attribute is considered as an attribute by itself, where if a record satisfies that attribute-value the value 1 is assigned (the value 0 is assigned, otherwise). For continuous attributes, the median value is used to assign 1 to attribute-values higher than the median value and 0 otherwise.

Evaluation Approach. We measure the classifiers' performance in terms of their accuracy. Classifier accuracy is assessed using 10-fold cross-validation. It is worth noting that the accuracy of ε-DP classifiers is affected by the randomness introduced both by the partitioning of the datasets and by the ε-DP noise, where the latter has an especially large effect on accuracy. To mitigate the effect of randomness and to get a clear picture of the average accuracy, we have repeated each 10-fold cross-validation for 100 runs for each ε value for ε-DP Naïve Bayes and SVM classifiers. As ε-DP Decision Tree classifiers showed a more stable behaviour, we repeated the experiments 10 times for Decision Tree. The parameters of classifiers have been tuned to their highest performance with respect to each selected dataset in order to allow for a fair comparison.

Criteria for RQ1: Research question RQ1 aims to understand the effect of dataset properties on classifier accuracy in an ε-DP setting. To this end, we investigate how classifier accuracy varies for datasets with different sizes and number of attributes.

Criteria for RQ2: To investigate the effect of built-in properties of classifiers on their accuracy when trained in an ε-DP setting, we study the performance of classification algorithms when used in an ε-DP setting independently from the dataset. To this end, we compute the average classifier accuracy over all datasets.

[8]https://archive.ics.uci.edu/ml/datasets/SPECT+Heart

[9]https://archive.ics.uci.edu/ml/datasets/skin+segmentation

To verify whether the accuracy difference between classification algorithms used in an ε-DP setting is statistically significant, we use a non-parametric statistical test, named the *Wilcoxon* test (Wilcoxon, 1945). The Wilcoxon test can be adapted to our problem as follows.

Definition 4.1 (Wilcoxon Test)**.** *Given two classification algorithms, let d_i be the signed difference between the performance scores of the classifiers obtained by applying each algorithm on a given dataset for a given privacy level. The differences d_i ($1 \leq i \leq N$ where N is the number of possible combinations of datasets and privacy levels to which the classification algorithms are applied) are ranked based on the absolute values (average rank is assigned for equal performances). Let R^+ denote the sum of the ranks for datasets and privacy level on which $d_i > 0$, and let R^- be the sum of the ranks for datasets and privacy level on which $d_i < 0$ (dividing the sum of the ranks for which $d_i = 0$ evenly), i.e.,*

$$R^+ = \sum_{d_i>0} rank(d_i) + \frac{1}{2}\sum_{d_i=0} rank(d_i) \tag{20}$$

$$R^- = \sum_{d_i<0} rank(d_i) + \frac{1}{2}\sum_{d_i=0} rank(d_i) \tag{21}$$

Let $T = \min(R^+, R^-)$, then

$$z = \frac{T - \frac{1}{4}N(N+1)}{\sqrt{\frac{1}{4}N(N+1)(2N+1)}} \tag{22}$$

is approximately distributed normally. Under this condition, the difference between the accuracy distribution of the two classification algorithms is statistically significant (i.e., the null hypothesis is rejected) if the p-value is less than or equal to a given significance level σ.

In our experiments, we require a 95% confidence interval, which corresponds to $\sigma = 0.05$.

To get insight into the performance of classification algorithms when used in an ε-DP setting compared to the non-private setting, we compute

- *Accuracy (no privacy):* the classifier accuracy in the non-private setting.
- *Average accuracy (ε-DP):* the average accuracy of ε-DP classifiers over all ε values.
- *Ratio:* the effect size of employing ε-differential privacy compared to a non-private learning setting computed as the average accuracy of ε-DP classifiers over all privacy level $\varepsilon \in \mathcal{E}$ divided by the classifier accuracy obtained in a non-private setting.

