A Study on Negotiation for Revealed Information with Decentralized Asymmetric Multi-objective Constraint Optimization

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Abstract: The control of revealed information from agents is an important issue in cooperative problem solving and negotiation in multi-agent systems. Research in automated negotiation agents and distributed constraint optimization problems address the privacy of agents. While several studies employ the secure computation that completely inhibits to access the information in solution process, a part of the information is necessary for selfish agents to understand the situation of an agreement. In this study, we address a decentralized framework where agents iteratively negotiate and gradually publish the information of their utilities that are employed to determine a solution of a constraint optimization problem among the agents. A benefit of the approach based on constraint optimization problems is its ability of formalization for general problems. We represent both problems to determine newly published information of utility values and to determine a solution based on published utility values as decentralized asymmetric multi-objective constraint optimization problems. As the first study, we investigate the opportunities to design constraints that define simple strategies of agents to control the utility values to be published. For the objectives of individual agents, we also investigate the influence of several social welfare functions. We experimentally show the effect and influence of the heuristics of different criteria to select the published information of agents.

1 INTRODUCTION

The control of revealed information from agents is an important issue in cooperative problem solving and negotiation in multi-agent systems. Research in automated negotiation agents (Kexing, 2011) and distributed constraint optimization problems (Yeoh and Yokoo, 2012; Fioretto et al., 2018) address the privacy of agents. While several studies employ the secure computation that completely inhibits to access the information in solution process (Léauté and Faltings, 2013; Grinshpoun and Tassa, 2016; Tassa et al., 2017; Tassa et al., 2019), a part of the information is necessary for selfish agents to understand the situation of an agreement.

A class of asymmetric constraint optimization problems (Grinshpoun et al., 2013) has been proposed to represent the situation where agents have different individual utility functions (Petcu et al., 2008; Matsui et al., 2018a; Matsui et al., 2018b). This problem can be extended as the basis of a negotiation framework that publishes/reveals selected partial information of constraints and solves the problems with the published information. A benefit of the approach based on constraint optimization problems is its ability of formalization for general problems.

In a previous work (Matsui, 2019), a solution framework consisting of asymmetric constraint optimization problems with the publication of private utility values and a centralized local search method as a mediator has been presented. With the approach, agents gradually reveal their information until they agree on the first globally consistent solution. While the agents locally select utility values to be published by referring the information from the mediator, published utility values of individual agents are globally aggregated with the traditional summation operator in the optimization process based on the published information.

However, there are opportunities to design decentralized framework without the central node. In addition, for the problems that consider the utilities and the cost values of published information for individual agents, criteria of multiple objectives should be employed. For this class of negotiation, there are opportunities to apply the decentralized constraint optimization with multiple objectives for individual agents (Matsui et al., 2018a).

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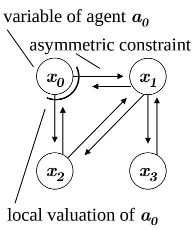


x_i	u_i	p_i
d^0	0	0
d^1	u^a	p^a
d^2	u^b	p^b

x_i	x_j	$u_{i,j}$	$u_{j,i}$	$p_{i,j}$	$p_{j,i}$
d^0	d^0	0	0	0	0
d^0	d^1	0	0	0	0
d^0	d^2	0	0	0	0
...	...	...	...	...	...
d^2	d^0	0	0	0	0
d^2	d^1	u^e	u^g	p^e	p^g
d^2	d^2	u^f	u^h	p^f	p^h

Figure 1: Asymmetric multi-objective DCOP with publication cost.

For the problem settings similar to the previous study, we address a decentralized framework where agents iteratively negotiate and gradually publish the information of their utilities that are employed to determine a solution of a constraint optimization problem among the agent. In the proposed framework, utility values in constraints for agents are incrementally published to restrict revealed information, and problems with only published information are solved until the agreement or termination condition of the negotiation.

We represent both problems to determine newly published information of utility values and to determine a solution based on published utility values as decentralized asymmetric multi-objective constraint optimization problems. As the first study, we investigate the opportunities to design constraints that define simple strategies of agents to control the utility values to be published. For the objectives of individual agents, we also investigate the influence of several social welfare functions.

Our contribution is the fundamental design of the negotiation framework that applies a decentralized constraint optimization approach with criteria for multiple objectives among individual agents, and investigation of composite constraints of fundamental strategies to determine newly published utility values. Our experimental results demonstrate the influence and effect of criteria to select published information of constraint.

The rest of the paper is organized as follows. In the next section, we present preliminaries of our study including fundamental problem definitions, criteria of optimization, and basis of solution methods by referring the previous study. Then we present a framework with complete decentralized constraint optimization methods that iterates rounds of negotiation process to gradually publish utility values considering publication cost in Section 3. We experimentally investigate the proposed approach in Section 4. We address discussions in Section 5 and conclude our study in Section 6.

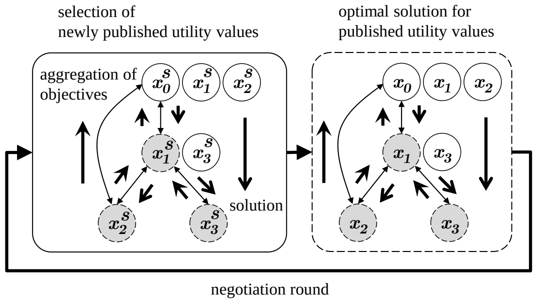


Figure 2: A negotiation framework.

2 PRELIMINARY

2.1 Asymmetric Constraint Optimization Problem with Publication of Private Information

In a previous study, a fundamental solution framework based on an asymmetric constraint optimization problem with publication of constraints and a central processing that performs a local search has been proposed (Matsui, 2019). Here, we address a similar problem with a decentralized solution framework.

In an asymmetric constraint optimization problem with publication of constraints, agents select the publicity of each utility value in the constraints by considering their privacy cost values. This problem is defined by $\langle A, X, D, C, P, X^p, D^p \rangle$. Here, A is a set of agents, X is a set of variables, D is a set of the domains of variables, and C is a set of constraints that define the utility values for the assignments to several variables in X.

Agent $a_i \in A$ has variable $x_i \in X$ and unary constraint $u_i \in C$ for the variable. For several pairs of variables x_i and x_j, asymmetric binary constraints $u_{i,j}, u_{j,i} \in C$ are defined. It is assumed that each agent knows the domains of peer agents related to its asymmetric binary constraints. Agent a_i has $u_{i,j}$, and a_j has $u_{j,i}$. Each agent aggregates its own utility value for unary and binary constraints that are related to the agent. The utility values are defined as $u_i : D_i \to \mathbb{N}_0$ and $u_{i,j} : D_i \times D_j \to \mathbb{N}_0$. The functions are also denoted by $u_i(d)$ and $u_{i,j}(d_i, d_j)$.

