

Energy Consumption Modeling for Specific Washing Programs of Horizontal Washing Machine using System Identification

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Abstract: This paper presents the application of an energy consumption modeling technique using a system identification method regarding the washing program settings for a horizontal washing machine. The observer/Kalman filter identification/eigensystem realization algorithm (OKID/ERA) method is employed to identify the linear discrete state-space model by choosing the system order computed by the significant singular values. The identified model is used as an estimator to figure out the energy consumption level for washing programs with the full loading condition, and results show the feasibility of the method in energy consumption modeling.

1 INTRODUCTION

The electricity and the water consumption in the washing machines are mainly dependent on the usage pattern of an end-user such as the washing program, the temperature setting, the program duration, the auxiliary functions and the laundry amount as well as the capacity of the washing machine (Schmitz and Stamminger, 2014; Afzalan and Jazizadeh, 2019). In the European Union, horizontal washing machines are commonly used for the laundry, while vertical washing machines are mostly populated in the North America, Asia and Australia. A vertical washing machine uses more water than a horizontal one, while the latter consumes more power to control the water temperature via a heater which is a high power consumption device (Pakula and Stamminger, 2010; Bertocco et al., 2020). In general, researches are mainly focused on the total energy consumption to provide the energy-policy direction either in the residential buildings or in the household appliances. Richardson (Richardson et al., 2010) presented the annual energy demand for the household appliances using the statistics between the energy use and the occupant activity. In references (Bourdeau et al., 2019; Li and Wen, 2014), authors reviewed a data-driven method for the purpose of the modeling and forecasting in a building sector and pointed out the popular approaches such as statistical regression, k-nearest neighbors, de-

cision tree, support vector machines, artificial neural-network, etc. A simplified model of the energy consumption for horizontal washing machines was proposed using a linear relationship regarding the age of the end-user, the temperature setting, the capacity of washer and the energy efficiency (Milani et al., 2015). Recently, a modeling framework was shared by using a bottom-up activity to estimate the accurate energy consumption in residential buildings (Leroy and Yannou, 2018). However, these researches have been conducted to create the energy model for all types of household appliances over a year or daily-base to figure out the optimal energy saving purpose. In household appliance sector, monitoring the power and the energy consumption in real-time per unit will give more flexibility to give the efficient product design and development strategy.

Addressing the modeling strategy for new product development, the system identification methodology is the most favourable framework by system designers. For several decades, this method has been an emerging research topic to characterize the system behavior using the experimental data to overcome the knowledge gap from the physics-based modeling in the engineering fields (Ljung, 1999; Van Overschee and De Moor, 1994; Juang and Pappa, 1985). However, the limited studies were reported in a washing machine sector using this approach. Therefore we propose an innovative approach to develop the mathematical model in a systematic way and the prediction performance of the energy consumption from the measured data for specific washing programs subjected

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to the washing program type, the temperature setting, the drum speed profile, the laundry amount, the unbalanced load, the amount of detergent and the water intake volume (Boyano et al., 2020). The goal of this research is to develop a framework identifying the mathematical model from the measured input-output data sets during the washing cycles and to estimate the energy consumption without a power sensor in order to reduce the product cost.

2 PROBLEM STATEMENT

In order to predict and analyze the energy consumption in a washing machine, a mathematical model is necessary to clarify its characteristics from the measured data sets. Therefore, an identification process is required to relate how the input affects the output. In this research, we consider that a washing machine is a black-box system for an energy consumption modeling induced by multiple input variables such as a washing program type (P), a temperature setting (T), a profile of motor speed (ω_d), a laundry amount (m_l), an amount of detergent (m_d), an amount of water (V), and so forth in equation (1). In order to address the multiple inputs and the single output relationship, we employ an observer/Kalman filter identification (OKID) working on the time-domain in Figure 1.

$$E = f(P, T, \omega_d, m_l, m_d, V) \quad (1)$$

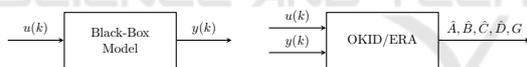


Figure 1: The system identification process.

