# Turning Rate Estimation in Roundabouts: Analysis and Validation of Different Estimation Methods

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Abstract: The knowledge of turning rates in roundabouts is a crucial element of traffic modeling. Measuring the turning movements is often carried out by manual traffic counts (noting on paper or using handheld devices), which is a labor-intensive, therefore expensive process. The aim of this paper is the examination and comparison of different estimation methods used for turning rates in roundabouts. Traditional iteration based approach as well as estimators adopted from control theory are discussed, benchmarked, and validated on real-world traffic data. For the estimation procedures, the traffic flows (measured at each leg of the intersection) are the input. In this way, the traditional origin-destination traffic count at an intersection can be substituted by automated traffic detection at the cross-sections together with the adequately implemented estimation process (suggested in the paper). The calibration of estimation methods is of crucial importance as well. The calibration is demonstrated based on real-world traffic counts at roundabouts. The different methods have been compared using different error metrics. As a main finding of the research, it is shown that, given the right tuning, constrained Kalman Filtering outperforms the unconstrained Kalman Filtering and the traditional iterative procedure.

## 1 INTRODUCTION

Road traffic infrastructure planning or development is initiated based on reliable traffic modeling. The input of the modeling is the knowledge of vehicular flows on road links and turning rates at intersections. Traffic volumes at cross-sections can be straightforwardly measured manually or with help of a wide variety of traffic sensors. At the same time, turning flows or turning rates can be collected by human resources solely, which is quite costly. Therefore, if turning flows are collected, typically more than one person is needed in order to perceive all movements. The more, observing turnings in roundabouts is extremely problematic due to the special geometry and size of this type of junction (Cao and Zöldy, 2020).

Fig. 1 demonstrates the possible turning movements at a roundabout for vehicles arriving at Entrance 1. $V_{1j}$ is the turning traffic flow from Entrance 1 to exit $j$, whereas $V_{1,in}$ and $V_{1,out}$ are the total traffic volumes entering and exiting at the corresponding junction leg. Using the volumes in Fig. 1, turning rates can be defined as follows:

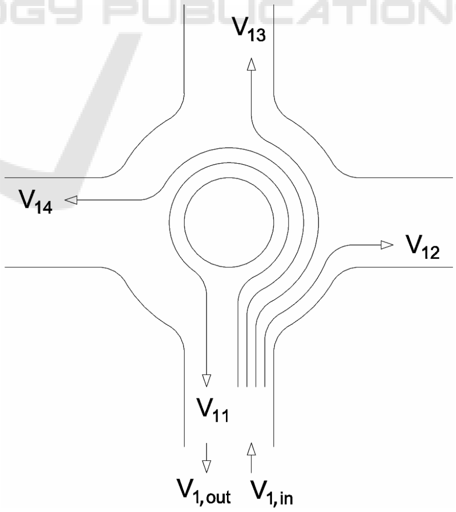

Figure 1: Turning movements at a roundabout.

$$x_{ij} = \frac{V_{ij}}{\sum_{j=1}^{n_D} V_{ij}} = \frac{V_{ij}}{V_{i,in}} \tag{1}$$

where $n_D$ is the number of exits.

There exist, in fact, automated methods for turn-

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ing flow counts. For instance, it can be carried out by installing cameras on the spot and evaluating the footage subsequently by using artificial intelligence (Taylor et al., 2016). However, placing cameras or shooting aerial videos with drones (Salvo et al., 2014) can be quite costly as well. Moreover, the legal background of drones is yet to be simplified for this to be a real alternative (Budinska, 2019). On the other hand, in the case of cross-sectional counts, there is a wide range of solutions for automation (e.g. inductive loop detectors, cameras, ultrasonic detectors).

Counting traffic on the legs of a roundabout and adequately estimating turning rates based on the collected data has the potential to substitute labor-intensive turning flow counts. This could reduce the cost of determining turning rates at an intersection significantly. In this paper, cross-sectional counts are used as a basis to estimate turning rates at a roundabout. This proposes a possible solution to overcome the obstacles posed by turning movement observation.