P is a set of privacy cost values for utility values in constraints. $p_i \in P$ corresponds to u_i, and $p_{i,j} \in P$ corresponds to $u_{i,j}$. The privacy cost values are individually evaluated by each agent to select its related utility value to be published. The cost values of constraints are defined as $p_i : D_i \to \mathbb{N}_0$ and $p_{i,j} : D_i \times D_j \to \mathbb{N}_0$. The functions are also denoted by $p_i(d)$ and $p_{i,j}(d_i, d_j)$.

An example of directed constraint graph and related function tables for above basic parts of a problem is shown in Figure 1.

X^p and D^p are a set of binary variables and a set of the domains of the variables that represent the publication of the utility values, respectively. When a utility value in a constraint is published, its corresponding variable takes 1. Otherwise, the variable takes 0. Variable $x^p_{i,(d)} \in X^p$ corresponds to utility value $u_i(d)$ in unary constraint u_i. Similarly, variable $x^p_{i,j,(d,d')} \in X^p$ corresponds to utility value $u_{i,j}(d,d')$ of assignments $d \in D_i$ and $d' \in D_j$ in asymmetric binary constraint $u_{i,j}$. If at least one of $x^p_{i,j,(d,d')}$ takes 1, $x^p_{i,(d)}$ is set to 1.

For each variable $x_i \in X$, a value $d^0 \in D_i$ represents a special case where an agent does not cooperate with any other agents. Value d^0 is selected by several agents when any solution cannot be globally evaluated with utility functions whose utility values have been partially published. For assignment d^0 to a variable x_i, the corresponding utility value $u_i(d^0)$ and privacy cost value $p_i(d^0)$ take zero. Similarly, $u_{i,j}(d^0,d)$, $u_{i,j}(d,d^0)$, and corresponding privacy cost values take zero. Value d^0 is given as a common value in the domains of all variables, and corresponding zero utility/privacy-cost values are also defined as common knowledge. Unpublished utility values related to assignment d^0 are defined as $-\infty$. The evaluations $v_i(d)$ and $v_{i,j}(d,d')$ for a unary constraint and an asymmetric binary constraint are represented by the followings.

$$v_i(d) = \begin{cases} u_i(d) & \text{if } x^p_{i,(d)} = 1 \\ -\infty & \text{otherwise} \end{cases} \tag{1}$$

$$v_{i,j}(d,d') = \begin{cases} u_{i,j}(d,d') & \text{if } x^p_{i,j,(d,d')} = 1 \\ -\infty & \text{otherwise} \end{cases} \tag{2}$$

With an assignment to related variables in X^p, the local valuation $f_i(\mathcal{D}_i)$ for agent a_i and an assignment $\mathcal{D}_i$ to related variables in X is defined as the summation of the evaluations for its own unary constraint and asymmetric binary constraints for neighborhood agents in Nbr_i that are related to i by constraints:

$$f_i(\mathcal{D}_i) = v_i(d_i) + \sum_{j \in Nbr_i} v_{i,j}(d_i,d_j) \ , \tag{3}$$

where d_i and d_j are the value of x_i and x_j in assignment $\mathcal{D}_i$.

The goal the of problem is the maximization of the globally aggregated valuations for all f_i.

$$\text{maximize} \quad \oplus_{a_i \in A} f_i(\mathcal{D}_i) \tag{4}$$

While a typical aggregation operator of the valuations is the summation of the values, different operators can be applied in the case of multi-objective problems among individual utilities of agents as shown in Section 2.2.

To avoid the utility value of $-\infty$ shown in Equations (1) and (2), value d^0 can be assigned to several agents' variables if any other solutions cannot be evaluated with published part of utility functions. However, such a situation will also be avoided, since the utility value corresponds to value d^0 is zero.

The agent can partially publish the utility values of its own utility functions. The total published cost of agent a_i is defined as follows.

$$\sum_{d \in D_i \text{ s.t. } x^p_{i,(d)}=1} p_i(d) + \sum_{j \in Nbr_i, d \in D_i, d' \in D_j \text{ s.t. } x^p_{i,j,(d,d')}=1} p_{i,j}(d,d') \tag{5}$$

A problem is to determine the utility values to be published within a budget. We consider a scheme of negotiation process where agents iteratively publish their utility values based on situations in each round of the process.

2.2 Social Welfare Criteria

When we consider objective of individual agents, several operators $\oplus$ and corresponding criteria to aggregate/compare the objectives as shown in Equation (4). For multi-objective problems, several types of social welfare (Sen, 1997) and scalarization methods (Marler and Arora, 2004) are employed to handle the objectives. In addition to the traditional summation operator and the comparison on scalar values, we employ leximin and maximin with a tie-break by a summation value, that are also employed in a solution method (Matsui et al., 2018a). Since we employ a solution method in the previous study to solve our problem, we also inherit these operators and criteria. While those operators and criteria are designed for maximization problems of utilities, those can be easily modified for minimization problems.

Summation $\sum_{a_i \in A} f_i$ only consider the total utilities. While maximin $\max\min_{a_i \in A} f_i$ improves the worst case utility value, it does not consider the whole utilities, and is not Pareto optimal. Therefore, ties are additionally broken by comparing summation values. Leximin can be considered as an extension of maximin where the objective values are represented as a vector whose values are sorted in ascending order, and the comparison of two vectors is based on a dictionary order of values in the vectors. The maximization with leximin is Pareto optimal and relatively improves fairness among objectives. See the literature for details (Sen, 1997; Marler and Arora, 2004; Matsui et al., 2018a).

In addition, to evaluate the fairness among agents in a resulting solution, we also employ the Theil index that is defined as an inverted value of entropy. $T = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_i}{\bar{x}}\ln\frac{x_i}{\bar{x}}\right)$, where $\bar{x}$ denotes the average value for all x_i. T takes zero if all x_i are equal.

2.3 Decentralized Dynamic Programming Method for Asymmetric Multi-objective Constraint Optimization Problems

A class of asymmetric constraint optimization problems with individual objectives of agents and decentralized complete solution methods have been presented (Matsui et al., 2018a). The goal of this study was the application of a social welfare criteria called leximin to pseudo-tree-based solution methods to improve the fairness among individual agents. As mentioned above section, with the leximin, individual objectives of agents are aggregated and optimized considering the worst case and fairness among the objectives. In addition to the leximin, several social welfare criteria including traditional summation were compared.

