

# Method for Quantitative Assessment of the Eyes Pulse Blood Flow with Linear Axisymmetric Model

Sergey B. Kazakov, Petr V. Luzhnov and Irina D. Davydova  
*Bauman Moscow State Technical University, Russian Federation*

**Keywords:** Ophthalmoplethysmography, Oscillations Modelling, Diagnostic Algorithm, Eye, Blood Flow.

**Abstract:** The work examines a linear axisymmetric model of the eyes pulse blood flow. The main purpose of the model is the determination of mathematical dependence of linear changes in the eye thickness of vascular layer with the pulse. The main parameter is the volume of the input eye pulsation of blood flow. Ocular hemodynamic parameters depend from various individual anatomical values of a patient's eye. In the paper all assumptions of axisymmetric model are described. Calculations of the mathematical dependence equation are based on geometric parameters of the eyeball. The analytical expression of volume changes in the measuring ophthalmoplethysmography is presented. Accordance to our calculations, linear change in the eyeball layers thickness varies within tens and units of microns after the input blood pulse volume. Advanced technical methods that can register such a small range of length units are considered. Such a registration and subsequent analysis of micro-displacement data will allow diagnosing different pathological states of the eye hemodynamic in ophthalmology and neurology.

## 1 INTRODUCTION

Vascular factor is the essential and sometimes the crucial aspect in the development of eye pathologies: age-related macular degeneration (Mori F. et al., 2001), diabetic retinopathy (Geyer O. et al., 1999; Langham M.E. et al., 1989), glaucoma (Abegao Pinto L. et al., 2016), circulatory disorders in the retinal blood vessels (Tultseva S.N. et al., 2016). For example, in the study of glaucoma, one theory has been put forward that this pathology does not contain only an increasing in intraocular pressure (IOP). Moreover, it sometimes does not exclude a violation of intraocular blood flow too. It was found that at least 80% of people with increased IOP did not develop optic nerve damage, while approximately 30% of patients with glaucoma never experienced increasing in IOP (Abegao Pinto L. et al., 2016). These results allow us to understand that one of the significant risk factors for such diseases is a pathological change in the ocular hemodynamic (Schmetterer L., Kiel J., 2012). Nowadays, current technologies do not allow us to use reliable and high-precision quantitative non-invasive measurement of ocular pulse blood flow (Langham M.E. et al., 1989).

Nevertheless, it is necessary to pay an attention to the diagnostic calculations of eye blood circulation due to using mathematical modelling in order to improve its accuracy. New knowledge in the field of eye hemodynamic will allow improving the diagnostic algorithms of various eye diseases, to create new therapeutic methods in ophthalmology and neurology. Mathematical modelling of complex ocular hemodynamics or its individual segments has already been used to solve many different problems (Shamaev D.M. et al., 2017). Even significantly simplified models were useful for ophthalmologists.

We have already studied the hemodynamic parameters of the eye in a normal, non-pathological state in our previous work (Luzhnov P.V. et al., 2020). Therefore, it is possible to form parameters and technical characteristics of linear axisymmetric model of the eyes pulse blood flow together with a physical stand for ocular pulse modelling.

It is known that functional hemodynamic changes are based on pathomorphological eye conditions. The study of which can be useful in the early diagnosis of eyes diseases. Thus, the purpose of our work is the development of eye pulse mathematical model. The main task of that model will be the determination of mathematical dependence of linear changes in the ocular

geometrical sizes on the volume of the input eye blood pulsation.

## 2 MATERIALS AND METHODS

The simplest linear axisymmetric model was chosen for modelling the ocular hemodynamics. The main assumptions of this model are presented below:

- The eyeball is considered as an object isolated from surrounding biological tissues.
- The simplest model of the eye vascular layer is an isotropic thin-walled spherical shell with a thickness  $d$ , with a constant inner radius  $R_1$  and variable outer radius  $R_2$ .
- The incompressible vitreous body is inside the spherical shell.
- The eye anterior-posterior axis is  $D$  (23.0 mm). The initial value of the external radius  $R_2$  of the eye vascular layer is  $D/2$  mm.
- The initial thickness of the eye vascular layer  $d$  is 70.0 mm.
- The elasticity coefficient of a thin-walled spherical shell must be ignored.
- The input volumes of eye blood flow pulsation are set as constant data  $\Delta Q$ .  $\Delta Q$  is the volume of input ocular blood flow pulsation.
- The vascular layer of the eye is modelled as a shaded shell that has an inner radius  $R_1$  and an outer radius  $R_2$ .

