An Adjustable Robust Formulation and a Decomposition Approach for the Green Vehicle Routing Problem with Uncertain Waiting Time at Recharge Stations

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Abstract: We investigate the problem of routing a fleet of electric vehicles in an urban area, which must serve a set of customers within predefined time windows. We allow partial battery recharge to any available recharge stations, located in the area. Since the recharge stations could be busy when a driver arrives for charging operations, the time spent can be seen as the sum of recharge times and waiting times. We model the waiting times as uncertain parameters and we assume to do not know their distribution. Hence, we address the problem under the robust optimization framework by modelling the realization of the uncertain parameter with the budget of uncertainty polytope. We propose an adjustable robust formulation, then we implement a row-and-column generation solution approach based on a two-stage reformulation of the problem, to provide a robust routing against infeasibility with respect to time collect requirements. We test the proposed approach on benchmark instances and we analyze the behavior of the considered transportation system with respect to the uncertain parameter. In addition, we investigate the price of robustness and the reliability of the robust solutions obtained.

1 INTRODUCTION

The worrying effects of the road traffic emissions on the air quality has become an issue of global importance. Hence, the need to provide sustainable transportation plans is the main objective of many countries. In recent years, several political decisions and regulations concerning the reduction of the greenhouse and polluting emissions have been proposed and have already become law (i.e., Kyoto Protocol (1997) and the European plan on climate change (2008)). Certainly, the greenhouse issue has become a political topic with high priority and the definition of sustainable logistics systems is an essential choice. Hence, a “green” management of transportation systems plays a key role. Among the strategies that can be adopted, the use of Battery Electric Vehicles (BEVs) represents a promising alternative to the traditional Internal Combustion Engine Vehicles (ICEVs) for reducing negative externalities, such as noise and pollution. In fact, one of the benefits associated with BEVs is the absence of the CO_2 emissions. However, there are also several critical aspects related to some technical issues. The BEVs have a poor battery life compared to the ICEVs autonomy, the vehicles need to be recharged during the routes and the number of available recharge stations is still low. In addition, the time needed to fulfil a complete recharge is very high and the stops during the routes increase the total time spent to complete a trip. Fortunately, the automotive sector is interested in BEVs development and invests in this research area, hence, the innovation is fast and continuous. For this reason, considering the introduction of BEVs in transportation is a strategic investment.

The introduction of BEVs in the classical Vehicle Routing Problem (VRP), requires the modeling and insertion of some specific constraints. Hence, several restrictions must be taken into account, especially the limited battery capacity of the vehicles and the possibility to recharge the batteries at the Alternative Fuel Stations (AFSs). (Erdoğan and Miller-

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Hooks, 2012) introduced the first routing model that considers AFSs. In particular, the authors modeled a Mixed Integer Linear Programming (MILP) of a VRP in which a fleet is composed of Alternative Fuelled Vehicles (AFVs) with a limited fuel capacity. They considered the possibility to stop the vehicles and recharge their batteries at the available AFSs and named this problem Green VRP (GVRP). In that paper, the vehicles are uncapacitated and time window constraints are not included. Several techniques are developed to find a solution that minimizes the total distance traveled. (Schneider et al., 2014) and (Felipe et al., 2014) extended the model presented in (Erdoğan and Miller-Hooks, 2012) introducing the Electric-VRP (E-VRP), in which the fleet is composed of BEVs. In particular, (Schneider et al., 2014) studied the E-VRP with time windows and recharge stations, which considers capacity constraints on vehicles and time windows on customers. The vehicles can stop at any available AFS for fulfilling a complete battery recharge. The time spent to recharge the battery depends on the battery charge of the vehicle on arrival at the AFS. The authors proposed a meta-heuristic that combines variable neighbourhood search and tabu search to solve their problem. (Felipe et al., 2014) introduced more realistic elements to the E-VRP. In particular, the authors considered the possibility of partial recharging at the stations. Furthermore, a battery recharge can be done with different technologies, in this case different recharging times and costs should be taken into account. For example the overnight depot recharging is the cheapest and slowest technology. The authors propose a constructive algorithm based on a greedy generation method, a deterministic local search, and a Simulated Annealing algorithm.