The solution methods are extended versions of dynamic programming and tree-search based on pseudo-trees of constraint graphs. A pseudo tree is a structure on a constraint graph that is employed to decompose problems. A pseudo tree is based on a depth-first search tree on a constraint graph, and optimization methods are performed along the spanning tree with back edges that are not included in the edges of the spanning tree. For asymmetric problems, the pseudo-tree and solution methods have been modified so that both two agents that relate each other by a pair of asymmetric constraints can evaluate assignments to opposite agent's variable. In part of Figure 2, simple examples of the modified pseudo trees are illustrated. See the literature for details (Matsui et al., 2018a).

In this study, we employ a variant of pseudo-tree-based dynamic programming method DPOP (Petcu and Faltings, 2005) that has been extended in the previous study (Matsui et al., 2018a), because it is relatively simple as a basis of our extension. Note that its scalability is limited for complex problems. When problems are densely constrained, the tree width of pseudo trees that corresponds to the number of combinations of variables in partial problems for agents grows and cause a combinational explosion. However, for the fundamental investigation, we concentrate on the problems that can be addressed with this class of complete solution methods. We develop two variants of the solution method that are adjusted for our problem definitions.

3 A DECENTRALIZED SOLUTION FRAMEWORK THAT ALTERNATES SELECTING PUBLISHED UTILITY VALUES AND SOLVING PUBLISHED PROBLEMS

3.1 Basic Framework

We propose a framework that alternates two optimization methods to select utility values to be published and to solve the current problem with published utilities. For both optimization methods, we employ a decentralized dynamic programming method based on pseudo trees as mentioned above. For simplicity, two methods employ a common pseudo tree. For the first investigation, we focus on the process that incrementally publishes the utility values of constraints/functions. We assume that arbitrary conditions and parameters to control and stop the publication process can be defined.

The framework iterates rounds of negotiation among agents and utility values are incrementally published. In each round, the utility values that are published for the next problem are determined, and the next problem with published utility values is solved.

Similar to the previous methods, in the optimization for the problem with published utility values, the total utility for all agents is maximized.

3.2 Selection of Utility Values to be Published

In the phase of selecting utility values to be published, agents iteratively publish a part of utility values of their own constraints in each round of negotiation. As defined in Section 2.1, the decision variables of agent a_i in this problem are originally $x_i^p, x_{i,j}^p \in X^p$ for Nbr_i. In the initial state, all the variables in X^p are initialized by 0, and they are set to 1 when their corresponding utility values are published. However, the solution space for the variables in X^p is too huge to explore for the complete solution method that is employed in this study. Therefore, we define another problem for this negotiation to restrict solutions in the original solution space.

The problem is defined by $\langle A, X^s, D, C^s \rangle$. Here, we introduce new decision variables X^s. For agent a_i, a decision variable $x_i^s \in X^s$ is defined. Variable x_i^s takes a value from $D_i \in D$ that is the same as a part of decision variables in the original problem. Assigning d and d' to x_i^s and x_j^s represents that the utility values $u_i(d)$ and $u_{i,j}(d,d')$ are published in the next round of negotiation. When x_i^s takes the special variable value d^0, it means that no utility values relating x_i are newly published. In addition, if a utility value corresponding to d and d' have been published, the published utility value does not change the current situation. In the end of each round, several original variables in X^p are set to 1 by translating the solution for X^s. If an assignment to a variable $x_i^p \in X^p$ or $x_{i,j}^p \in X^p$ changed from 0 to 1, its corresponding utility value is newly published.

The structure of this problem determining the utility values to be published for the next problem resembles the maximization problem with published utility values. The variables and their domains are identical to the maximization problem for utility values. However, it is defined as a minimization problem with cost values for published utility values. The constraints $c_i \in C^s$ are asymmetrically defined for each agent a_i. Here, cost values related to a constraint can take real number to represent ratio values. Several cost values for the constraints can be defined based on different publication strategies, and the cost values can be integrated as vectors with a structure. We investigate fundamental strategies as shown in Section 3.3 and 3.4.

3.3 Criteria to Evaluate Newly Published Utility Values

The following criteria evaluate the situation of newly published utility values that are represented by an assignment to variables in X^s. We assume that, some information about status of published utility values is available in the evaluation. The information can be collected by simple additional protocols.

3.3.1 R) Degree of Privacy Cost for Published Utility Values

Ratio of revealed privacy cost: we assume that the information of publish cost values can be employed in the negotiation of agents by modifying them to ratio values of locally aggregated cost values so that the ratio values only represent a normalized degree of unsatisfactory. For each agent, the ratio of privacy cost values between the total values for published utilities and total values for all constraints is defined as a criterion.

$$c_i^R(\mathcal{D}_i^s) = \\ (x_{i,(d_i)}^{p+} * p_i(d_i) + \sum_{j \in Nbr_i} x_{i,j,(d_i,d_j)}^{p+} * p_{i,j}(d_i,d_j)) / \\ (\sum_{d \in D_i} p_i(d) + \sum_{j \in Nbr_j, d \in D_i, d' \in D_j} p_{i,j}(d,d')) , \quad (6)$$

where, d_i is the value of x_i^s in assignment $\mathcal{D}_i^s$. Values of variables $x_{i,(d)}^{p+}$ and $x_{i,j,(d,d')}^{p+}$ are equal to $x_{i,(d)}^p$ and $x_{i,j,(d,d')}^p$ respectively if x_i^s and x_j^s does not represent the publication of corresponding utility values. Otherwise, $x_{i,(d)}^{p+}$ and $x_{i,j,(d,d')}^{p+}$ are set to 1.

Since agents might relate different numbers of constraints and utility values, the above ratio value is also considered.

3.3.2 A) Degree of Unpublished Utility Values

Agreement opportunity: for each value d of each agent's variable x_i^s, the number of unpublished utility values that are related to the variable's value is considered as a criterion. For $x_i^s = d_i$,

$$c_i^A(d_i) = \\ 2|\{(d_i,d_j)|j \in Nbr_i, d_j \in D_j, d_i = d^0 \vee d_j = d^0\}| + \\ |\{x_{i,j,(d_i,d_j)}^p | j \in Nbr_i, d_j \in D_j, d_i \neq d^0 \wedge d_j \neq d^0, \\ x_{i,j,(d_i,d_j)}^p = 0\}| \quad (7)$$

With this criterion, we consider that when the number of unpublished utilities that relates the assignment for newly published utility values is relatively large, the opportunity of agreement among agents with the assignment is relatively small. For the assignment of d^0 that represents no publication for a pair of variables' values, the count of an unpublished utility for the assignment is doubled for emphasis.

3.3.3 U) Degree of Progress in Publication Process

Utility publication progress: since, the cost values for revealed information restrict the publish of utility values, a counter part of the cost values is necessary to continue publication process. Here, we employ a ratio of published utility values.

$$c_i^U(\mathcal{D}_i^s) = |\{x_{i,j,(d_i,d_j)}^p | j \in Nbr_i, x_{i,j,(d_i,d_j)}^p = 1\} / \\ \sum_{j \in Nbr_i} |D_i| \times |D_j| \quad (8)$$

As addressed below, this cost value is combined with other criteria so that it has a higher priority than others.