Criteria for RQ3: To assess the impact of privacy levels on classifier accuracy, we analyze, for each ε value in $\mathcal{E}$, the distribution of the classifier accuracy over all selected datasets and classification algorithms.

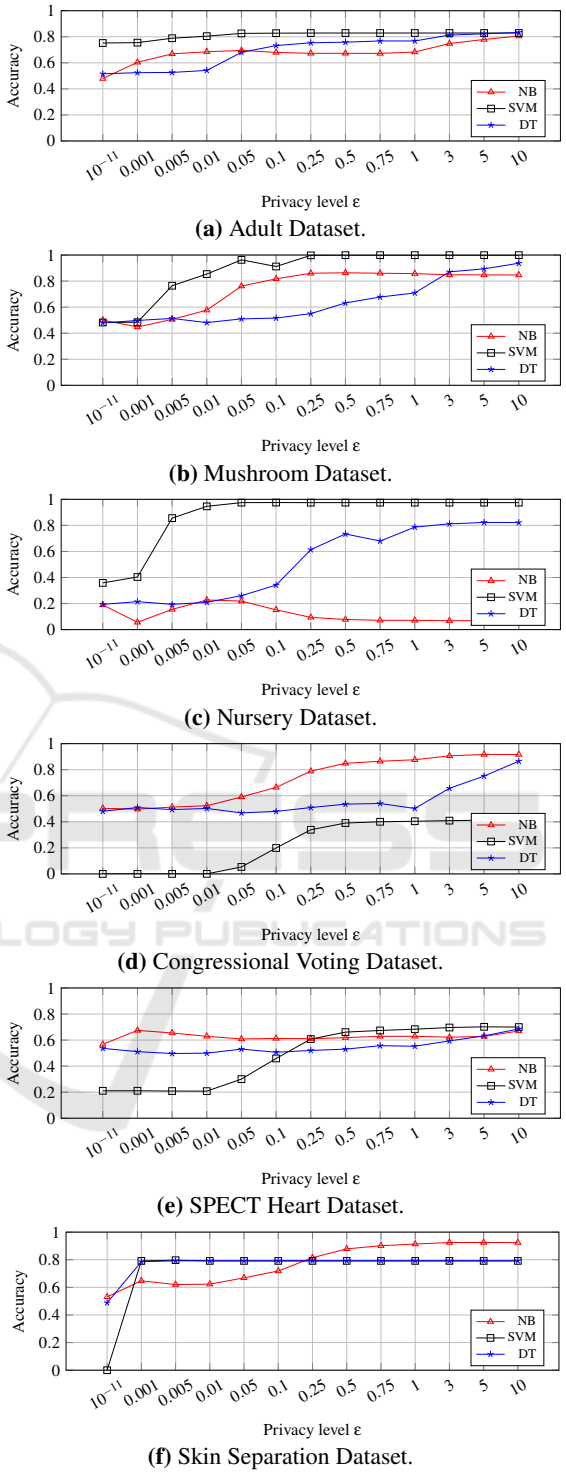
To measure the effect size of ε on the classifier accuracy, we use the classifier accuracy obtained in the non-private setting (Accuracy (no privacy)) as the baseline and compute, for each ε-DP classifier, the *classifier accuracy ratio* as the ratio between the classifier accuracy in the ε-DP setting and the accuracy of the corresponding baseline classifier. Intuitively, the classifier accuracy ratio represents to what extent enforcing a given privacy level affects classifier accuracy compared to the non-private setting.

5 RESULTS

We computed the accuracy of ε-DP Naïve Bayes, SVM, and Decision Tree classifiers on each selected dataset for all privacy levels in $\mathcal{E}$. The results of classifiers accuracy over Adult, Mushroom, Nursery, Congressional Voting, SPECT Heart, and Skin datasets, are respectively shown in Figures 2a, 2b, 2c, 2d, 2e, and 2f.

