Properties and Bounds for the Single-vehicle Capacitated Routing Problem with Time-dependent Travel Times and Multiple Trips

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Abstract: This paper deals with a problem where the same vehicle performs several routes to serve a set of customers and arc traversal times vary along the planning horizon. The relationship with its time-invariant counterpart is investigated and a procedure to compute lower and upper bounds on the optimal solution value is developed. Computational results on instances, based on the Paris (France) road graph, show the effectiveness of this approach.

1 INTRODUCTION

In this work, we consider a variant of the capacitated vehicle routing problem, where the same vehicle can perform several routes during its workday and the travel times depends on the departure times. This problem arises in e-groceries, where customers can order goods through the internet and have them delivered at home. In particular in the home delivery of perishable goods, like foods, routes are of short duration and must be combined to form a complete workday. Indeed in urban areas, physical street structures essentially allow only small-sized vehicles for delivery. This leads to routes shorter than the workday. At the time of writing, this type of problem is becoming more and more important: Coronavirus disease (COVID-19) pandemic has rapidly accelerated the shift toward online grocery shopping and the digitization of stores. The idea is that, once people will get used to order online their groceries, they will stick with this habit even after the pandemic. A comprehensive review on the variants of multi-trip vehicle routing problem can be found in (Cattaruzza et al., 2016). In urban areas, taking into account time-dependencies of travel times helps to capture congestion phenomena and improve route design and logistics costs. Some existing researches have studied vehicle routing problems under time-dependent settings (Gendreau et al., 2015). To the best of our knowledge, only in (Sun et al., 2018), (Pan et al., 2020) and (Karoonsoontawong et al., 2020) it has been considered both time-dependent travel time and multiple use of vehicles together. In (Sun et al., 2018) the authors report about a tabu search heuristic that can efficiently handle different types of constraints including time windows and multiple uses of vehicles. The authors take into consideration the time-dependent travel times between different customers in order to satisfy time windows constraints, and also minimize the total scheduling time of all vehicles. They adopted a piece-wise linear travel speed model which leads to a quadratic travel time function, characterized by complicated calculations of travel times. In (Pan et al., 2020) and (Karoonsoontawong et al., 2020) the time-dependent setting is modeled by the widely used piecewise linear travel time function paradigm (Ichoua et al., 2003) only in order to satisfy time windows constraints, whilst arc costs are assumed to be constant.

In this paper, we investigate some properties of the time dependent capacitated single-vehicle routing problem with multiple trips (TD-CSVRPMT). In particular we investigated the relationship between TD-CSVRPMT and its time-independent counterpart. We exploit some results recently provided by (Adamo et al., 2020), where the authors studied a fundamental property of time-dependent graphs called *path ranking invariance*. A time-dependent graph is path ranking invariant if the ordering of its paths w.r.t. travel duration is not dependent on the start travel time. (Adamo et al., 2020) proved that this property can be exploited to solve a large class

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of time-dependent routing problems including the Time-Dependent Travelling Salesman Problem and the Time-Dependent Rural Postman Problem. We extended these results to the TD-CSVRPMT by proving that the optimal solution of a time-independent capacitated vehicle routing problem (CVRP) provides both a lower bound and an upper bound for the original TD-CSVRPMT. The paper is organized as follows. Section 2 introduces the notation used throughout the paper. Section 3 presents a procedure to compute lower and upper bounds on the optimal solution value. Section 4 is devoted to computational experiments. Finally some conclusions follow in Section 5.

2 NOTATION AND PROBLEM DEFINITION

The problem considered is defined on a time-dependent directed complete graph $G := (V \cup \{0\}, A, \tau, q_i, Q)$, where $V = \{1, \ldots, n\}$ is the set of customers, vertex 0 is the depot and $A := \{(i, j) : i \in V, j \in V\} \bigcup \{(0,i) : i \in V\} \bigcup \{(i,0) : i \in V\}$ is the set of arcs. We have a single vehicle of capacity Q delivering goods from the depot to the set of customer nodes V. The vehicle workday corresponds to a route made up of a set of R trips, where each trip starts and ends at the depot (some of these trips might be empty). We assume, without loss of generality, that the trips are served in the order $1, 2, \ldots, R$. Let denote with $[0, T]$ the time interval associated to a single working day. We denote with $\tau : A \times \mathbb{R}^+ \to \mathbb{R}$ a function that associates to each arc $(i, j) \in A$ and starting time $t \in [0, +\infty)$ the traversal time when a vehicle leaves the vertex i at time t. Without loss of generality we suppose that the travel time functions are constant in the long run, that is $\tau(i, j, t) := \tau(i, j, T)$ with $t \geq T$. For the sake of notational simplicity, we use $\tau_{ij}(t)$ to designate $\tau(i, j, t)$. We suppose that traversal time $\tau_{ij}(t)$ satisfy the *first-in-first-out* (FIFO) property, i.e., leaving the vertex i later implies arriving later at vertex j. Each customer $i \in N$ is characterized by a demand q_i, which is deterministic, known in advance and cannot be split.