3.3.4 T) A Measurement of Trade-off between Publication Cost and Utility

Trade-off measurement: above criteria do not employ the information of utility value. To consider the expected gain of utility by publishing utility values, the information of utility values should be considered. However, originally the utility values should be hidden until their publication. As abstract information, we employ the ratio of trade-off values which are locally aggregated as difference values between expected utility values and cost values of published utility values.

Since the optimal utility value is unknown, each agent employs rough upper and lower bound values of trade-off.

$$trd_i(\mathcal{D}_i^s)^\top = u_i(d_i) + \sum_{j \in Nbr_i} u_{i,j}(d_i, d_j) - rvl_i(\mathcal{D}_i^s) \quad (9)$$

$$trd_i(\mathcal{D}_i^s)^\perp = u_i(d_i) - rvl_i(\mathcal{D}_i^s) \quad (10)$$

$$rvl_i(\mathcal{D}_i^s) = x^{p+}_{i,(d_i)} * p_i(d_i) + \sum_{j \in Nbr_i} x^{p+}_{i,j,(d_i,d_j)} * p_{i,j}(d_i, d_j) \quad (11)$$

While arbitrary estimation values between the bounds can be employed, we investigate simple cases for the fundamental analysis. The estimation trade-off value $trd_i(\mathcal{D}_i^s)$ is set to $trd_i(\mathcal{D}_i^s)^\perp$, $(trd_i(\mathcal{D}_i^s)^\top + trd_i(\mathcal{D}_i^s)^\perp)/2$ or $trd_i(\mathcal{D}_i^s)^\top$. Then the ratio of the trade-off value is employed as a criterion.

$$c_i^T(\mathcal{D}_i^s) = 1 - (trd_i(\mathcal{D}_i^s) - trd_i^\perp)/(trd_i^\top - trd_i^\perp) \quad (12)$$

$$trd_i^\top = u_i(d_i) + \sum_{j \in Nbr_i} \max_{d_i \in D_i, d_j \in D_j} u_{i,j}(d_i, d_j) \quad (13)$$

$$trd_i^\perp = -(\sum_{d \in D_i} p_i(d) + \sum_{j \in Nbr_j, d \in D_i, d' \in D_j} p_{i,j}(d, d')) \quad (14)$$

3.3.5 S) Degree of Newly Published Utility Values

Switch of trade-off measurement: the above cost values except U need the energy to continue publication process. We modified the previous criteria T to conditionally continue publication process. The modified cost $c_i^S(\mathcal{D}_i^s)$ is defined as follows. First, we evaluate the cost value for newly published utility values

$$rvl_i^{new}(\mathcal{D}_i^s) = (x^{p+}_{i,(d_i)} - x^{p}_{i,(d_i)}) * p_i(d_i) + \sum_{j \in Nbr_i} (x^{p+}_{i,j,(d_i,d_j)} - x^{p}_{i,j,(d_i,d_j)}) * p_{i,j}(d_i, d_j) \quad (15)$$

If $rvl^{new} = 0$, then $c_i^S(\mathcal{D}_i^s) = 0$. Otherwise, if $trd_i(\mathcal{D}_i^s) > 0$,

$$c_i^S(\mathcal{D}_i^s) = -(trd_i(\mathcal{D}_i^s) - trd_i^\perp)/(trd_i^\top - trd_i^\perp). \quad (16)$$

In other cases, $c_i^S(\mathcal{D}_i^s) = c_i^T(\mathcal{D}_i^s)$.

With this criterion, the publication is preferred by a negative cost value when $trd_i(\mathcal{D}_i^s) > 0$.

3.3.6 H) Hard Constraint for Termination

Hard constraint for termination: we also introduce a cost value $c^H(\mathcal{D}_i^s)$ of hard constraint for the assignments that should be avoided. The value of $c^H(\mathcal{D}_i^s)$ is 1 or 0 that represents violation or satisfaction. This cost value should have the first priority. Here, we employ $c^H(\mathcal{D}_i^s)$ for a termination condition based on above trade-off value. $c^H(\mathcal{D}_i^s) = 1$ if $rvl_i^{new}(\mathcal{D}_i^s) > 0 \wedge trd_i(\mathcal{D}_i^s) < 0$. Otherwise, $c^H(\mathcal{D}_i^s) = 0$. This constraint inhibits the publication with negative trade-off value where an estimation utility value is less than the total publish cost.

3.4 Integrating Criteria for Published Utility Values

The above criteria can be combined as hierarchically structured cost vectors, and the optimization method solves a minimization problem on the vectors to determine the utility values to be newly published. Hard constraint H is most prioritized. Since criteria R, A, T with higher priorities will cause earlier convergence without publication of sufficient utility values, cost U should be secondary prioritized. S can be used without U. T and S consider a trade-off between an estimation utility and to total publish cost, while R and A does not evaluate utility values. Considering these properties, we investigate the following combinations of criteria.

- S: $c^H \gg c^S$
- UAR: $c^H \gg c^U \gg c^A \gg c^R$
- URA: $c^H \gg c^U \gg c^R \gg c^A$
- UT: $c^H \gg c^U \gg c^T$

Here, any aggregated cost values do not exceed a cost value with a higher priority.

For two hierarchically structured cost vectors $\boldsymbol{v} = [v_1, \cdots, v_k]$ and $\boldsymbol{v}' = [v'_1, \cdots, v'_k]$, the aggregation of the vectors is defined as $\boldsymbol{v} \oplus \boldsymbol{v}' = [v_1 \oplus v'_1, \cdots, v_k \oplus v'_k]$. The comparison $\boldsymbol{v} < \boldsymbol{v}'$ is defined as $\exists t, \forall t' < t, v_{t'} = v'_{t'} \wedge v_t < v'_t$.

We also investigate the cases where the elements of the vectors are aggregated and compared with several criteria for multiple objectives for individual agents. Here, the lexicographic augmented weighted Tchebycheff function (Marler and Arora, 2004) where ties of maximum values are broken by summation values, and leximax that is a modified leximin for minimization problems are applied in addition to traditional summation.

3.5 Solution with Published Utility Values

In each round of the negotiation to determine newly published utility value, agents can solve a problem with currently published utility values as shown in Section 2.1. While this result can be employed as a feedback to agents' strategies, we only evaluate the anytime property for rounds of negotiation as the first study.