The paper is divided into 5 sections. A detailed description of the examined estimation methods follows the introduction. Next, the test sites are introduced, where the traffic counts were carried out for the research. Then, the testing of different methods (a traditional iterative procedure and modern Kalman Filter based methods) and the methodology of determining the optimal tuning settings are presented. The methods are then compared using different error metrics. Finally, a summary of findings and recommendations for future research is given.

## 2 ESTIMATION METHODS

This section covers different methods used for turning rate estimation. Biproportional procedure is discussed as a traditional iterative algorithm, then the Kalman Filter and its extension with constraint handling are introduced.

### 2.1 Biproportional Procedure

The biproportional procedure (BP) is an iterative algorithm (Ben-Akiva et al., 1985), where the variation of two coefficients ($a$ and $b$) causes the variation of turning flows in each iteration. Two sets of input data are necessary for this procedure. A preliminary origin-destination matrix ($t$) and the traffic flows on each leg of the roundabout ($O_i$ entering and $D_j$ exiting counts in case of entrance $i$ and exit $j$). $t_{ij}$ is the traffic volume from $i$ to $j$, and there exist $n_O$ entrances and $n_D$ exits. The accuracy of the BP estimation depends largely on the accuracy of prior matrix $t$ (Dixon and Rilett, 2005).

The BP procedure aims to estimate the elements of the current OD-matrix $T$, based on the current flows on each leg and prior matrix $t$. Therefore, the resulting matrix of the procedure contains traffic volumes, which then can be converted into turning rates. This assists the comparison of estimation procedures.

The estimated $T$ has to satisfy the following constraints:

$$O_i = \sum_{j=1}^{n_D} T_{ij}, \quad (2)$$

$$D_i = \sum_{i=1}^{n_O} T_{ij}. \quad (3)$$

To meet the constraints in Eq. (2) and Eq. (3), iterations are executed. Each iteration alters the proportions $a$ and $b$. These proportions from the previous iteration are marked as $a^*$ and $b^*$. Estimated matrix $T$ has a minimal difference from prior matrix $t$, whilst satisfying the constraints (Dixon et al., 2007).

The initial conditions for the BP procedure are that $a_i^*$, $b_j$, and $b_j^*$ are set to 1, while $T_{ij}$ is set equal to $t_{ij}$ for all turning movements. For a stopping criterion, a sufficiently small value of ε needs to be reached by the changes in $a_i$ and $b_j$.

The steps of the algorithm are detailed as follows.

1. Calculation of $a_i$:

$$a_i = \left( \frac{O_i}{\sum_{j=1}^{n_D} T_{ij}} \right) a_i^*. \quad (4)$$

2. Calculation of $T_{ij}$:

$$T_{ij} = t_{ij}\, a_i\, b_j. \quad (5)$$

3. Calculation of $b_j$:

$$b_j = \left( \frac{D_j}{\sum_{i=1}^{n_O} T_{ij}} \right) b_j^*. \quad (6)$$

4. Calculation of $T_{ij}$ using Eq. (5).
5. End of iteration. If the changes in $a_i$ and $b_j$ are greater than the previously defined ε, the iteration starts over from Step 1. If the changes are less than or equal to ε, the last estimated $T_{ij}$ is the result of the current interval.

The algorithm above depicts only one measurement period. While implementing the BP procedure, turning flows need to be estimated in each interval. After the stopping criterion is met, all elements of $T$ are rounded to the nearest integer. $T$ then becomes the prior matrix for the next period as the volumes of $O_i$ and $D_j$ are updated as well.

An advantage of the BP procedure is its relatively low computational requirements. Also, OD-matrices are estimated based on 8 cross-sectional counts instead of 16 turning movement observations. Another benefit that derives merely from the characteristics of the algorithm is that if U-turns are assumed to be zero in the prior matrix, the estimated matrices also have zeros in the main diagonal. A disadvantage of the procedure is its heavy dependence on the accuracy of the prior matrix.

## 2.2 Kalman Filter

State space based estimators include a model of the system and noises. Some procedures are apt to manage constraints concerning the estimated values (e.g. for each turning rate to be non-negative). Moreover, these methods estimate the mean and standard deviation for all states in each interval.