In our model (see Fig.1)  $d_0$  – the initial thickness of the eye vascular,  $R_1$  – the constant inner radius of eye vascular layer,  $\Delta d$  – the linear change in the thickness of eye vascular layer,  $R_2$  – the outer radius of eye vascular layer,  $R_3$  – the outer radius of eye vascular layer after the volume of the input eye pulsation of blood flow.

The main task of this model is the determination of mathematical dependence of linear changes in the eye thickness of vascular layer on the volume of the input eye pulsation of blood flow.

The ophthalmoplethysmography method was chosen as a method of recording and measuring the variations in volume of the eyeball. Therefore, it was necessary to determine the dependence of the volume changes in the measuring chamber (i.e. in the measuring eyecup) on the changes in the linear size of our linear axisymmetric model.

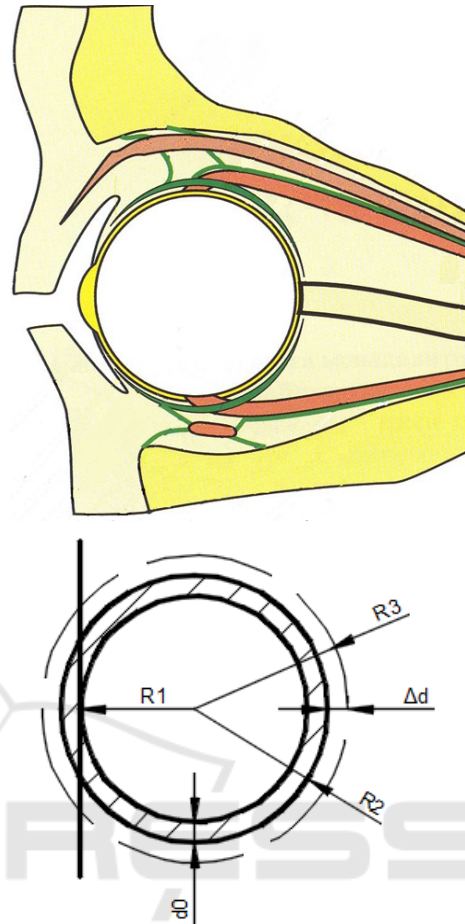


Figure 1: Linear axisymmetric model of the eyeball.

Firstly, it is necessary to output an analytical equation that defines the value of the outer radius of eye vascular layer  $R_3$  (see Fig.1). The initial volume of the vascular layer  $V_0$  is defined as

$$V_0 = \frac{4}{3} \pi \cdot (R_2^3 - R_1^3) \quad (1)$$

The volume of the vascular layer after the input ocular pulsation of blood flow is defined as

$$V_0 + \Delta Q = \frac{4}{3} \pi \cdot (R_3^3 - R_1^3) \quad (2)$$

Due to this equation it is possible to determine the outer radius of eye vascular layer:

$$R_3 = \sqrt[3]{R_2^3 + \frac{3 \cdot \Delta Q}{4\pi}} \quad (3)$$

Finally, due to previous calculations it is possible to determine the linear change in the thickness of the eye vascular layer, which is determined by the following equation:

$$d = \sqrt[3]{R_2^3 + \frac{3 \cdot \Delta Q}{4\pi}} - R_2 + d_0 \quad (4)$$

It is the value of the thickness of the vascular layer after the input eye pulsation.

Secondly, it is necessary to output analytical dependence of the change in the volume of the measuring eyecup on the change in the linear size of the thickness of the eye vascular layer. It can be accomplished due to the equation for calculating the volume of the ball segment

$$\Delta V = \pi \cdot R_1^2 \cdot \left( \sqrt[3]{R_2^3 + \frac{3 \cdot \Delta Q}{4\pi}} - R_2 \right) \quad (5)$$

The volume of the measuring eyecup  $\Delta V$  will change as much as the volume of the segment of the eyeball, which immersed in this measuring chamber.

### 3 RESULTS

Thus, by setting various individual values of a patient:

- The value of the anterior-posterior axis of an eye  $D$  (in mm).
- The volume of input ocular blood flow pulsation  $\Delta Q$  (in  $\mu\text{l}$ ).

And, moreover, considering that  $d_0$  is given as the set of constant:  $d_1, d_2, d_3$ , it will be possible to display on a three-dimensional graph the dependence of the final thickness of the eye vascular layer on the previous values (see Fig.2).