4 EVALUATION

4.1 Settings

We empirically evaluated the proposed approach. Due to the limitation of dynamic programming on a pseudo-tree, we solved relatively small scale and sparse problems. A problem instance has n variables/agents/unary-constraints and c pairs of asymmetric binary constraints. For a pair of variables/agents, a pair of asymmetric constraints were defined. The size of variables' domain including d^0 was commonly set to four. Utility values of constraints were set to random integer values in $[10,50]$ based on uniform distribution. Privacy cost values were set as follows.

- equ: A privacy cost value for a utility value u is an integer values $\lfloor \max(1, u/10) \rfloor$ so that there is a positive correlation.
- inv: A privacy cost value for a utility value u is an integer values $\lfloor \max(1,(50-u+1)/10) \rfloor$ so that there is a negative correlation.
- rnd: Random integer values in $[1,5]$ based on uniform distribution.

As shown in Section 3.4, we compared the influence of several combinations of criteria for selecting published utility values that are denoted by S, UAR, URA and UT. In addition, as shown in Section 3.3.4, the estimated trade-off values for the criterion T, S, and H is set to the minimum/average/maximum values of lower and upper bounds that are denoted by tmin/tave/tmax.

We also applied different aggregation/comparison operators of objectives, including summation, maximum/minimum value with tie-break by summation, and leximin/leximax, on utility/privacy-cost values/vectors.

- sum: Summation for both minimization problems determining published utility values and maximization problems of utilities under published utility values.
- ms: The maximum/minimum value with a tie-break by summation for the minimization/maximization problems for publication/utility.
- lxm: Leximax/leximin for the minimization/maximization problems for publication/utility.
- summs: Summation value for the minimization problems for publication and the minimum value for the maximization problem for utility.
- sumlxm: Summation value for the minimization problems for publication and leximin for the maximization problem for utility.

Here, we assumed a simple termination condition where an accumulated publish cost exceeds estimated utility for next publication. In this case, each agent does not select the corresponding assignment. When all agents do not select new publication, the negotiation process terminates.

For each setting, results are averaged over ten instances.

4.2 Results

Figures 3-6 show typical anytime-curves of 'utility' and 'trade-off'. Each curve corresponds to an agent. While the utility values almost converge in earlier rounds, trade-off values decrease until a termination. Therefore, an issue is the selection of published utility values within a budget, and we investigate the such opportunities. In these problem settings, the trade-off values are better in earlier steps for most agents due to the scale of accumulated publication cost values are relatively greater than that of utility values. This situation basically depends on publication cost parameters of the problem settings, and by simply setting smaller cost values, the peaks will move to later rounds. Here, we do not focus on this issue because the parameter design and more sophisticated strategies of agents to handle such peaks will be included our future work based on this investigation.

Table 1 shows the results in the final round of publication process in the case of $n = 10$ variables/agents, the number of pairs of asymmetric binary constraints $c = 20$, equ, and sum. Here, the following results are

Table 1: Influence of priorities on criteria for utility values to be published ($n = 10$, $c = 20$, equ, sum).

alg.	term.	utiliy				trade-off			ratio. rvl. num.			ratio. rvl. cost		
	round	sum.	min.	ave.	max	min.	ave.	max.	min.	ave.	max.	min.	ave.	max.
sum, S, tmin	6.7	1620.9	97.4	162.1	225.0	**76.9**	**134.1**	**194.2**	**0.181**	**0.291**	**0.429**	**0.178**	**0.292**	**0.435**
sum, S, tave	21.2	1763.2	**110.2**	176.3	248.8	46.7	88.3	127.9	0.763	0.856	0.934	0.805	0.884	0.961
sum, S, tmax	21.8	**1773.1**	103.0	**177.3**	**254.3**	30.9	77.3	115.4	1	1	1	1	1	1
sum, UAR, tmin	6.1	1560.6	95.6	156.1	223.4	72.8	124.2	187.3	0.245	0.357	0.482	0.211	0.330	0.478
sum, UAR, tave	13.9	1770.1	106.3	177.0	251.9	39.2	84.4	123.0	0.859	0.929	0.985	0.854	0.929	0.988
sum, UAR, tmax	11.9	**1773.1**	103.0	**177.3**	**254.3**	30.9	77.3	115.4	1	1	1	1	1	1
sum, URA, tmin	6.0	1526.6	90.2	152.7	219.0	64.7	121.1	181.9	0.248	0.359	0.488	0.212	0.328	0.472
sum, URA, tave	14.8	1762.1	107.1	176.2	253.1	38.5	83.0	124.3	0.881	0.938	0.991	0.865	0.934	0.987
sum, URA, tmax	13.7	**1773.1**	103.0	177.3	**254.3**	30.9	77.3	115.4	1	1	1	1	1	1
sum, UT, tmin	5.5	1598.4	97.6	159.8	225.4	76.2	130.5	190.5	0.193	0.307	0.438	0.191	0.305	**0.453**
sum, UT, tave	16.3	1764.2	103.2	176.4	251.9	41.3	86.8	129.4	0.754	0.873	0.950	0.800	0.898	0.964
sum, UT, tmax	13.4	**1773.1**	103.0	**177.3**	**254.3**	30.9	77.3	115.4	1	1	1	1	1	1

Table 2: Influence of priorities on criteria for utility values to be published ($n = 10$, $c = 20$, equ, S).

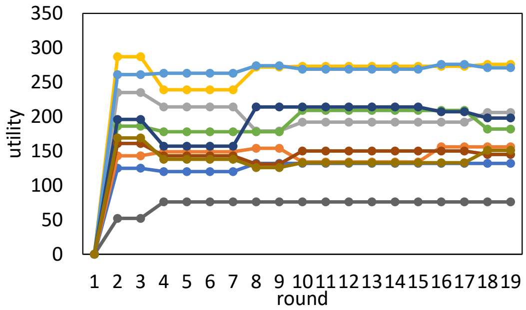
alg.	term.	utiliy					trade-off				ratio. rvl. num.				ratio. rvl. cost			
	round	sum.	min.	ave.	max	theil	min.	ave.	max.	theil	min.	ave.	max.	theil	min.	ave.	max.	theil
sum, S, tmin	6.7	1620.9	97.4	162.1	225	0.032	76.9	**134.1**	194.2	0.039	**0.181**	**0.291**	**0.429**	0.032	**0.178**	**0.292**	**0.435**	0.035
sum, S, tave	21.2	1763.2	110.2	176.3	248.8	0.031	46.7	88.3	127.9	0.041	0.763	0.856	0.934	0.002	0.805	0.884	0.961	0.002
sum, S, tmax	21.8	**1773.1**	103	**177.3**	**254.3**	0.035	30.9	77.3	115.4	0.061	1	1	1	**0**	1	1	1	**0**
ms, S, tmin	6.6	1617.9	110.5	161.8	226.9	0.026	**86.9**	133.3	**194.5**	0.031	0.185	0.298	0.434	0.034	0.184	0.299	0.457	0.037
ms, S, tave	20.7	1704.4	126.5	170.4	231.3	0.019	56.9	82.2	118.3	**0.025**	0.755	0.853	0.940	0.002	0.807	0.885	0.955	0.001
ms, S, tmax	19.7	1710.2	**126.6**	171.0	232.5	0.020	44.9	71.0	102.8	0.033	1	1	1	**0**	1	1	1	**0**
lxm, S, tmin	6.5	1600.3	109.2	160.0	218.9	0.025	84.4	131.5	185.8	0.031	0.183	0.295	0.439	0.036	0.181	0.301	0.471	0.040
lxm, S, tave	20	1657.2	125.4	165.7	224	0.020	51.5	78.1	113.1	0.032	0.741	0.849	0.935	0.002	0.784	0.879	0.957	0.002
lxm, S, tmax	19.8	1653.8	**126.6**	165.4	219.4	**0.015**	38.6	65.4	90.3	0.033	1	1	1	**0**	1	1	1	**0**