The E-VRP has been widely studied and, starting from these works, many authors proposed several extensions and variants and developed solutions methods, see, e.g., (Ding et al., 2015; Keskin and Çatay, 2016; Desaulniers et al., 2016; Goeke, 2019; Löffler et al., 2020) for variants which consider partial recharges, and (Lin et al., 2009; Hiermann et al., 2016; Hiermann et al., 2016; Joo and Lim, 2018; Basso et al., 2019) for versions of the problem which consider full recharge only. A particular variant of the E-VRP is the mixed-fleet GVRP, in which the fleet is composed of both BEVs and ICEVs, see, e.g. (Gonçalves et al., 2011; Sassi et al., 2014; Macrina et al., 2019a; Macrina et al., 2019b; Hiermann et al., 2019). Since the battery charge level is a non-linear function, recently several authors focused on this specific feature and proposed mathematical formulations which consider this issue (see, e.g., (Montoya et al., 2017; Froger et al., 2019)).

Another important aspect to take into account is the limited number of recharge stations, located in the urban area. Hence, on the one hand, combining the optimization of recharge stations location and BEVs routing could be a leading strategy (see, e.g., (Yang and Sun, 2015; Paz et al., 2018; Zhang et al., 2019)), on the other one, it is necessary to take into account the possibility that a charger is not available when a vehicle stops at a recharge station. Focusing on the latter problem, we propose a variant of the E-VRP with time windows and partial recharges, which considers an uncertain waiting time at the recharge stations. A similar framework has been studied by (Keskin et al., 2019) and (Keskin et al., 2021). In particular, (Keskin et al., 2019) considered time-dependent queuing times at the stations, they split the time horizon into predetermined number of time intervals and assigned an average queue length to each recharge station, for different time intervals. They formulated the problem as a MILP and proposed a math-heuristic based on Adaptive Large Neighborhood Search (ALNS) to solve it. (Keskin et al., 2021) studied an E-VRP with time windows and stochastic waiting times at recharge stations. They proposed a two-stage simulation-based heuristic using ALNS to solve their problem. In particular, in the first stage they used the expected waiting time values at the stations for determining the routes. After the arrival of the vehicles at the recharge station, their queuing times are revealed, hence, the second stage corrects the infeasible solutions in case the actual waiting time at a station exceeds its expected value.

In our work, we propose a mathematical formulation based on (Schneider et al., 2014), but we extend it by introducing the possibility to perform partial recharges and waiting time at recharge stations. We assume the waiting time uncertain and suppose to not know the distribution of the realization of the uncertain parameters. Thus, we address the problem under the robust optimization framework. We propose a two-stage robust formulation defining routing first stage variables and scheduling second stage ones where the latter depend on the realization of the uncertain waiting time. We define a decomposition approach based on a row-and-column generation strategy in which second stage variables and the related constraints are added on the fly within an iterative procedure.

The paper is organized as follows. Section 2 presents the problem and the mathematical formulations. Section 3 defines the proposed solution strategy based on a decomposition approach. Section 4 shows the computational results. Section 5 concludes the paper.