For any given path $p_k := (i_0, i_1, \ldots, i_k)$, the corresponding duration $z(p_k, t)$ can be computed recursively as:

$$z(p_k, t) := z(p_{k-1}, t) + \tau_{i_{k-1} i_k}(z(p_{k-1}, t)), \qquad (1)$$

with the initialization $z(p_0, t) := 0$. The TD-CSVRPMT aims to determine the optimal multi-trip route on $G := (V \cup \{0\}, A, \tau, q_i, Q)$ used by a single vehicle, based at the depot, to serve the set of customers. Only the capacity restriction for the vehicle is imposed, and the objective is to minimize the total travel time needed to serve all the customers when the vehicle leaves the depot at a time instant $t = 0$, that is:

$$\min_{p \in P} z(p, 0).$$

where P denotes the set of paths feasible for TD-CSVRPMT. It is worth noting that the time independent counterpart of the TD-CSVRPMT is the classical CVRP. Indeed, in the classical CVRP routes do not need to correspond to vehicles. In other words, any feasible solution of the CVRP may be used to model a real-world situation where a single vehicle will perform all routes in sequence. Algorithms developed for the CVRP are not able to consider time-varying travel times without essential structural modifications. Nevertheless, we observe that the absence of time constraints implies that time-varying travel times have an impact on the ranking of solutions of the TD-CSVRPMT, but they do not pose any difficulty for feasibility check of solutions. In particular, one can assert that there always exists a time-invariant (dummy) cost function $c : A \to \mathbb{R}^+$ such that a least duration route of TD-CSVRPMT is also a least cost solution of the time-invariant CVRP, defined on the time-invariant graph $G_c = (V \cup \{0\}, A, c, q_i, Q)$.

Definition 2.1 (**Valid Cost Function**). *A time-invariant cost function $c : A \to \mathbb{R}^+$ is valid for the TD-CSVRPMT defined on $G = (V \cup \{0\}, A, \tau, q_i, Q)$, if the least duration solution $p^* = \min_{p \in P} z(p, 0)$ corresponds to a least cost solution of the time-invariant CVRP defined on $G_c = (V \cup \{0\}, A, c, q_i, Q)$.*

If we are given a cost function *valid* for an instance of the TD-CSVRPMT defined on a time-dependent $G = (V \cup \{0\}, A, \tau, q_i, Q)$, then we can determine the least duration solution p^* by exploiting algorithms developed for CVRP. For this purpose we introduce a property of time-dependent graphs called *path ranking invariance*.

Definition 2.2 (**Path Ranking Invariance**). *A time-dependent graph G is path ranking invariant, if the path dominance rule holds true for any pair of paths p' and p'' of G, it results that:*

$$z(p', t) \geq z(p'', t) \quad \forall t \geq 0.$$

Since travel time function are constant in the long run, if a time-dependent graph $G = (V \cup \{0\}, A, \tau, q_i, Q)$ is path ranking invariant then a *valid* cost function is $c(i, j) = \tau_{ij}(T)$. In the following section we exploit the path ranking invariance property in order to devise a procedure to compute lower and upper bounds for the TD-CSVRPMT.