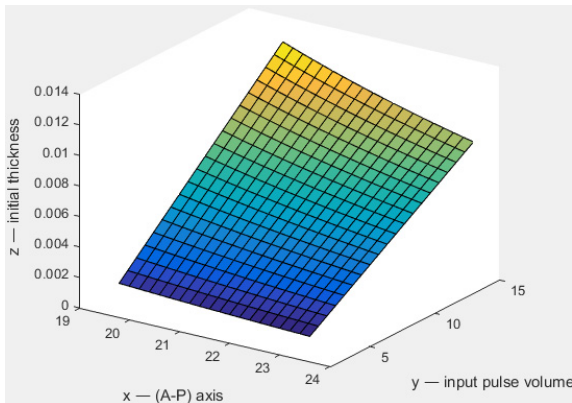


Figure 2: Graph of the dependence of linear changes in the thickness of the eye vascular layer on the individual values of a patient: the A-P axis of the eye, the value of the input pulse volume, the initial thickness of the eye.

The necessary ranges of changes in values were taken from the following literature sources. The values of the eye anterior-posterior (A-P) axis are in the range of 19.69 to 23.50 mm. The average value is 21.60 mm. In the work (Tultseva S. N. et al., 2017) the values of normal systolic increase in pulse volume are ranged from 2.35  $\mu\text{l}$  to 14.96  $\mu\text{l}$ . The average value is 7.27  $\mu\text{l}$ . The values of constant are:  $d_1=0.065 \mu\text{m}$ ,  $d_2=0.070 \mu\text{m}$ ,  $d_3=0.075 \mu\text{m}$ .

Due to the equation (4), which defines the plane, a three-dimensional graph was constructed for a variable parameter  $d_0$  (see Fig. 2).

At a large approximation three planes are visible at different values of the Fig. 2 (see Fig. 3).

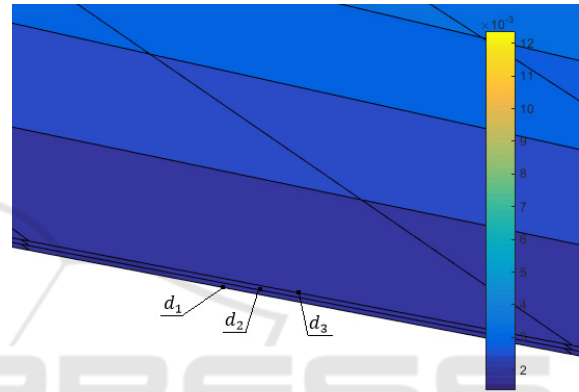


Figure 3: Approximate view of linear changes in the thickness of the eye vascular layer.

Verification of our linear axisymmetric model can be made by a technique which is described in previous work (Kiseleva A.A. et al., 2020).

### 4 DISCUSSION AND CONCLUSIONS

Accordance to our calculations, linear changes in the eye thickness of vascular layer after the input pulse volume varies within tens and units of  $\mu\text{m}$ . Nowadays, there are several advanced technical methods which can register such a range of length units.

In the work (Zhu T. et al., 2012) the development of fiber-optic sensors (FOS) is considered. The vast majority FOS obtain an external primary converter. Such a scheme works when measured physical quantity (pressure, temperature, acceleration, etc.) causes a mechanical movement of a certain sensitive element (for example, a membrane or an inertial mass), which in

turn leads to a modulation of the light intensity. Movements in the FOS can be recorded by using an interferometric measuring circuit. One of the simplest devices of such a type can be considered a fiber-optic end interferometer (FOEI). During using quartz single-mode fiber and laser with emission wavelength of 1.55  $\mu\text{m}$  in FOEI, range of detected linear motion of the mirror relative to the end face of the optical fiber is in the range from 0.000025  $\mu\text{m}$  to 640  $\mu\text{m}$ , with an accuracy of  $\pm 0.000025 \mu\text{m}$ .

Moreover, there are methods for registering both vibrations and displacement of surfaces using laser radiation that directly probes a biological object. For example, in work (Casaccia S. et al., 2015) describes the technique of laser Doppler myography (LDM), which is used as a non-contact method for measuring the signal of mechanomyography (MMG) from the biceps of a shoulder. The LDM signal was measured by using a Polytec PDV100 laser Doppler vibrometer, which uses a laser beam with a wavelength of 633 nm, which corresponds to the second class of the laser equipment (harmless to the eye). Polytec PDV100 obtains the following technical characteristics: a wide range of measurement parameters of frequency fluctuations from infra-low 0.05 Hz to ultrasonic 22 kHz, measurement accuracy  $\pm 0.05 \text{ mm/s}$ .

The methods described above and our linear axisymmetric model will allow us to measure linear displacements in the necessary range of length units.

## CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

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