2 PROBLEM DEFINITION

Let $G(\mathcal{V},\mathcal{A})$ be a directed graph, where $\mathcal{V}$ is the set of nodes and $\mathcal{A}=\{(i,j),\forall i,j\in\mathcal{V}\}$ is the set of arcs. The set of nodes $\mathcal{V}$ contains customers, recharge stations, and nodes s, which is the depot, and t, a copy of s, where vehicles routes start and end, respectively. Hence, $\mathcal{V}=\mathcal{N}\cup\mathcal{R}\cup\{s,t\}$, where $\mathcal{N}$ and $\mathcal{R}$ are the set of customers and recharge stations, respectively. A demand q_i and a service time s_i are associated with each customer $i\in\mathcal{N}$, expressed in kg and hours, respectively. Each customer must be visited by a single vehicle. Each node $i\in\mathcal{V}$ is characterized by a time window $[e_i,l_i]$. Furthermore, the vehicles have limited transportation capacity and battery capacity; thus, let Q be the maximal capacity of the vehicle (expressed in tonne), and B be the maximal battery capacity (expressed in kWh). For each arc $(i,j)\in\mathcal{A}$, d_{ij} refers to the distance from i to j (expressed in km), t_{ij} denotes the travel time (expressed in hours), and c_{ij} the cost per unit of distance. It is assumed that $d_{ij}\le d_{ih}+d_{hj}$ and $t_{ij}\le t_{ih}+t_{hj}$ for all $i,h,j\in\mathcal{V}$, hence the triangle inequality holds for both the distance and the time. Each recharge station $i\in\mathcal{R}$ is characterized by a recharge speed ρ_i (expressed in kWh per hour), a waiting time w_i, and a recharge cost $\bar{c}_i$ per unit of kWh. In particular, w_i is the time that a vehicle must wait before starting the recharge at station $i\in\mathcal{R}$. It is worth observing that if the vehicle does not recharge, then no waiting time is considered. The value π denotes the coefficient of energy consumption, that is assumed to be proportional to the distance travelled (expressed in $\frac{KWh}{Km}$). We assume to have a limited number of available vehicles to perform the deliveries, equal to E. In the following, we describe the variables used to model the problem:

- x_{ij} is equal to 1 if the vehicles travel from i to j, zero otherwise, $\forall(i,j)\in\mathcal{A}$;
- y_j is equal to 1 if the vehicle recharge at station j, zero otherwise, $\forall j\in\mathcal{R}$;
- z_{ij} amount of energy available when arriving at node j from the node i [kWh], $\forall(i,j)\in\mathcal{A}$;
- g_{ij} amount of energy recharged by the vehicle at the node i for travelling to j [kWh], $\forall i\in\mathcal{R},\forall j\in V$;
- τ_j arrival time of the vehicle to the node j [h], $\forall j\in V$;
- u_i amount of load available at node i [kg], $\forall i\in V$.

The problem is formulated as follows:

$$\min\sum_{i\in\mathcal{R}}\bar{c}_i\sum_{j\in\mathcal{V}}g_{ij}+\sum_{(i,j)\in\mathcal{A}}c_{ij}x_{ij} \quad (1)$$

s.t.

$$\sum_{j\in\mathcal{V}}x_{ij}=1,\forall i\in\mathcal{N} \quad (2)$$

$$\sum_{j\in\mathcal{V}}x_{ij}\le 1,\forall i\in\mathcal{R} \quad (3)$$

$$\sum_{j\in\mathcal{V}\setminus\{s\}}x_{ij}-\sum_{j\in\mathcal{V}\setminus\{s,t\}}x_{ji}=0,\forall i\in\mathcal{V}\setminus\{t\} \quad (4)$$

$$\sum_{j\in\mathcal{V}\setminus\{s\}}x_{sj}\le E, \quad (5)$$

$$\sum_{i\in\mathcal{V}\setminus\{t\}}x_{it}\ge 1, \quad (6)$$

$$\sum_{i\in\mathcal{V}\setminus\{s\}}x_{si}-\sum_{j\in\mathcal{V}\setminus\{t\}}x_{jt}=0 \quad (7)$$

$$u_j\ge u_i+q_jx_{ij}-Q(1-x_{ij}),$$
$$\forall i\in\mathcal{V}\setminus\{s,t\},j\in\mathcal{V}\setminus\{s\} \quad (8)$$