State space based estimators have been applied to predict turning flows for traditional intersections (Papapanagiotou et al., 2019), (Kulcsár et al., 2005). At the same time, by studying the relevant scientific literature, it can be stated that state space based methods have not been used to estimate turning rates in roundabouts so far. Accordingly, in this paper, Kalman Filter and its constrained extension are implemented for roundabout traffic flow estimation. First, the algorithms of these approaches are discussed in the sequel.

The basis of Kalman Filtering is the following discrete time-invariant measurement equation (Kalman, 1960):

$$y(k) = C(k)\,x(k) + z(k), \qquad (7)$$

where the variables are as follows:

- $y(k)$ - output or measurement vector;
- $x(k)$ - unknown state vector that varies over time;
- $C(k)$ - a weighing matrix called output matrix;
- $z(k)$ - a vector of measurement noise.

Eq. (7) represents that states cannot always be measured directly and that the measurement is affected by some level of noise (considered to be white noise). The objective of Kalman Filtering is to estimate the state vector as accurately as possible in each interval.

Next, diagonal covariance matrix $R$ is defined containing the variances of measurement noises. State error covariance matrix $P$ is introduced consisting of variances concerning the estimated states. $G$ is a gain-matrix which plays a role in calculating $P$.

In the case of dynamic systems, defining the model solely with the measurement equation can result in rigidity when applying the estimation for longer time periods. Thus, the system itself and the noise affecting it need to be modeled as well. The Kalman Filter is a recursive algorithm containing the system model and the concerning noises.

Eq. (8) is the state equation describing the system in discrete linear time varying case:

$$x(k+1) = A(k)\,x(k) + B(k)\,u(k) + v(k), \qquad (8)$$

where the variables are as follows:

- $x(k)$ - state vector;
- $u(k)$ - input vector or control vector;
- $A, B$ - system matrices;
- $v(k)$ - vector of state noise (the error of the system model).

$Q$ is a state noise covariance matrix for vector $v(k)$, just as $R$ is a measurement noise covariance matrix for vector $z(k)$.

The relation of $Q$ and $R$ matrices play a major role in the operation of the Kalman Filter. Their values are to be determined empirically prior to the start of the algorithm. These can be described as tuning matrices. If the values of $Q$ are far larger than that of $R$, the algorithm relies heavily on the current measurements. If the values of $R$ exceed that of $Q$, the Kalman Filter rather accepts the last interval's estimation as opposed to the measurements.

The Kalman Filter algorithm is detailed as follows (where in time-variant case $A$, $B$, and $C$ are varying matrices, i.e. $A(k)$, $B(k)$, and $C(k)$).

1. Project state ahead:
$$\hat{x}^{-}(k) = A\,\hat{x}(k-1) + B(k-1)\,u(k-1). \qquad (9)$$
2. Project the error covariance ahead:
$$P^{-}(k) = A\,P(k-1)\,A^{T} + Q. \qquad (10)$$
3. Execute the measurement providing $y(k)$.
4. Compute the Kalman gain:
$$G(k) = P^{-}(k)\,C^{T}\,(C\,P^{-}(k)\,C^{T} + R)^{-1}. \qquad (11)$$
5. Update estimate with measurement $y(k)$:
$$\hat{x}(k) = \hat{x}^{-}(k) + G(k)\,(y(k) - C\,\hat{x}^{-}(k)). \qquad (12)$$
6. Update the error covariance:
$$P(k) = (I - G(k)\,C)\,P^{-}(k)). \qquad (13)$$
7. Increment $k$, and go to Step 1 of the algorithm:
$$k := k + 1.$$

The estimation algorithm is divided into two parts. The prediction (Steps 1 and 2) is the projection of state vector $\hat{x}^{-}(k)$ and error covariance matrix $P^{-}(k)$ based on the previous estimations. The correction (Steps 3-7) is updating the state estimates and error covariance matrix knowing the current measurement values.