Taking into consideration of the real application, we relate the input physical quantities in equation (1) to the low-level mechanical actuators subjected to a heater on-time, a pre-wash valve on-time, a main-wash on-time, a pump on-time, and a profile of motor speed.

3 DATA AND METHOD

3.1 Data

Firstly, the washing program types were selected based on widely used programs in the European Union via Amazon Web Services (AWS) connected by HomeWhiz IoT ecosystem developed by Arcelik. The QUICKWASH and the BEDDING programs were popularly chosen washing programs by the customers, therefore we have collected the input-output

data sets for these washing programs from the same washing machine with the full load case (9 kg of etamine fabric) described in Table 1 and the test setup environment in Figure 2.

Table 1: A washing machine configuration for the test.

Washing Program		BEDDING	QUICKWASH
Test Condition	Load Amount (kg)	9	9
	Load Type	etamine (70×70cm)	etamine (70×70cm)
	Spin Speed (rpm)	1000	1400
	Temperature (°C)	40	40
Current	Total Max. Current (A)	8.4	8.4
	Washing Motor Current (A)	0.52	0.6
	Spinning Motor Current (A)	2.67	2.5
Power	Washing Motor Power (W)	110	86
	Total Max. Power (W)	1912	1870
Water Level	Main Wash (lt)	21.17	21.10
	1. Rinse (lt)	19.44	-
	2. Rinse (lt)	19.55	-
	Softener (lt)	19.50	21.10
	Total Water Consumption (lt)	89.60	43.12
Spin Speed	Washing RPM	54	75
	Main Wash Spinning RPM	300-617	840
	1. Rinse Spin RPM	300-615	-
	2. Rinse Spin RPM	300-618	-
	Final Spin RPM	300-1020	839
Duration	Total Program Duration (min)	110	40



Figure 2: Overview of the test station.

Figures 3-4 show the measured data set regarding the washing program selection. In the both figures, during the heater activation to reach the targeted water temperature (40°C), it consumes most of the energy between (3-8) minutes and (20 – 22) minutes for the QUICKWASH program, and between (14-33) minutes for the Bedding program. Afterwards, the second highest energy consumption is caused by the motor run, and also the amount of water volume in the drum affects the motor power consumption. Additionally, the amount of water volume in a washing machine is determined by the amount of detergent dosage, the pre-wash valve on-time, the main-wash valve on-time and the drain pump on-time. Therefore, some of the inputs are dependent to the others, and this effect will be simplified via the linear system identification process. In this research, a model to be identified is the multiple-inputs and the single-output (MISO) system subjected to $u \in \mathbb{R}^5$, $y \in \mathbb{R}^1$. In order to apply the system identification, we collected following input and output data sets as follow.

$u = [\text{drum speed, heater on-time, prewash valve on-time, main-wash valve on-time, pump on-time}]$
 $y = \text{energy consumption}$

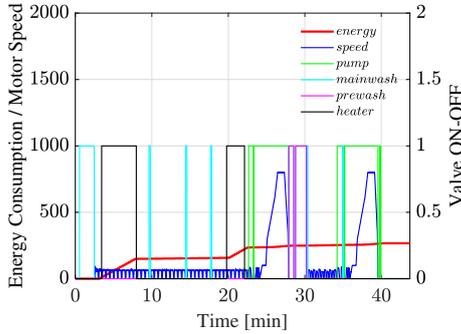


Figure 3: Input-output data for the QUICKWASH program.

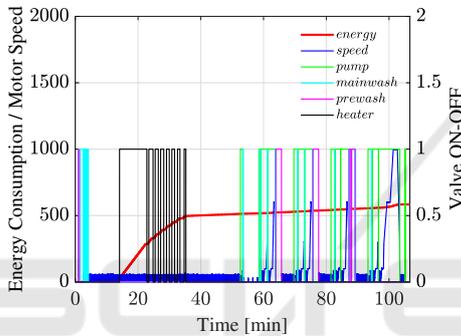


Figure 4: Input-output data for the BEDDING program.