RQ1: Which Dataset Properties Influence the Accuracy of ε-DP Classifiers? From Figure 2 we can observe that SVM classifiers are typically accurate when trained over datasets with a large number of records (Adult, Mushroom, Nursery, and Skin Separation), while it returns low accuracy when trained over small datasets (SPECT Heart and Congressional Voting). This is due to the SVM structure in which the hyperplane is determined based on the support vectors' distances. When the dataset comprises a large number of records, the noises added through differential privacy negligibly affect the hyperplane location. On the other hand, the accuracy of Naïve Bayes classifiers depends neither on the number of attributes nor on the number of records. As shown in Figure 2, for datasets with an equal number of attributes (Mushroom and SPECT Heart) or a large number of records (Mushroom and Nursery), the Naïve Bayes classifier returns different trends of accuracy. This could be because the accuracy of Naïve Bayes classifier mainly depends on: *i)* the distribution of attributes' values, and *ii)* the independence of attributes (Jiang et al., 2007). Similarly, the results show that the accuracy of Decision Tree classifiers does not depend on the number of attributes and dataset size.

RQ2: How Does the Accuracy of Different Classification Algorithms Change when Trained in an ε-DP Setting? The average accuracy of the classification algorithms when used in an ε-DP setting over all datasets is reported in Figure 3. It can be observed that, on average, SVM (for ε values higher than 0.005 and lower than 3) outperforms the other two classification algorithms. However, for small ε values, Naïve Bayes and Decision Tree show slightly better performances.



(a) Adult Dataset.

(b) Mushroom Dataset.

(c) Nursery Dataset.

(d) Congressional Voting Dataset.

(e) SPECT Heart Dataset.

(f) Skin Separation Dataset.

Figure 2: Accuracy of Naïve Bayes, SVM, Decision Tree classifiers trained in an ε-DP setting for different values of ε.

For low privacy levels (ε higher than 3), Decision Tree returns the most accurate results. Overall, Decision Tree classifiers show, in general, a higher improvement

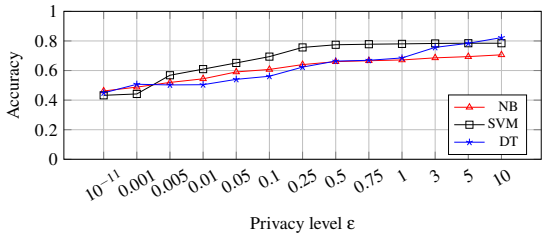


Figure 3: Average accuracy of the classification algorithms when used in an ε-DP setting over all datasets.

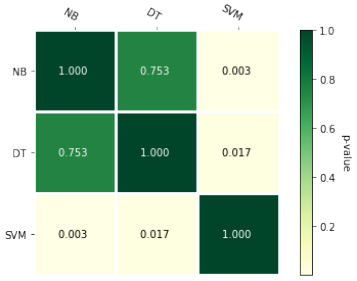
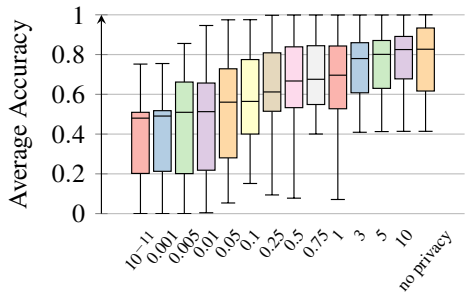
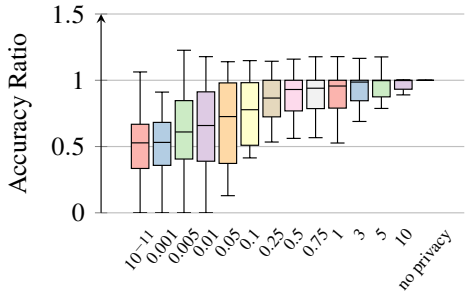


Figure 4: Heatmap of Wilcoxon test on mutual comparison of performance between classification algorithms applied in an ε-DP setting in terms of p-values.