In case of estimating turning rates in roundabouts based on the traffic flow on the legs, the elements of the state vector in the Kalman Filter are the turning rates (Tettamanti et al., 2019). $A$ in state equation (8) is an identity matrix, whereas $B$ can be substituted with 0 as there is no control vector, i.e. the state vector to be estimated is as follows:

$$\hat{x}(k) = \begin{pmatrix} \hat{x}_{11} \\ \hat{x}_{12} \\ \vdots \\ \hat{x}_{n_O,n_D} \end{pmatrix}, \tag{14}$$

where $\hat{x}_{ij}$ denotes the estimated turning rate from entrance $i$ ($i = 1, 2, ..., n_O$) to exit $j$ ($j = 1, 2, ..., n_D$). $C(k)$ in measurement equation (7) contains the measured entering traffic flows (marked by $q_m$ where $m$ denotes the $m^{th}$ leg of the roundabout).

$$C(k) = \begin{pmatrix} q_1(k) & & & q_2(k) & & & \\ & \ddots & & & \ddots & & \\ & & q_1(k) & & & q_2(k) & \cdots \end{pmatrix}. \tag{15}$$

Thus, exiting traffic flows appear in vector $y(k)$ as measured parameters.

### 2.3 Kalman Filter with Constraints

Assume that the modeled system satisfies the following constraints:

$$A_{eq} x(k) = b_{eq}, \tag{16}$$

$$A_{in} x(k) \leq b_{in}, \tag{17}$$

where $A_{eq}$ and $A_{in}$ are known matrices as well as $b_{eq}$ and $b_{in}$ are known vectors. In this case, estimated states also need to satisfy these conditions:

$$A_{eq} \hat{x}(k) = b_{eq}, \tag{18}$$

$$A_{in} \hat{x}(k) \leq b_{in}. \tag{19}$$

Compliance with these constraints can be reached by projecting the state to lie in the constrained space at each estimation interval (Gupta and Hauser, 2007). This means that the unconstrained filter runs in a normal way, but at each iteration the updated state estimate is forced to lie in the constrained space. In this approach, the analytic solution is no longer available for filtering. Thus, numerical optimization is needed to be applied.

The projection is carried out via the following constrained optimization problem (Simon, 2010), (Gupta and Hauser, 2007):

$$\tilde{x}(k) = \text{argmin}_x \, (x - \hat{x}(k))^T \, W \, (x - \hat{x}(k)), \tag{20}$$

$$\text{s.t. (16) and (17)},$$

where $\tilde{x}$ is the projected state estimate and $W$ is a weighing matrix.

$W$ can be chosen as an identity matrix (hereinafter referred to as cKF-I). The result is then the least square estimate subject to the constraints, which means that estimates necessarily get closer to the real state values. If noises are assumed to be white and $W$ is set to $P(k)^{-1}$ in each interval (hereinafter referred to as cKF-P), the result is the maximum probability estimate of the state subject to state constraints (Simon, 2010).

The issue of managing constraints by (20) can be tackled easily by any standard optimization package. For this research, MATLAB Optimization Toolbox was applied.

Initial vector $x_0$ for the optimization is state vector $\hat{x}(k)$ estimated by the Kalman Filter without constraints. Adequately defined $A_{in}$ and $b_{in}$ results in an inequality constraint enforcing turning rates to be non-negative. Suitable equality constraint matrix $A_{eq}$ and vector $b_{eq}$ can set all U-turn rates to zero while ensuring that the aggregate of turning rates arriving at the roundabout at a given entrance is 1 at all times.

The procedure of managing constraints is the following. The Kalman Filter algorithm outputs an $\hat{x}(k)$ vector in interval $k$. The optimization subject to constraints (18) and (19) is then executed on this estimated state vector using MATLAB optimization function (*quadprog*). In interval $(k+1)$ the Kalman Filter uses the constrained state vector estimated in interval $k$ as input data.

## 3 TEST FIELD

The knowledge of real turning movement volumes is necessary for the comparison of estimated and real turning rates. For this research, turning flow counts were conducted at two different roundabouts in Kecskemét, Hungary (Fig. 2 and Fig. 3).


Figure 2: Aerial footage of Roundabout 1 at Kecskemét, Hungary (GPS coordinates: 46.92971298057884, 19.663997128931193).