3 PROPERTIES AND BOUNDS

Given a time-dependent graph $G = (V \cup \{0\}, A, \tau, q_i, Q)$, we define an auxiliary path ranking invariant graph $\underline{G} = (V \cup \{0\}, A, \underline{\tau}, q_i, Q)$ where $\underline{\tau}_{ij}(t)$ is a lower approximation of the original traversal time $\tau_{ij}(t)$, that is:

$$\underline{\tau}_{ij}(t) \le \tau_{ij}(t),$$

with $(i, j) \in A$ and $t \in [0, T]$. We suppose that the traversal time function is generated by the travel time model proposed in (Ichoua et al., 2003) (IGP model for short), in which each arc $(i, j) \in A$ is characterized by a constant stepwise speed function $v_{ij}(t)$ and a length L_{ij}. We suppose that the horizon is partitioned into H subintervals $[T_h, T_{h+1}]$ $(h = 0, \ldots, H-1)$, with $T_0 = 0$ and $T_H = T$. We assume that all arcs of the auxiliary graph share a common speed function, such that

$$v_{ij}(t) = v_h,$$

with $t \in [T_h, T_{h+1}]$ and $h = 0, \ldots, H-1$. According to the IGP model, given a start time t the travel time value $\underline{\tau}_{ij}(t)$ is computed by the following iterative procedure.

Algorithm 1: Computing the travel time $\underline{\tau}_{ij}(t)$.

```
k ← h : t_h ≤ t ≤ t_{h+1}
d ← L_ij;
t' ← t + d/v_k;
while t' > T_{k+1} do
    d ← d − v_h(T_{k+1} − t);
    t ← T_{k+1};
    t' ← t + d/v_{k+1};
    k ← k + 1
return t' − t
```

In the IGP model the speed of a vehicle is not a constant over the entire length of arc $(i, j) \in A$ but it changes when the boundary between two consecutive time periods is crossed. The relationship between the input parameters and the output value of the IGP model can be expressed in a compact fashion as follows:

$$L_{ij} = \int_t^{t+\underline{\tau}_{ij}(t)} v(\mu) d\mu. \qquad (2)$$

We denote with $\underline{z}(p_k, t)$ the traversal time of a path p_k at time instant t on the time-dependent graph $\underline{G}$, that is

$$\underline{z}(p_k, t) = \underline{z}(p_{k-1}, t) + \underline{\tau}_{i_{k-1} i_k}(\underline{z}(p_{k-1}, t)), \qquad (3)$$

with the initialization $\underline{z}(p_0, t) = 0$.

Proposition 3.1. *The time dependent graph $\underline{G} = (V \cup \{0\}, A, \underline{\tau}, q_i, Q)$ is path ranking invariant.*

Proof. We observe that from (2) it follows that given a path p we have that:

$$\sum_{(i,j)\in p} L_{ij} = \int_t^{t+\underline{z}(p,t)} v(\mu) d\mu,$$

where the notation $(i, j) \in p$ means that the arc $(i, j) \in A$ is traversed by the path p. This implies that if a path p' is shorter that a path p'' then p' is also quicker that p'' for any start time $t \in [0, T]$:

$$\sum_{(i,j)\in p'} L_{ij} \le \sum_{(i,j)\in p''} L_{ij} \Leftrightarrow \underline{z}(p', t) \le \underline{z}(p'', t),$$

which proves the thesis. □

In order to determine the IGP parameters we follow the two steps procedure proposed in (Adamo et al., 2020).

Step 1 - Determining the Potential Speed Breakpoints.
Let $\{t_{ijk}, k = 0, \ldots, K_{ij} - 1\}$ be the set of breakpoints of the travel time function $\tau_{ij}(t)$ and let $\Gamma_{ij}(t)$ be the arrival time function, i.e. $\Gamma_{ij}(t) = t + \tau_{ij}(t)$, with $(i, j) \in A$. In the first phase, we determine a set $\Omega = \{T_0, \ldots, T_H\}$ of speed breakpoints as $\Omega = \bigcup_{(i,j)\in A} \Omega_{ij}$, where each Ω_{ij} is an ordered set determined by means of an iterative procedure (Algorithm 2) composed of a main while loop in which each travel time breakpoint t_{ijk} is added to Ω_{ij}. Moreover, for each t_{ijk}:

1. Ω_{ij} is iteratively enriched by the arrival time $\Gamma_{ij}(t_{ijk})$ associated to a starting time equal to t_{ijk}, by the arrival time $\Gamma_{ij}(\Gamma_{ij}(t_{ijk}))$ associated to a starting time equal to $\Gamma_{ij}(t_{ijk})$, etc, until no speed breakpoint less than or equal to $t_{ij,K_{ij}-1}$ can be generated;
2. finally, Ω_{ij} is iteratively enriched by the starting time $\Gamma_{ij}^{-1}(t_{ijk})$ associated to an arrival time equal to t_{ijk}, by the starting time $\Gamma_{ij}^{-1}(\Gamma_{ij}^{-1}(t_{ijk}))$ associated to an arrival time equal to $\Gamma_{ij}^{-1}(t_{ijk})$, etc, until no speed breakpoint greater than or equal to $t_{ij0} = 0$ can be generated.