Table 3: Influence of priorities on criteria for utility values to be published ($n = 10$, $c = 20$, equ, S).

alg.	term.	utiliy					trade-off				ratio. rvl. num.				ratio. rvl. cost			
	round	sum.	min.	ave.	max	theil	min.	ave.	max.	theil	min.	ave.	max.	theil	min.	ave.	max.	theil
sum, S, tmin	6.7	1620.9	97.4	162.1	225	0.032	76.9	**134.1**	**194.2**	0.039	**0.181**	**0.291**	**0.429**	0.032	**0.178**	**0.292**	**0.435**	0.035
sum, S, tave	21.2	1763.2	110.2	176.3	248.8	0.031	46.7	88.3	127.9	0.041	0.763	0.856	0.934	0.002	0.805	0.884	0.961	0.002
sum, S, tmax	21.8	**1773.1**	103	**177.3**	**254.3**	0.035	30.9	77.3	115.4	0.061	1	1	1	**0**	1	1	1	**0**
summs, S, tmin	6.7	1576.5	99.8	157.7	216.4	0.029	**79.1**	129.6	184.4	0.036	**0.181**	**0.291**	**0.429**	0.032	**0.178**	**0.292**	**0.435**	0.035
summs, S, tave	21.2	1697.5	**126.6**	169.8	227	0.019	54.7	81.7	114.3	0.026	0.763	0.856	0.934	0.002	0.805	0.884	0.961	0.002
summs, S, tmax	21.8	1710.2	**126.6**	171.0	232.5	0.020	44.9	71.0	102.8	0.033	1	1	1	**0**	1	1	1	**0**
sumlxm, S, tmin	6.7	1573.4	99.8	157.3	214.6	0.028	**79.1**	129.3	182.6	0.034	**0.181**	**0.291**	**0.429**	0.032	**0.178**	**0.292**	**0.435**	0.035
sumlxm, S, tave	21.2	1673.8	**126.6**	167.4	216.8	0.016	53.7	79.3	106.9	**0.022**	0.763	0.856	0.934	0.002	0.805	0.884	0.961	0.002
sumlxm, S, tmax	21.8	1653.8	**126.6**	165.4	219.4	**0.015**	38.6	65.4	90.3	0.033	1	1	1	**0**	1	1	1	**0**

Table 4: Influence of priorities on criteria for utility values to be published ($n = 10$, $c = 20$, inv, S).

alg.	term.	utiliy					trade-off				ratio. rvl. num.				ratio. rvl. cost			
	round	sum.	min.	ave.	max	theil	min.	ave.	max.	theil	min.	ave.	max.	theil	min.	ave.	max.	theil
sum, S, tmin	8.6	1703.0	107.2	170.3	235.4	0.029	87.1	**145.6**	**205.4**	0.034	0.218	**0.381**	**0.540**	0.037	**0.206**	**0.356**	0.541	0.041
sum, S, tave	22.6	**1773.1**	103	**177.3**	**254.3**	0.035	50.8	105.4	159.6	0.052	0.962	0.990	1	0.0001	0.940	0.986	1	0.0003
sum, S, tmax	21.8	**1773.1**	103	**177.3**	**254.3**	0.035	49.2	104.3	159	0.054	1	1	1	**0**	1	1	1	**0**
ms, S, tmin	8.5	1668.5	114.5	166.9	233.2	0.027	94.4	141.7	202.5	0.031	0.217	0.387	0.546	0.036	0.208	0.363	0.548	0.040
ms, S, tave	21.7	1711.7	**126.6**	171.2	232.5	0.020	69.2	99.2	142.1	0.024	0.963	0.991	1	0.0001	0.942	0.987	1	0.0003
ms, S, tmax	19.6	1710.2	**126.6**	171.0	232.5	0.020	68.1	98.0	141.3	0.025	1	1	1	**0**	1	1	1	**0**
lxm, S, tmin	8.3	1656.8	116.3	165.7	228.7	0.023	**96**	140.9	199.5	0.027	**0.216**	0.386	0.556	0.037	**0.206**	0.357	**0.537**	0.039
lxm, S, tave	21.4	1653.8	**126.6**	165.4	219.4	**0.015**	64	93.4	129.5	**0.021**	0.965	0.990	1	0.0001	0.945	0.987	1	0.0003
lxm, S, tmax	19.8	1653.8	**126.6**	165.4	219.4	**0.015**	63.3	92.4	128.7	0.022	1	1	1	**0**	1	1	1	**0**



(Each curve corresponds to an agent.)

Figure 3: Utility of agents ($n = 10$, $c = 20$, equ, S, lxm).

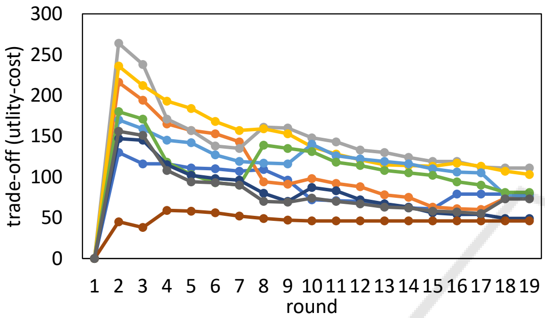


Figure 4: Trade-off of agents ($n = 10$, $c = 20$, equ, S, lxm).