$$u_s=0 \quad (9)$$

$$y_i\ge\frac{\sum_{j\in\mathcal{V}}g_{ij}}{B},\forall i\in\mathcal{R} \quad (10)$$

$$\tau_j\ge\tau_i+(t_{ij}+s_i)x_{ij}-M1(1-x_{ij}),$$
$$\forall i\in\mathcal{N}\cup\{s\},\forall j\in\mathcal{V} \quad (11)$$

$$\tau_j\ge\tau_i+t_{ij}x_{ij}+\frac{1}{\rho_i}g_{ij}+w_iy_i-M1(1-x_{ij}),$$
$$\forall i\in\mathcal{R},\forall j\in\mathcal{V} \quad (12)$$

$$e_j\le\tau_j\le l_j,\forall j\in\mathcal{N} \quad (13)$$

$$z_{ij}\le(z_{hi}+g_{ij})-\pi d_{ij}x_{ij}+M_1(1-x_{ij})+$$
$$+M_1(1-x_{hi}),$$
$$\forall h\in\mathcal{V},\forall i\in\mathcal{V}\setminus\{s\},\forall j\in\mathcal{V},$$
$$i\ne j,i\ne h,j\ne h \quad (14)$$

$$z_{sj}\le B-\pi d_{sj}x_{sj}+M_1(1-x_{sj}),\forall j\in\mathcal{V} \quad (15)$$

$$g_{ij}\le B-z_{hi}+M_1(1-x_{ij})+M_1(1-x_{hi}),$$
$$i\in\mathcal{R},h\in\mathcal{V},j\in\mathcal{V} \quad (16)$$

$$x_{ij}\in\{1,0\},\forall i\in\mathcal{V},\forall j\in\mathcal{V};$$
$$y_i\in\{1,0\},\forall i\in\mathcal{R};u_i\ge 0,\forall i\in\mathcal{V};$$
$$z_{ij}\ge 0,\forall i\in\mathcal{V},\forall j\in\mathcal{V};$$
$$g_{ij}\ge 0,\forall i\in\mathcal{R},\forall j\in\mathcal{V};\tau_i\ge 0,\forall i\in\mathcal{V} \quad (17)$$

The objective function minimizes the sum of the total recharge cost and the total travel cost, expressed by the equation (1). Constraints (2) impose that each customer is visited exactly once, while (3) guarantee that each recharge station can be visited at most once. Flow conservation is given by constraints (4) and they ensure that for each vertex, the number of incoming arcs is equal to the number of outgoing arcs. Constraints (5) guarantee that the number of routes does not exceed the number of available vehicles. Constraints (6) and (7) model the number of vehicles that incoming and outgoing to/from the depot s and its copy t. Constraints (8) ensure that the capacity of each vehicle is not exceeded and constraint (9) ensures that the vehicle is empty at the depot. Constraints (10) define the values of variables y. In particular, y_i is forced to assume value equal to 1 if $g_{ij} > 0$, for some $j \in \mathcal{V}$. Constraints (11) and (12) link the arrival times to the routing variables for the customers and recharge stations, respectively. Time windows are given by constraints (13). Constraints (14) and (15) model the battery capacity, while constraints (16) ensure that the energy recharged at the available station does not exceed the battery capacity. Constraints (17) define the domain of the decision variables. It is worth observing that since we assumed the triangle inequality holds for the time, a vehicle visits a recharge station only if it need to recharge its battery. Hence, we can exclude from the model variable y. It follows that constraints (10) are removed and variable y_i is dropped from constraints (12).

2.1 Robust Formulation

In our setting, we suppose to have not precise information about the waiting time $w \in \mathbb{R}^{|\mathcal{R}|}$, and we assume to know a mean value $\bar{w}_i$ and a deviation $\hat{w}_i$ at each recharge station $i \in \mathcal{R}$. Hence, $\bar{w}_i - \delta_i^- \hat{w}_i \le w_i \le \bar{w}_i + \delta_i^+ \hat{w}_i, \forall i \in \mathcal{R}$, with $\delta_i^-, \delta_i^+ \in (0,1), \forall i \in \mathcal{R}$.