Step 2 - Determining the Speed Levels and the Length of the IGP Model.
We start by observing that $\underline{\tau}$ is a lower approximation of τ, if the following relationships holds true for each arc $(i, j) \in A$ and time instant $t \in [0, T]$:

$$\int_t^{t+\tau_{ij}(t)} v(\mu) d\mu \ge \int_t^{t+\underline{\tau}_{ij}(t)} v(\mu) d\mu = L_{ij}. \qquad (4)$$

Algorithm 2 : Determine a set of speed breakpoints Ω_{ij} given the set of time breakpoints $\{t_{ij0},\ldots,t_{ijK_{ij}-1}\}$.

$\Omega_{ij}=\emptyset$
for all $t \in \{t_0,\ldots,t_{ijK_{ij}-1}\}$ **do**
  if $t \notin \Omega_{ij}$ **then**
    $\Omega_{ij} \leftarrow t$
    $t' \leftarrow t$
    while $(t' \leq t_{ijK_{ij}-1}) \wedge (\Gamma_{ij}(t') \notin \Omega_{ij})$ **do**
      $\Omega_{ij} \leftarrow \Gamma_{ij}(t')$
      $t' \leftarrow \Gamma_{ij}(t')$
    $t' \leftarrow t$
    while $(t' \geq \Gamma_{ij}(t_0)) \wedge (\Gamma^{-1}_{ij}(t') \notin \Omega_{ij})$ **do**
      $\Omega_{ij} \leftarrow \Gamma^{-1}_{ij}(t')$
      $t' \leftarrow \Gamma^{-1}_{ij}(t')$
return Ω_{ij}

Theorem 3.1. *(Adamo et al., 2020) Given two time-dependent graphs $G=(V,A,\tau)$ and $\underline{G}=(V,A,\underline{\tau})$, if the relationships (4) holds true for any arc $(i,j)\in A$ and time instant $t\in\Omega$, then the traversal time function $\underline{\tau}$ is a lower approximation of the original travel time function τ.*

Let $a_{ijkh}=\min(T_{h+1}-T_h,\max(0,\Gamma_{ij}(t_{ijk})-T_h))$ if $k\leq h$, 0 otherwise, with $(i,j)\in A$, $h=0,\ldots,H-1$, $k=0,\ldots,|\Omega_{ij}|-1$. Since $v(t)$ is a constant stepwise function the relationship (4) can be expressed by the following linear equality:

$$\sum_{h=0}^{|\Omega|-1} a_{ijkh}\times v_h - s_{ijk} = L_{ij}, \tag{5}$$

where s_{ijk} denotes the *surplus* of the right-hand-side of (4) with respect to L_{ij}, with $(i,j)\in A$, $h=0,\ldots,H-1$, $k=0,\ldots,|\Omega_{ij}|-1$. We observe that the *maximum fitting deviation* between the original travel time function $\tau_{ij}(t)$ and its lower approximation $\underline{\tau}_{ij}(t)$), depends on the quantity

$$\zeta_{ij}=\max_{t_k\in\Omega_{ij}} s_{ijk}-\min_{t_k\in\Omega_{ij}} s_{ijk},$$

with $(i,j)\in A$.

Remark 3.2. *If ζ_{ij} is equal to zero, then the travel time function $\underline{\tau}_{ij}(t)$ is a perfect fit, for all arcs $(i,j)\in A$. In this case, the original graph $G=(V\cup\{0\},A,\tau,q_i,Q)$ is path ranking invariant and the optimal solution of TD-CSVRPMT can be determined by solving a classical CVRP on $G_c(V\cup\{0\},A,c,q_i,Q)$ where $c_{ij}=\tau_{ij}(T)$, with $(i,j)\in A$.*

In all other cases (i.e. $\zeta_{ij}>0$ for some $(i,j)\in A$) the value of $\zeta=\sum_{(i,j)\in A}\zeta_{ij}$ represents a measurement of the *distance* from this special case.