3.2 Method

This section reviews the OKID/ERA method (Juang, 1994) to identify the system characteristics using the input-output data histories of a horizontal washing machine. Consider a discrete linear time-invariant system in a state-space form as below,

$$\begin{aligned}
 x(k+1) &= Ax(k) + Bu(k) \\
 y(k) &= Cx(k) + Du(k)
 \end{aligned} \quad (2)$$

where $x(k) \in R^n$ is the system state, $y(k) \in R^m$ is the output, $u(k) \in R^r$ is the input, and the system matrices are defined as $A \in R^{n \times n}$, $B \in R^{n \times r}$, $C \in R^{m \times n}$, $D \in R^{m \times r}$.

By assuming the zero initial condition of the state, the sequence of the above equations can be written as,

$$\begin{aligned}
 x(k) &= \sum_{i=1}^k A^{i-1} Bu(k-i) \\
 y(k) &= \sum_{i=1}^k CA^{i-1} Bu(k-i) + Du(k)
 \end{aligned} \quad (3)$$

Then, the output $y(k)$ can be decomposed into the system Markov parameters (Y) and the upper triangular input matrix (U) as below,

$$y = YU \quad (4)$$

where $y \in R^{m \times l}$, $Y \in R^{m \times rl}$, $U \in R^{rl \times l}$, and $k = l - 1$

From the above equation (4), Y represents the matrix composed of the pulse responses known as the system Markov parameters to be identified in equation (5).

$$Y = \begin{bmatrix} D & CB & CAB & \cdots & CA^{l-2}B \end{bmatrix} \quad (5)$$

The upper triangular input matrix is defined as

$$U = \begin{bmatrix} u(0) & u(1) & u(2) & \cdots & u(l-1) \\ & u(0) & u(1) & \cdots & u(l-2) \\ & & u(0) & \cdots & u(l-3) \\ & & & \ddots & \vdots \\ & & & & u(0) \end{bmatrix}$$

and the output vector y is measured as

$$y = [y(0) \quad y(1) \quad \cdots \quad y(l-1)]$$

Equation (5) can be directly derived from equation (2), however, it is not easy to measure the full states of the system and it does not guarantee the fast computation and also robust convergence if the data length l is too large. To solve these issues, the observer gain matrix G is employed to the state equation (5) to reshape the system eigenvalues so that one can obtain the desired system behavior.

$$\begin{aligned}
 x(k+1) &= Ax(k) + Bu(k) + Gy(k) - Gy(k) \\
 &= (A + GC)x(k) + (B + GD)u(k) - Gy(k)
 \end{aligned} \quad (6)$$

Then, we can design the new system containing the observer gain G in the system below,

$$x(k+1) = \bar{A}x(k) + \bar{B}v(k) \quad (7)$$

where $\bar{A} = A + GC$, $\bar{B} = [B + GD \quad -G]$, $v(k) = [u(k) \quad y(k)]^T$.

The observer gain matrix G is chosen to make the system matrix \bar{A} to be Hurwitz, and this means that for some sufficiently large p , $\bar{A}^k \approx 0$ for time steps $k \geq p$. The Kalman filter makes the computation faster to obtain the observer gain matrix G such that $G = -K$, where K is the Kalman gain matrix.

The output equation from the updated system including the non-zero initial condition can be written as

$$\bar{y}(k) = C\bar{A}^k x(0) + \sum_{i=1}^k C\bar{A}^{k-i} \bar{B}v(k-i) + Du(k) \quad (8)$$

Similarly, we can decompose output as below since the initial condition is negligible due to $\bar{A}^k \approx 0$

$$\bar{y} = \bar{Y}\bar{V} \tag{9}$$

where $\bar{y} \in \mathbb{R}^{m \times (l-p)}$, $\bar{Y} \in \mathbb{R}^{m \times [(m+r)p+r]}$, $\bar{V} \in \mathbb{R}^{[(m+r)p+r] \times (l-p)}$.