(a) Distribution of classifier accuracy for different ε



(b) Distribution of classifier accuracy ratio for different ε

Figure 5: Distribution of classifier accuracy and classifier accuracy ratio for different ε values.

when the privacy requirements are relaxed compared to the other types of classifiers, *i.e.*, the accuracy shows a more noticeable increase for increasing ε values.

We used the Wilcoxon test to verify the statistical significance of these differences. Figure 4 depicts the heatmap of mutual comparison of ε-DP classifiers' performance in terms of p-values. The lower p-value (lighter color) shows more confidence in rejecting the null hypothesis (*i.e.*, more different performance). Figure 4 shows that the null hypothesis of the Wilcoxon test is rejected in the mutual comparison of SVM with both Decision Tree ($p = 0.017$) and Naïve Bayes ($p = 0.003$) classification algorithms, *i.e.*, SVM shows a different behaviour compared to the other algorithms. On the other hand, the Wilcoxon test fails to reject the null hypothesis when comparing the accuracy of Naïve Bayes and Decision Tree, *i.e.*, these algorithms show similar performance.

A comparison between the accuracy achieved by the classification algorithms when trained in a non-private setting and in an ε-DP setting along with the effect size of the accuracy difference between these settings (*Ratio*) is reported in Table 2. The results in the non-private setting show that for the selected datasets, on average, the Decision Tree classification algorithm outperforms the other two classification algorithms. However, SVM shows better performance than the other two algorithms when used in an ε-DP setting. The ratio shows that the Naïve Bayes classification algorithm is the most stable between the ε-DP and non-private settings, indicating that it is the one less affected by the ε-DP noise.

RQ3: How Is Classifier Accuracy Affected by the Privacy Level Enforced? Figure 2 and Figure 3 show that classifier accuracy decreases with the decrease of ε (*i.e.*, for higher privacy requirement). SVM classifiers perform poorly for some datasets (Nursery, Congressional Voting and Skin Separation) when trained using very small values of ε (Figure 2), although on average perform only slightly worse than Decision Tree and Naïve Bayes classifiers (Figure 3). Decision Tree classifiers show sudden increments of accuracy for some ε values, which can be due to the recursive structure of this algorithm. In the construction of ε-DP Decision Tree classifiers, the noise is added at every level and each subtree can be constructed using a set of data with a different distribution of values.

Figure 5 shows the distribution of classifier accuracy and classifier accuracy ratio for different ε values over all classification algorithms and datasets. Each box represents the distribution over 18 classifiers (3 classification algorithms applied to 6 datasets) for the associated ε value.

Figure 5a shows a high variation in the accuracy of ε-DP classifiers (represented by the size of boxes and length of whiskers) for every ε value. This variation is especially notable for ε values lower than (equal to) 0.1. For ε values greater than 3, the distributions are similar. This suggests the selection of a higher privacy level in this range results in a small accuracy cost.

Table 2: Accuracy comparison between classification algorithms when applied in a non-private learning setting (*Accuracy (no privacy)*) and in an ε-DP learning setting (*Average accuracy (ε-DP)*). The *ratio* measures the effect size of training a classifier in an ε-DP setting compared to a non-private setting.