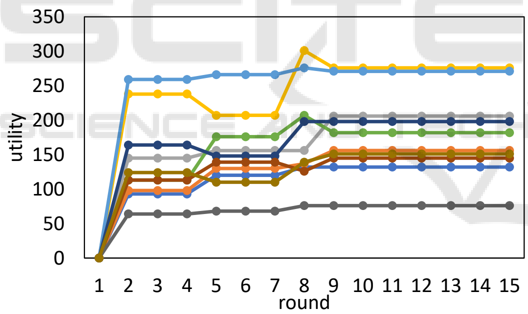


Figure 5: Utility of agents ($n = 10$, $c = 20$, equ, URA, lxm)

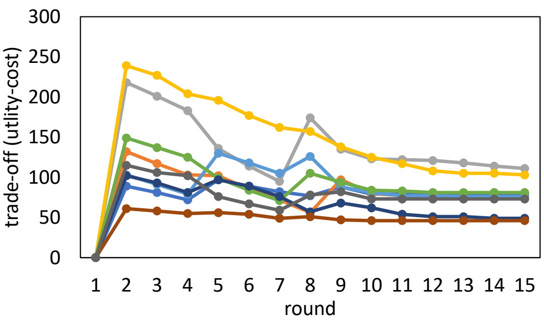


Figure 6: Trade-off of agents ($n = 10$, $c = 20$, equ, URA, lxm).

evaluated. The minimum/average/maximum value is evaluated for agents.

- utility: The optimal utility value of the problem with published utility values.
- trade-off: The actual trade-off values that is the difference between 'utility' and the total privacy cost value for published utility values.
- ratio. rvl. num.: The ratio of revealed number of utility values.
- ratio. rvl. cost.: The ratio of total privacy cost values for revealed utility values.

The result shows that the publication process continued until all the utility values are published when the case of estimation trade-off tmax, since this estimation value is too optimistic. In this problem setting, earlier termination of publication process is relatively better to save the privacy cost obtaining some utility. 'Trade-off' values of criteria S and UT that consider trade-off between the estimation utility and the total cost of published utility value are relatively greater than that of other criteria in the case of tmin that terminates in relatively earlier rounds. In the case of tmin, 'utiliy' values of UAR is relatively better than URA, since it considers the opportunities of aggregation. Basically, U enforces publication and dominates other criteria. As a result, the total publication cost is relatively greater than S.

Table 2 shows the results in the final round of publication process in the case of $n = 10$, $c = 20$, equ, and S. Here, the Theil index that is a measurement of in equality among agents is also evaluated. When all agents have the same value, the Theil index value is zero. Note that there is an inherent trade-off between the summation/average value and fairness, while it is often preferred for selfish agents without other mechanisms to trade their profits. The result shows that the Theil index values of 'trade-off' in ms and lxm are relatively smaller than that of sum, since those criteria consider the improvement of the worst case. However, lxm did not overcome ms although lxm consider the inequality, it reveals that the difficulty to design appropriate estimate trade-off value to be well optimized. Table 3 shows the results of the same problem settings, while the optimization criteria for publication process is sum. Since the publication process is the same, the result shows that lxm is the fairest criterion for 'utility' and it affects 'trade-off'.

Table 4 shows the results in the final round of publication process in the case of $n = 10$, $c = 20$, inv, and S. In this case, the Theil index of lxm for 'trade-off' was relatively better.

Table 5 shows the results in the case of $n = 10$, $c = 20$, rnd, and S. Due to the problem settings, 'trade-off' values were negative in a few instances in the case of tave and tmax. For such cases, the Theil index of

Table 5: Influence of priorities on criteria for utility values to be published ($n = 10$, $c = 20$, rnd, S).

alg.	term.	utiliy					trade-off				ratio. rvl. num.				ratio. rvl. cost			
	round	sum.	min.	ave.	max	theil	min.	ave.	max.	theil	min.	ave.	max.	theil	min.	ave.	max.	theil
sum, S, tmin	6.4	1649.5	102.8	165.0	237.5	0.03414	**80.4**	**137.4**	207.2	0.043	**0.172**	**0.261**	**0.379**	0.032	**0.158**	**0.249**	**0.364**	0.035
sum, S, tave	18.6	1760.7	105	176.1	250.6	0.03417	46.3	88.1	132	0.052	0.660	0.767	0.862	0.003	0.648	0.758	0.862	0.004
sum, S, tmax	22.1	**1770.9**	101.8	**177.1**	**260.7**	0.037	20.6	61.2	100.5	(0.061)	0.962	0.993	1	**0.0001**	0.954	0.992	1	0.00022
ms, S, tmin	6.6	1619.2	102.5	161.9	236.5	0.032	75	133.3	203.8	0.042	0.184	0.274	0.410	0.034	0.161	0.259	0.388	0.039
ms, S, tave	18.4	1675.8	117.6	167.6	236.9	0.026	46.3	79.5	119.2	**0.040**	0.644	0.772	0.872	0.004	0.641	0.760	0.865	0.004
ms, S, tmax	20.4	1673.8	**120.5**	167.4	229	0.0227	14.4	51.5	84.8	(0.048)	0.952	0.992	1	0.0002	0.944	0.991	1	0.0003
lxm, S, tmin	6.5	1624	101.6	162.4	240.6	0.033	74.3	133.4	**207.6**	0.045	0.183	0.278	0.419	0.037	0.162	0.262	0.404	0.041
lxm, S, tave	17.7	1644.5	117.6	164.5	232.7	0.0231	44.6	77.2	117	**0.040**	0.660	0.767	0.859	0.003	0.639	0.752	0.845	0.004
lxm, S, tmax	20.8	1642.1	**120.5**	164.2	230.8	**0.020**	11.8	48.2	79.9	(0.043)	0.958	0.993	1	0.0002	0.952	0.992	1	**0.00025**

Table 6: Influence of priorities on criteria for utility values to be published ($n = 20$, $c = 20$, equ, S).