Figure 3: Aerial footage of Roundabout 2 at Kecskemét, Hungary (GPS coordinates: 46.88150317109579, 19.707799625939572).

In accordance with the drone's maximal flight time, 26-minute aerial video recordings were taken at the two four-leg intersections. The counts took place at different times of the day (morning and afternoon). The 26-minute counts are adequate to be divided into 1, 2, and 5 minute intervals (in the latter case, only 25 minutes are examined). The traffic count was therefore conducted for 1-minute intervals, so that 2 and 5-minute intervals could be calculated afterward.

The estimation algorithms introduced in the paper work based on the counted number of vehicles expressed in passenger car equivalent (PCE, i.e., the different types of road vehicles expressed as the ratio of the private car (Lay, 2009)). The traffic counts thus included the differentiation of vehicle categories.

# 4 BENCHMARKING THE ALGORITHMS WITH REAL-WORLD DATA

In this section, the steps and circumstances of applying the estimation procedures are detailed; then, a comparison is made between the different methods. The real-world traffic counts provide input data for the estimators as well as a basis for determining their accuracy, i.e. for validation. The latter is carried out by comparing state estimates with the real turning rates, using error metrics. All concerning estimators are tested for all the collected data with different intervals.

## 4.1 Error Metrics

Two different error metrics (Chen et al., 2017) have been applied during the evaluation of estimation procedures. The first is the mean absolute error (MAE), for which the formula is as follows:

$$MAE = \frac{\sum_{k=1}^{n} |\hat{x}_k - x_k|}{n} \tag{21}$$

where $n$ is the number of samples (intervals), $\hat{x}_k$ is the state estimation in interval $k$, and $x_k$ is the actual state.

The second error metric used to describe the accuracy of the estimators is the root mean square error (RMSE). The formula for the RMSE is the following:

$$RMSE = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(\hat{x}_k - x_k)^2} \tag{22}$$

MAE and RMSE have the same unit of measure as the examined quantity. In the case of turning rates, this is a unitless value between 0 and 1. The direction of the error is neglected in both cases. MAE is an absolute measure and RMSE contains the squared error, which always gives a non-negative value. As the expression under the root symbol is non-negative, RMSE will always have a real solution.

## 4.2 Tuning the Kalman Filter

When applying the Kalman Filter, the objective is to adjust the tuning parameters precisely. The tuning was carried out through the comparison of estimated and real turning rates, searching for the smallest error measures when varying the tuning parameters. The attributes of the estimation depend on state noise and measurement noise covariance matrices $Q$ and $R$. These set the weighting between the current measurements and previous estimates. In practice, $Q$ and $R$ can be defined as diagonal matrices with constant values. Tuning of the Kalman Filter depends on the ratio of the values of the two matrices. Thus, $R$ can be defined as an identity matrix, and $Q$ needs to be altered after each run. In this way, the value of $Q$ is equal to the $Q/R$ ratio. During the search for the optimal settings, this ratio is set to be $10^{20}$, and is divided by 10 after each run, until it reaches $10^{-10}$. The corresponding error measures are calculated for each run and tabulated. The minimum of errors designates the optimal tuning parameter ratio. The appropriate $Q/R$ ratio for the Kalman Filter for the examined traffic data is $10^{-3}$.

In the case of the constrained Kalman Filter (cKF), weighing matrix $W$ is set to be an identity matrix (cKF-I). This leads to $10^{-2}$ as the optimal $Q/R$ ratio.

Giving $W$ values that are different from an identity matrix is worth examining to establish if it can improve the accuracy of the estimation. For this purpose, $W$ is altered subject to a fixed $Q/R$ ratio. This process has revealed that the errors cannot be decreased substantially; thus, a fixed $W$ can optimally be chosen to be an identity matrix.

In order to achieve a more accurate estimation, weighing matrix $W$ can be defined to vary over time.

A well functioning solution is to set $W$ to the inverse of the state error covariance matrix $P$ in each interval (cKF-P) (Simon, 2010). In this case, a local minimum in errors is forming around that of the unconstrained Kalman Filter. However, increasing the $Q/R$ ratio leads to a significant improvement in accuracy. The optimal ratio is determined to be $10^6$ for the cKF-P.