We determine the auxiliary graph $\underline{G}$ by determining the lower approximation that minimize the value of ζ. For this purpose we formulate the linear program (6)-(14), where $\underline{s}_{ij}$ and $\bar{s}_{ij}$ model, respectively, the minimum and maximum value of the surplus variable s_{ijk}, with $(i,j)\in A$ and $k=0,\ldots,|\Omega_{ij}|-1$. A solution of such linear programming model represents the parameters of a constant piecewise function $y(t)$ and the constant values x_{ij}, with $(i,j)\in A$. The continuous variable y_h represents the value of $y(t)$ during the $h-th$ time interval, that is:

$$y(t)=y_h,$$

with $t\in[t_h,t_{h+1}]$ and $h=0,\ldots,|\Omega|-1$. The set of feasible solutions of the linear program (6)-(14) represents the IGP input parameters for generating a family of lower approximations of the travel time function τ.

$$\zeta^* := \min \sum_{(i,j)\in A}\zeta_{ij} \tag{6}$$

s.t.

$$\sum_{h=0}^{|\Omega|-1} a_{ijkh}\cdot y_h - s_{ijk} = x_{ij} \tag{7}$$

$$,\quad k=0,\ldots,|\Omega_{ij}|-1 \quad (i,j)\in A$$

$$\zeta_{ij}\geq \bar{s}_{ij}-\underline{s}_{ij} \qquad (i,j)\in A \tag{8}$$
$$\underline{s}_{ij}\leq s_{ijk} \qquad k=0,\ldots,|\Omega_{ij}|-1,(i,j)\in A \tag{9}$$
$$\bar{s}_{ij}\geq s_{ijk} \qquad k=0,\ldots,|\Omega_{ij}|-1,(i,j)\in A \tag{10}$$
$$y_h\geq\rho \qquad h=0,\ldots,|\Omega|-1 \tag{11}$$
$$s_{ijk}\geq 0 \qquad k=0,\ldots,|\Omega_{ij}|-1,(i,j)\in A \tag{12}$$
$$\zeta_{ij}\geq 0 \qquad (i,j)\in A \tag{13}$$
$$x_{ij}\geq 0 \qquad (i,j)\in A \tag{14}$$

The objective function (6) states that the optimization model aims to determine a constant stepwise function $y^*(t)$, such that it is minimized the total maximum fitting deviation between the original travel time function τ and its lower approximation $\underline{\tau}$. Constraints (7) state the relationship between $y(t)$, x_{ij} and s_{ijk} at time instant $t_{ijk}\in\Omega_{ij}$. Constraints (8) state the relationship between the objective function and the range value of ζ_{ij}, modeled as the difference between $\bar{s}_{ij}$ and $\underline{s}_{ij}$. Constraints (9) and (10) state the relationship between $\underline{s}_{ij}$, $\bar{s}_{ij}$ and the continuous variables s_{ijk}. In order to cut off the trivial (pointless) solution $y(t)=0$ for $t\geq 0$, constraints (11) state that the constant stepwise linear function $y(t)$ has to be greater or equal than the input parameter $\rho > 0$. Constraints (12), (13) and (14) provide the non-negative conditions of the remaining decision variables.

Let $y^*(t)$ and x^* denote, respectively, the step function and the x's values associated with the optimal solution of the the linear program (6)-(14). The lower approximation $\underline{\tau}_{ij}(t)$ is generated by the IGP model with the following input parameters:

$$v(t) = y^*(t), \quad L_{ij} = x^*_{ij},$$

with $(i, j) \in A$ and $t \in [0, T]$.

Summing up the proposed lower bounding procedure consists of three main steps.

- **STEP 1.** Solve linear program (6)-(14). Set the travel speed function $v(t)$ equal to $y^*(t)$. Similarly we set the L_{ij} to x^*_{ij} for each $(i, j) \in A$.
- **STEP 2.** Determine the solution $\underline{p}^*$ as the least cost solution of the following time-independent CVRP:
$$\min_{p \in \mathcal{P}} \sum_{(i,j) \in p} L_{ij}$$
- **STEP 3.** Compute the lower bound $\underline{z}^*$ by evaluating $\underline{p}^*$ w.r.t. $\underline{\tau}$ obtained as output of the IGP model with input parameters set according the optimal solution of the linear program (6)-(14) determined at **STEP 1**, that is:
$$\underline{z}^* = \underline{z}(\underline{p}^*, 0)$$

We finally observe that since the path $\underline{p}^*$ belongs to the set of feasible solutions P, we also generate a parameterized family of upper bound $\bar{z}$ obtained by evaluating $\underline{p}^*$ w.r.t. the original travel time function τ :

$$\bar{z} := z(\underline{p}^*, 0).$$

4 COMPUTATIONAL RESULTS

The algorithms have been implemented in Java and run on a Linux machine clocked at 2.8 GHz and equipped with 16GB of RAM. We used IBM ILOG CPLEX 12.10 as a black-box solver to find the solution of the linear program (6)-(14), and VRPSolver from (Pessoa et al., 2020) as exact solver for the Asymmetric CVRP. We imposed a time limit of 3600 seconds for both stages.