alg.	term.	utiliy					trade-off				ratio. rvl. num.				ratio. rvl. cost			
	round	sum.	min.	ave.	max	theil	min.	ave.	max.	theil	min.	ave.	max.	theil	min.	ave.	max.	theil
sum, S, tmin	7.2	2166.6	54.4	108.3	183.5	0.057	38.3	**84.7**	**150.3**	0.066	0.254	0.463	**0.683**	0.031	0.262	0.472	**0.707**	0.032
sum, S, tave	15.1	2231.9	57.2	111.60	195.8	0.055	27.8	63.2	104.9	0.053	0.715	0.889	0.992	0.00347	0.771	0.913	0.996	0.0024
sum, S, tmax	17.1	**2232.3**	57	**111.62**	**198**	0.056	24.4	58.3	96.6	0.052	0.940	0.992	1	**0.00024**	0.965	0.995	1	**0.00008**
ms, S, tmin	7.2	2150.1	61.8	107.5	182.7	0.049	46	84.1	150.3	0.055	**0.251**	0.461	0.692	0.034	0.258	0.470	0.710	0.0331
ms, S, tave	15.1	2169.6	69.3	108.5	189.8	0.044	27.4	60.2	96.2	0.044	0.715	0.883	0.995	0.00357	0.771	0.907	0.998	0.00247
ms, S, tmax	16.4	2164.6	**69.5**	108.2	190.3	0.044	21	55.0	86.6	0.050	0.924	0.990	1	0.000335	0.956	0.994	1	0.000117
lxm, S, tmin	7	2108.9	61.8	105.4	171.4	0.040	**46.6**	82.2	139.0	0.045	0.254	**0.459**	0.692	0.033	**0.256**	**0.468**	0.710	0.0332
lxm, S, tave	14.2	2098.8	68.9	104.9	173.8	0.033	30.3	57.0	88.5	**0.035**	0.707	0.873	0.992	0.00354	0.760	0.900	0.997	0.00255
lxm, S, tmax	15.6	2102.9	**69.5**	105.1	171.8	**0.031**	24.6	51.9	79.5	0.039	0.925	0.990	1	0.000329	0.957	0.994	1	0.000116

'trade-off' is evaluated only for positive values and denoted by parentheses. In this case, the benefit of lxm seems to small. These results reveal the influence of correlation between utility values and their publish-cost values. Table 6 shows the results in the case of $n = 20$, $c = 20$, equ, and S. The results resemble in the case of $n = 10$ and $c = 20$.

The average execution time of our experimental implementation on a computer with g++ (GCC) 4.4.7, Linux version 2.6, Intel (R) Core (TM) i7-3770K CPU @ 3.50GHz and 32GB memory was 973 seconds in the case of $n = 10$, $c = 20$, inv, lxm, S, and tave.

5 DISCUSSION

In the previous work (Matsui, 2019), similar problem was solved using a mediator agent that performs a centralized local search. The goal of the study is to find the first solution where all agents can agree with published utility values. Therefore, the solution process only finds one of combinations of parts of constraints that involves an assignment to all variables, and other possible complete solutions are not explored. In addition, the criteria to aggregate and evaluate publish cost values and utility values is the summation.

Our study investigates the negotiation process on similar problem with a decentralized complete solution method employing several criteria that consider preferences of individual agents. We also allow to continue the search for other solutions with better utility values, because the complete solution method finds the first solution after the first round. On the other hand, due to the limitation of dynamic programming on a pseudo-tree, we solved relatively small scale and sparse problems in comparison to the problems in the previous work.

We assumed that it is accepted to reveal some abstract information to determine utility values to be published, since the aim of this work is a fundamental investigation of the proposed approach. There are opportunities to employ a secure computation in part of the negotiation process of publication. In such situations, the information to be finally published so that agents can understand the reason of an agreement on a solution will be an issue.

We employed complete solution methods to solve problems in each negotiation round so that local parts of negotiation are based on optimal solutions as a baseline. However, incomplete solution methods are necessary for practical and large-scale problems. There are opportunities to develop such solution methods for composite criteria considering social welfare among agents. For initial investigation, we concentrated on fundamental benchmark problems. Applying the proposed approach with more scalable solution methods to practical resource allocation and collaboration problems will be a goal of future study.

6 CONCLUSION

In this study, we addressed a negotiation framework based on asymmetric constraint optimization problems, where agents iteratively publish utility values of their constraints and solve the problem with published utility values. We studied applying a decentralized complete solution method to solve both phases in each negotiation round. The proposed approach employs two solution methods based on pseudo-trees to select utility values to be published and to solve the problem with the published utility values. As our first investigation, we evaluated the criterion of the dedicated optimization problems and aggregation operators, and demonstrated is influence and effect.

Since we employed a complete solution method based on pseudo-trees, the scalability for complex problems is limited. A focus of our future work will be decentralized solution methods for large scale problems in practical domains. Improvement of the proposed criterion and termination condition considering agreement among agents with dedicated pricing of privacy and utility will also be included our future work.

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REFERENCES

Fioretto, F., Pontelli, E., and Yeoh, W. (2018). Distributed constraint optimization problems and applications: A survey. *Journal of Artificial Intelligence Research*, 61:623–698.

Grinshpoun, T., Grubshtein, A., Zivan, R., Netzer, A., and Meisels, A. (2013). Asymmetric distributed constraint optimization problems. *Journal of Artificial Intelligence Research*, 47:613–647.

Grinshpoun, T. and Tassa, T. (2016). P-syncbb: A privacy preserving branch and bound dcop algorithm. *J. Artif. Int. Res.*, 57(1):621–660.

Kexing, L. (2011). A survey of agent based automated negotiation. In *2011 International Conference on Network Computing and Information Security*, volume 2, pages 24–27.

Léauté, T. and Faltings, B. (2013). Protecting privacy through distributed computation in multi-agent decision making. *Journal of Artificial Intelligence Research*, 47(1):649–695.

Marler, R. T. and Arora, J. S. (2004). Survey of multi-objective optimization methods for engineering. *Structural and Multidisciplinary Optimization*, 26:369–395.

Matsui, T. (2019). A study of cooperation with privacy loss based on asymmetric constraint optimization problem among agents. In *3rd International Conference on Advances in Artificial Intelligence*, pages 127–134.

Matsui, T., Matsuo, H., Silaghi, M., Hirayama, K., and Yokoo, M. (2018a). Leximin asymmetric multiple objective distributed constraint optimization problem. *Computational Intelligence*, 34(1):49–84.

Matsui, T., Silaghi, M., Okimoto, T., Hirayama, K., Yokoo, M., and Matsuo, H. (2018b). Leximin multiple objective dcops on factor graphs for preferences of agents. *Fundam. Inform.*, 158(1-3):63–91.

Petcu, A. and Faltings, B. (2005). A scalable method for multiagent constraint optimization. In *19th International Joint Conference on Artificial Intelligence*, pages 266–271.

Petcu, A., Faltings, B., and Parkes, D. C. (2008). M-DPOP: Faithful distributed implementation of efficient social choice problems. *Journal of Artificial Intelligence Research*, 32:705–755.

Sen, A. K. (1997). *Choice, Welfare and Measurement*. Harvard University Press.

Tassa, T., Grinshpoun, T., and Yanay, A. (2019). A Privacy Preserving Collusion Secure DCOP Algorithm. In *28th International Joint Conference on Artificial Intelligence*, pages 4774–4780.

Tassa, T., Grinshpoun, T., and Zivan, R. (2017). Privacy preserving implementation of the max-sum algorithm and its variants. *J. Artif. Int. Res.*, 59(1):311–349.

Yeoh, W. and Yokoo, M. (2012). Distributed problem solving. *AI Magazine*, 33(3):53–65.