Firstly, we compute the observer Markov parameter matrix \bar{Y} from equation (10) by taking the pseudo-inverse.

$$\bar{Y} = \bar{y}\bar{V}^\dagger \tag{10}$$

where

$$\begin{aligned} \bar{V}^\dagger &= \bar{V}^T [\bar{V}\bar{V}^T]^{-1} \\ \bar{y} &= [y(p) \quad y(p+1) \quad \dots \quad y(l-1)] \\ \bar{Y} &= [D \quad C\bar{B} \quad C\bar{A}\bar{B} \quad \dots \quad C\bar{A}^{p-1}\bar{B}] \\ \bar{V} &= \begin{bmatrix} u(p) & u(p+1) & \dots & u(l-1) \\ v(p-1) & v(p) & \dots & v(l-2) \\ v(p-2) & v(p-1) & \dots & v(l-3) \\ \vdots & \vdots & \ddots & \vdots \\ v(0) & v(1) & \dots & v(l-p-1) \end{bmatrix} \end{aligned}$$

Secondly, the system Markov parameters (Y) can be recovered from the observer Markov parameters (\bar{Y}), and the observer Markov parameters are also expressed with the system matrices and the observer gain matrix as below,

$$\begin{aligned} \bar{Y}_0 &= D \\ \bar{Y}_k &= C\bar{A}^{k-1}\bar{B} \\ &= [C(A+GC)^{k-1}(B+GD) \quad -C(A+GC)^{k-1}G] \end{aligned} \tag{11}$$

or

$$\bar{Y}_k = [\bar{Y}_k^{(1)} \quad -\bar{Y}_k^{(2)}] \tag{12}$$

By the induction process, the system Markov parameters are obtained in equation (13).

$$\begin{aligned} Y_0 &= D \\ Y_k &= \bar{Y}_k^{(1)} - \sum_{i=1}^k \bar{Y}_i^{(2)} Y_{k-i} \text{ for } k = 1, 2, \dots, p \\ Y_k &= - \sum_{i=1}^p \bar{Y}_i^{(2)} Y_{k-i} \text{ for } k = p+1, p+2, \dots \end{aligned} \tag{13}$$

Now, using the eigensystem realization algorithm proposed by Juang and Pappa (Juang and Pappa, 1985), The Hankel matrix composed of the observer and the system Markov parameters can be constructed in equation (14).

$$H(k-1) = \begin{bmatrix} Y_k & Y_{k+1} & \dots & Y_{k+\beta-1} \\ Y_{k+1} & Y_{k+2} & \dots & Y_{k+\beta} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{k+\alpha-1} & Y_{k+\alpha} & \dots & Y_{k+\alpha+\beta-2} \end{bmatrix} \tag{14}$$

The Hankel matrix can be also represented by using the system Markov parameters in equation (15).

$$\begin{aligned} H(k-1) &= \begin{bmatrix} C \\ C\bar{A} \\ \dots \\ C\bar{A}^{\beta-1} \end{bmatrix} \bar{A}^{k-1} [\bar{B} \quad \bar{A}\bar{B} \quad \dots \quad \bar{A}^{\beta-1}\bar{B}] \\ &= O\bar{A}^{k-1}C \end{aligned} \tag{15}$$

where C and O denote the controllability and the observability matrices, respectively. From the Hankel matrix, a singular value decomposition is performed to obtain the unitary matrices (U_n, V_n) and a singular value matrix (Σ_n) for $k = 1$.

$$H(0) = U_n \Sigma_n V_n^T \tag{16}$$

The singular value matrix (Σ_n) contains the n number of singular values whose magnitudes are bigger than zero such that $\sigma_1 \geq \sigma_2 \geq \dots \sigma_n > 0$. At this stage, one can check the relative magnitude of the singular values, and eliminate the values which are not significant to the system performance (i.e., characteristics), and determine the system order.

Therefore, the estimated system matrices ($\hat{A}, \hat{B}, \hat{C}, \hat{D}$) can be obtained in equation (17).

$$\begin{aligned} \hat{A} &= \Sigma_n^{-1/2} U_n^T H(1) V_n \Sigma_n^{-1/2} \\ \hat{B} &= \Sigma_n^{1/2} V_n^T E_r \\ \hat{C} &= E_m^T U_n \Sigma_n^{1/2} \\ D &= \bar{Y}_0 \end{aligned} \tag{17}$$

where E_m^T and E_r^T are consisted of the identity and the zero matrices, which have different matrix dimension.