Classifier	Measurement	Adult	Mushroom	Nursery	SPECT	Congress	Skin	**Average**
NB	Accuracy (no privacy)	0.8208	0.8472	0.0834	0.5337	0.9135	0.9239	0.6871
	Average accuracy (ε-DP)	0.6905	0.7458	0.1148	0.6204	0.7374	0.7763	0.6130
	Ratio	0.8412	0.8803	1.3772	1.1624	0.8073	0.8922	0.9934
SVM	Accuracy (no privacy)	0.8288	0.9990	0.9747	0.6995	0.4139	0.7914	0.7846
	Average accuracy (ε-DP)	0.8131	0.8892	0.8794	0.5014	0.2454	0.7918	0.6867
	Ratio	0.9811	0.8901	0.9022	0.7167	0.5929	1.0001	0.8473
DT	Accuracy (no privacy)	0.8450	1.0000	0.8248	0.7388	0.9538	0.7926	0.8592
	Average accuracy (ε-DP)	0.7059	0.6620	0.5427	0.5636	0.5893	0.7685	0.6387
	Ratio	0.8354	0.6620	0.6580	0.7628	0.6179	0.9696	0.7510

In Figure 5b, the small boxes for $\varepsilon \geq 0.25$, with median values close to 1, show that classifier accuracy is not considerably affected when the classifiers are trained in an ε-DP setting for this range of ε (for the selected datasets). For $0.005 \leq \varepsilon < 0.25$, the result shows more variation where the accuracy of ε-DP classifiers can be slightly or significantly worse than the one of classifiers trained in a non-private setting. For $\varepsilon \leq 0.001$, we can observe that the median value is close to 0.5, indicating that, on average, the accuracy of classifiers trained in the ε-DP setting halves compared to the classifiers trained in a non-private setting.

Discussion. In this work, we have selected three well-known classifiers, namely Naïve Bayes, SVM, and Decision Tree classifiers, and trained them in an ε-DP setting. We then explored the impact of dataset properties, classification algorithms, and privacy levels (in terms of differential privacy) on classifier accuracy.

Our analysis shows that none of the selected classifiers is a *one-size-fits-all* solution for all datasets and privacy levels. Nonetheless, based on their inherent structural properties, the required privacy level, and the dataset properties, we found some interesting results on classifiers' performance trained in the ε-DP setting:

- The ratio values reported in Table 2 show that the Naïve Bayes classifier returns the most similar accuracy between the private and non-private settings. This specifically suggests the application of the differentially private Naïve Bayes classifier in datasets in which the non-private version is accurate.
- The private SVM classifier is quite accurate when it is trained over *large* datasets due to its structure.
- The Decision Tree classifiers show increased accuracy when privacy constraints are relaxed. This could result from the fact that on the selected datasets, the non-private Decision Tree classifiers also return the most accurate results. Further investigation is required to study this trend.
- The mutual comparison of ε-DP classifiers' performance (in terms of the Wilcoxon test) shows that probability-based classification algorithms behave *almost* similarly when trained in an ε-DP setting.
- For the selected datasets, classifier accuracy does not change when the privacy level ε varies in a specific range of values. This suggests the data owner and analyst need to find the maximum privacy level for their dataset which will not significantly affect accuracy.

It should be noted that there exist several other alternative classification algorithms for Naïve Bayes, SVM and Decision Tree classifiers compared to the ones selected for this study. For instance, ID3 and C4.5 are two types of Decision Trees, the polynomial and RBF kernel-based SVM are other types of SVM classifiers, and the Bernoulli and Gaussian are two types of Naïve Bayes classifiers. Nonetheless, we expect that the selection of an alternative classification algorithm will not considerably affect our findings. This claim and the other aforementioned findings of this study needs more work to investigate the results on a wider range of datasets, different types of classifiers, and other classification algorithms in an ε-DP setting.

6 RELATED WORK

In recent years, privacy-preserving machine learning, including classification, regression, clustering, and dimensionality reduction, has received increasing attention (Ji et al., 2014). This attention has resulted in several solutions in the field of differential privacy classification. Existing approaches in this field usually ensure differential privacy by employing one of the following general methods:

1. Each row in the dataset is obfuscated, and the training algorithm is run on the resulting data.
2. Queries to the dataset originating from the training algorithm are answered with a noisy result set.
3. Once the classifier has been trained, noise is added to its parameters before its release.