During the tuning procedure, the optimal parameters were established for the Kalman Filter:

- without constraints (KF) - $Q/R = 10^{-3}$
- with constraints, while $W = I$ (cKF-I) - $Q/R = 10^{-2}$
- with constraints, while $W = P(k)$ (cKF-P) - $Q/R = 10^6$

### 4.3 Evaluation of the Estimation Methods

The tendency of error measures are similar in all cases irrespective of the location or the time of the day. Therefore, for the sake of transparency and a more general result, error values are averaged over the different traffic counts. The average values form the basis for the comparison of different estimation procedures.

Table 1: Comparison of estimation procedures.

| Method | Interval | MAE | RMSE | MAE rank |
|---|---|---|---|---|
| BP | 1 min | 0.1181 | 0.1760 | 9 |
| | 2 min | 0.0822 | 0.1230 | 5 |
| | 5 min | 0.0670 | 0.1050 | 2 |
| KF | 1 min | 0.1484 | 0.2122 | 12 |
| | 2 min | 0.1036 | 0.1505 | 8 |
| | 5 min | 0.0742 | 0.1118 | 4 |
| cKF-I | 1 min | 0.1431 | 0.2110 | 11 |
| | 2 min | 0.1026 | 0.1480 | 7 |
| | 5 min | 0.0692 | 0.1048 | 3 |
| cKF-P | 1 min | 0.1183 | 0.1765 | 10 |
| | 2 min | 0.0843 | 0.1276 | 6 |
| | 5 min | 0.0608 | 0.0945 | 1 |

Table 1 lists the average MAE and RMSE values for all examined estimation methods and all interval sizes. A ranking in the MAE values is also assigned to the procedures. Based on the order, it can be stated that the longer the interval, the more accurate the estimation. Whereas shorter intervals result in larger errors.

The 5-minute interval led to the smallest errors in the case of every examined method. A possible explanation for this is the following. If the intervals are short, it is more frequent that a specific turning movement is not executed during that brief time period. This can result in sharp fluctuations in turning rates, which is harder to track for an estimator.

The order of estimation procedures with 5-minute intervals based on MAE values is the following:

1. cKF-P (constrained Kalman Filter while $W = P(k)$)
2. BP - biproportional procedure
3. cKF-I (constrained Kalman Filter while $W = I$)
4. KF (Kalman Filter)

In the case of 5-minute intervals, the Kalman Filter with constraints outperforms the BP procedure (the improvement in error measures is approximately 10%) and the unconstrained Kalman Filter. It is also observable that the shorter estimation intervals of 1 or 2 minutes provide higher errors in every estimation procedures. This clearly means that on longer time intervals, the algorithms can better smooth their estimations.

It is also noted that the performance of the constrained Kalman Filter can be improved by tuning the parameters separately for each junction leg.

## 5 CONCLUSIONS

Different methods of turning flow counts exist with different benefits and drawbacks concerning roundabouts. A traditional iterative algorithm (biproportional procedure) and Kalman Filter based methods (never used before for roundabout turning rate estimation) have been benchmarked. The exact methodology to apply these procedures was also introduced in detail.

The main contribution of the paper is the validated comparison of different methods on real-world data sensed by drone and then counted manually. Analyzing the results, the following conclusions can be drawn:

- in general, longer intervals result in more accurate estimations;
- managing constraints improves the accuracy of the state space based estimators significantly;
- the adequately tuned constrained Kalman Filter outperforms the unconstrained Kalman Filter and the traditional iterative procedure.

The continuation of this research is twofold. On the one hand, another state space based estimator, the Moving Horizon Estimation (MHE) will be implemented for the same estimation problem, by which the current state can be estimated based on more than one previous step. On the other hand, the evaluation of the estimation procedures will be extended with the help of microscopic road traffic simulation. After the validation of simulation models using real-world traffic data, different traffic situations can be tested easily. Thereafter, changes in the accuracy and tuning of estimators can be further examined.

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