4 RESULTS

The system identification process has been performed with three measurements for both QUICKWASH and BEDDING programs from 9 kg capacity of a single washing machine. Two of the three measurements have been used to construct the discrete state-space model using an OKID/ERA method for each washing program. The third measurement was used for the validation of the identified model. The data processing, the algorithm implementation, and the simulation were carried out using MATLAB scripts (MATLAB R2019a) with Control System Toolbox.

4.1 Model Selection

In general, the singular value represents the characteristics of the system, and it is a reasonable criteria

Table 2: The identified discrete state-space model.

	QUICKWASH					BEDDING				
\hat{A}		0.8893	0.4491	-0.0631			1.0157	0.2100	-0.0806	
		0.2284	0.1618	-0.5788			-0.1125	0.2362	-0.8358	
		-0.1193	0.6037	-0.0331			-0.0155	0.5006	-0.2665	
\hat{B}	-0.0005	-0.3207	0.0574	0.0107	0.0243	-0.0002	-0.3592	0.0031	0.0055	-0.0029
	0.0010	-0.0727	0.0458	-0.0065	-0.0404	0.0004	-0.2068	0.0043	-0.0020	0.0120
	-0.0001	0.7456	0.2538	0.0133	-0.0082	-0.0005	0.6957	-0.0116	0.0113	0.0101
\hat{C}		-0.5708	0.3617	0.0367			-0.4659	0.2141	0.0726	
\hat{D}	-0.0002	0.1861	0.0015	-0.0065	-0.0646	0.0001	0.1439	-0.0023	0.0034	0.0010
G		1.7099	-0.0989	-0.1664	T		1.5168	-1.0481	-0.7498	T

to determine the system size. Therefore, we initially chose the system size as 10-th order as seen in Figures 5-6 and the singular values were computed from the Hankel matrix. Thus, one can choose the system order by checking the number of the dominant singular values. In this way, we can reduce the system order since the rest of the singular values have less impact to be the system behavior or it can be noise effects. For the QUICKWASH program in Figure 5, first three singular values are dominant compared to the others, and for the BEDDING program, we chose the first three values since the third and the fourth ones do not have big difference in magnitude. The identified mathematical models with observers (G) have the three-degree of freedom described in Table 2.

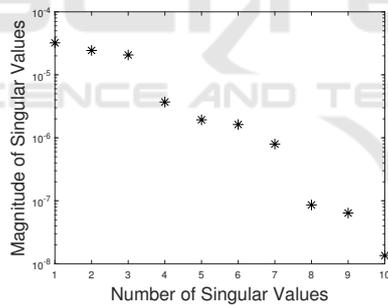


Figure 5: The singular values for the QUICKWASH.

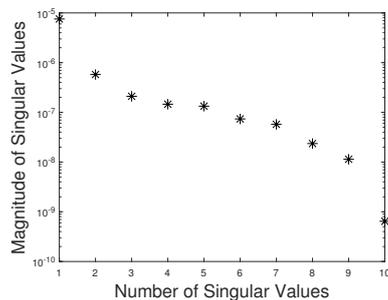


Figure 6: The singular values for the BEDDING.

Both the identified state-space models are controllable and observable since we can obtain the full rank from the controllability and the observability matrices, respectively. Tables 3 and 4 show the accuracy of the identified model in $RMSE$ and $MAPE$.

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (\hat{y}_i - y_i)^2} \quad (18)$$

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left| \frac{\hat{y}_i - y_i}{y_i} \right| \quad (19)$$

Table 3: Model accuracy for the QUICKWASH program.

Observer	Test No.	E_{rmse} (Wh)	E_{mape} (%)
Yes	Test 1	0.1467	7.71×10^{-4}
	Test 2	0.1407	7.67×10^{-4}
No	Test 1	13.9991	6.6575
	Test 2	12.5445	5.9351

Table 4: Model accuracy for the BEDDING program.