The first method, called *Local Differential Privacy* (Ka-

siviswanathan et al., 2011), provides strong privacy guarantee in training the classifiers in a differential privacy setting (Gong et al., 2020). For instance, the Naïve Bayes classifier has been implemented with Local Differential Privacy, *i.e.*, with a non-interactive obfuscated dataset (Yilmaz et al., 2019). While this method has the advantage that it does not require a trusted data aggregator, it comes at an undesirable utility cost (Arachchige et al., 2019). Accordingly, under the condition that data has already been collected by a trusted aggregator, this extra privacy guarantee is not needed.

The second method has been widely used as an effective tool in privacy-preserving classification, when one party owns the data and another party is interested in obtaining a classifier model on this sensitive non-public data (Fletcher and Islam, 2019). This approach has been used to enforce differential privacy on Naïve Bayes classifier (Vaidya et al., 2013), which replaces the dataset queries in the standard Naïve Bayes algorithm with differentially private ones. This methodology has been improved in (Zafarani and Clifton, 2020) by using smooth sensitivity, a differential privacy technique that lowers the amount of random noise on each query, while retaining the same level of privacy. Differential private SVM in a nonlinear environment has been addressed with the use of kernel methods based on random projections (Rahimi and Recht, 2008). The accuracy of these methods can be increased by perturbing and then solving the dual problem (Zhang et al., 2019). The methods in (Jain and Thakurta, 2013) offer a weaker form of privacy, namely (ε, δ)-differential privacy, but can be applied to a wider range of kernel functions. All these approaches result in a model that does not leak unwanted information about the training data. Depending on the precise implementation, these approaches may have the additional privacy guarantee that private information is kept from the analyst as well. An overview of differentially private Decision Tree algorithms is given in (Fletcher and Islam, 2019). In particular, the methodology proposed in (Blum et al., 2005) replaces the dataset queries in a non-private Decision Tree algorithm by differentially private equivalents. Since under differential privacy, having more queries decreases utility, one can improve upon this by using algorithms that require fewer dataset queries (Friedman and Schuster, 2010).

The last method adds noise to the model's parameters before the model is published e.g., the optimal hyperplane of the SVM classifier is perturbed (Chaudhuri et al., 2011), or a random forest is created independently of the database, and noise is then added to the leaves' class predictions taken from the database (Jagannathan et al., 2009). While this approach needs fewer queries per tree, one needs multiple trees to get decent accuracy. In this setting, in (Jayaraman and Evans, 2019) the evaluation of differential privacy mechanisms for two machine learning algorithms presented to understand the impact of different choices of ε and different relaxations of differential privacy on both utility and privacy. Adding noise to the classifier's parameters after it is trained usually results in lower accuracy compared to previous two methods.

Our work employs the second method in which noise is added to the analyst's queries during the training of the classifier. Specifically, we showed how Naïve Bayes, SVM, and Decision Tree classifiers can be constructed in an ε-DP setting and compared their performance. While some work in the literature compares the impact of privacy in the context of classifier learning, *e.g.*, the costs of training different classifiers using Homomorphic Encryption (Sheikhalishahi and Zannone, 2020), to the best of our knowledge no prior work has focused on the comparison of classifiers' performance in a differential privacy setting.

7 CONCLUSION

This paper provides a comparison of classifiers' performance when they are trained in an ε-DP setting. Three well-known classifiers, namely Naïve Bayes, SVM and Decision Tree, have been trained under the assumption that one party owns the data and the other party is interested in obtaining the classifier's model respecting ε-differential privacy. Our experimental results show that depending on dataset properties, classifier structure, and privacy level ε one classifier might outperform the other ones.

In future work, we plan to extend our work to a thorough comparison considering a wider range of well-known classifiers (*e.g.*, k Nearest Neighbor, Random Forest) including different types of each classifier (*e.g.*, different SVM algorithms) on a broader set of benchmark datasets trained in an ε-DP setting.

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