We have generated 7 classes of test instances each containing 10 individual instances, based on the Paris (France) road graph (Ghiani et al., 2020), with $|V| =$ 20, 30, 40, 50, 60, 70 and 80 nodes, respectively. We assigned a demand $q_i \in \{6, 8, 10, 12\}$ $(i \in V)$ to each customer. Therefore, customers can be partitioned in a family of 4 subsets sharing the same demand, i.e. $V = \bigcup_{c \in \{6,8,10,12\}} V_c$. The number of daily trip R is chosen to be 3, 4, or 5. Moreover we set the value of ρ equal to $1/\min_{h=0,\ldots,H-1}(T_{h+1} - T_h)$. Table 1 summarizes customers demands distribution and vehicles capacity Q according to the number of nodes. $Q_c = \sum_{i \in V} q_i$ is the total demand.

Table 1: Test instances.

$\|V\|$	$\|V_6\|$	$\|V_8\|$	$\|V_{10}\|$	$\|V_{12}\|$	Q_c	Q
20	5	5	5	4	168	66
30	8	7	7	7	262	97
40	10	10	10	9	348	126
50	13	12	12	12	442	157
60	15	15	15	14	528	186
70	18	17	17	17	622	217
80	20	20	20	19	708	246

The results are reported in Table 2. The headings are as follows:

- $TIME_0$: average computing time for the STEP 1 in seconds;
- ζ^*: average objective value determined at STEP 1;
- OPT: number of instances solved to optimality in STEP 2 by VRPSolver out of 10 ;
- $TIME_1$: average computing time for the VRPSolver in seconds;
- GAP: average optimality gap $\frac{\bar{z} - \underline{z}^*}{\underline{z}^*}$ (%).

Table 2: Computational results.

		STEP 1		STEP 2		
$\|V\|$	R	$TIME_0$	ζ^*	OPT	$TIME_1$	GAP
20	5	7.4	0.138	10	1.2	1.19
	4	6.7	0.117	10	1.3	1.03
	3	5.9	0.112	10	1.3	1.03
30	5	22.3	0.214	10	1.6	1.93
	4	20.7	0.172	10	1.5	1.39
	3	20.1	0.148	10	1.8	1.20
40	5	50.7	0.374	10	10.7	4.87
	4	49.8	0.343	10	3.7	4.42
	3	48.2	0.307	10	4.6	3.81
50	5	160.5	0.537	10	7.5	8.33
	4	164.6	0.514	10	8.7	8.05
	3	147.3	0.499	10	18.5	7.86
60	5	427.8	0.596	10	52.8	8.90
	4	452.6	0.586	10	57.1	8.99
	3	363.2	0.576	10	63.8	8.98
70	5	988.8	0.619	9	340.4	7.91
	4	988.8	0.619	10	580.6	8.98
	3	988.8	0.619	9	244.2	8.08
80	5	1749.8	0.628	10	223.6	8.73
	4	1749.8	0.628	10	226.8	8.81
	3	1749.8	0.628	10	405.5	8.90

The procedure does not exceeds the time limits for 208 out of 210 instances: in particular the algorithm fails to determine a lower bound for two test cases. Therefore, with reference to columns $TIME_1$ and GAP, each row is the average across instances solved to optimality by VRPSolver. The overall average time to solve the linear program (6)-(14) (STEP 1) is about 484 seconds with an average ζ^* equal to 0.427, while the overall average time required by VRPSolver (STEP 2) is about 108 seconds obtaining an average GAP of 5.88%. We underline that as ζ^* grows, the GAP also increases. In particular when ζ^* raises up to 0.5, GAP doubles its value.

5 CONCLUSIONS

This paper has introduced a procedure to compute lower and upper bounds of the optimal solution value of the time dependent capacitated single-vehicle routing problem with multiple trips. For the special case where the graph is path ranking invariant, we have shown that the upper bound computed in this way provides an optimal solution. Future work will focus on embedding the lower and upper bounding procedure introduced in this paper in an enumerative search algorithm.

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