Observer	Test No.	E_{rmse} (Wh)	E_{mape} (%)
Yes	Test 1	0.1331	1.48×10^{-4}
	Test 2	0.1291	2.00×10^{-4}
No	Test 1	10.2693	1.9780
	Test 2	10.2693	1.978

4.2 Model Validation

From the identified state-space model in Table 2, we used the third measurement data, which was not included for the system identification process, to validate the prediction accuracy of the energy consumption for both washing programs in Figure 7. Tables 5 and 6 indicate the errors in the $RMSE$ and the $MAPE$ defined in equations (18)-(19). Both tables show that adding an observer (G) provides the accurate energy estimation since the augmented input, $v(k) = [u(k) \ y(k)]^T$ in equation (7), contains the input measurement as well as the output via sensors. That means the observer generates the optimal system states by minimizing the error between the measured energy consumption and the estimated one.

In Figure 7, the estimated output is defined by \hat{y}_k at each time step and the errors are calculated as below,

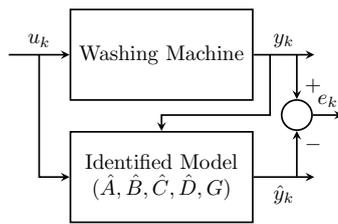


Figure 7: The model validation.

In this research, however, we focused on identifying the linear discrete state-space model and validating the accuracy of the model by comparing the predicted output to the measured one. The results show that the identified MISO model without an observer roughly follows the trend of the energy consumption with the accuracy of 91.2% and 94.2% in MAPE for the QUICKWASH and the BEDDING programs, respectively.

Table 5: Validation for the QUICKWASH program.

Observer	Test No.	E_{rmse} (Wh)	E_{mape} (%)
Yes	Test 3	0.1435	7.48×10^{-4}
No	Test 3	18.1276	8.8069

Table 6: Validation for the BEDDING program.

Observer	Test No.	E_{rmse} (Wh)	E_{mape} (%)
Yes	Test 3	0.1575	8.19×10^{-5}
No	Test 3	28.0675	5.8599

Figures 8-9 also graphically demonstrate that how well the model with and without an observer estimates the energy consumption, where \hat{E} is without an observer and \bar{E} with an observer.

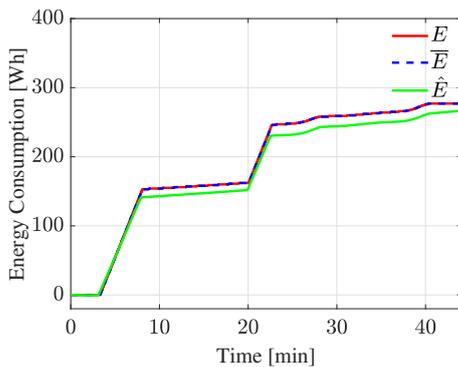


Figure 8: Comparison of the energy consumption for the QUICKWASH program.

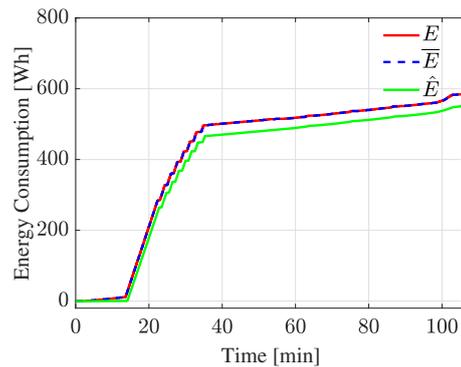


Figure 9: Comparison of the energy consumption for the BEDDING program.

5 CONCLUSION AND FUTURE WORK

In this research, we have studied the systematic modeling technique for the energy consumption in a horizontal washing machine using an OKID/ERA approach in the time-domain and the model reduction process was carried out to reduce the computational time by counting the dominant singular values obtained from the Hankel matrix. The discrete linear time-invariant state-space models with the three-degree of freedom were obtained and validated to see the feasibility of the framework for the energy consumption of the specific washing programs. From the simulation results, the method can successfully generate the accurate model with the input-output measurements. Especially, in the case of being used as an estimator with Kalman gain ($K = -G$) in a feedback system to adjust the optimal system states, the prediction accuracy in the energy consumption can be significantly improved. For the future study, we will consider medium and large capacity of washing machines under the different laundry amounts such as quarter, half and full loading. The proposed model will be integrated and deployed in the software stack of washing machine to estimate the energy consumption level without a power sensor to check the practical aspect.

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