The waiting time w and the recharge time g play a key role in the delivery process. In fact, recharging operations are necessary to ensure the completion of the tours, avoiding the out-of-battery, however, the recharge time and the waiting time have to be carefully managed. Recharge and waiting time too long can delay the delivery to the customers causing the infeasibility due to the time window constraints.

Since all customers have to be served within their own time windows, the solution must remain feasible by satisfying the time windows requirements for any realization of w. For this purpose we consider a robust formulation of the problem where the problem is optimized under a worst case scenario. Hence, the waiting time is assumed to be $w_i \le \bar{w}_i + \delta_i^+ \hat{w}_i, \forall i \in \mathcal{R}$.

We model the realization of the uncertain parameter w with a polytope. In particular, we consider the budgeted uncertainty set from (Bertsimas and Sim, 2003), which has attracted the attention of many researchers to handle uncertainty in several contexts, see, e.g. (Poss, 2014; Lu and Gzara, 2015; Pessoa et al., 2015; Bruni et al., 2017; Di Puglia Pugliese et al., 2019). For the problem at hand, the budgeted uncertainty polytope is defined as $\mathcal{U}^\Gamma = \{w \in \mathbb{R}^{|\mathcal{R}|} : w_i = \bar{w}_i + \delta_i^+ \hat{w}_i, \delta_i^+ \in (0,1), \forall i \in \mathcal{R}, \sum_{i \in \mathcal{R}} \delta_i^+ \le \Gamma\}$.

In order to take into account the uncertain waiting time, we formulate a route-based two-stage robust model.

Let $p = \langle i_0 = s, i_1, \ldots, i_{n-1}, i_n = t \rangle$ be a route defined as an ordered sequence of nodes, starting and ending from/to node s and t, respectively. Let $N(p)$ be the set of customers served by route p, $R(p)$ be the set of recharge stations visited by route p, and $A(p)$ be the set of arcs $A(p) = \{(i_h, i_{h+1}), h = 0, \ldots, n-1\}$ traversed by route p. Let $\mathcal{P}$ be a feasible solution to the problem, i.e., $\mathcal{P}$ is a set of feasible routes p with respect to the capacity and the avoiding-out-of-battery constraints, such that $\bigcup_{p \in \mathcal{P}} N(p) = \mathcal{N}$ and $\bigcap_{p \in \mathcal{P}} N(p) = \emptyset$ to guarantee constraints (2), and $\bigcup_{p \in \mathcal{P}} R(p) \subseteq \mathcal{R}$ and $\bigcap_{p \in \mathcal{P}} R(p) = \emptyset$ to guarantee constraints (3). In other words, $\mathcal{P}$ represents a solution to the routing problem (1)–(9), (14)–(16) defined by the first stage variables x, u, z, g. Given a route p, $c(p)$ defines the traveling cost, whereas $g(p)$ represents the recharge cost.The traveling cost $c(\mathcal{P})$ and the recharge cost $g(\mathcal{P})$ of a set $\mathcal{P}$ are defined as the sum of the traveling costs and the sum of the recharge costs of all routes $p \in \mathcal{P}$, respectively. Let $\bar{\mathcal{P}}$ be the set of all first stage feasible solutions $\mathcal{P}$, i.e., $\bar{\mathcal{P}} = \{(x^{\mathcal{P}}, u^{\mathcal{P}}, z^{\mathcal{P}}, g^{\mathcal{P}}) : (1)-(9), (14)-(16)\}$. The two-stage robust formulation is given below:

$$\min \quad c(\mathcal{P}) + g(\mathcal{P}) \tag{18}$$

s.t.

$$\mathcal{P} \in \bar{\mathcal{P}}, \tag{19}$$

$$\tau(w, \mathcal{P}) \in \mathcal{T}(\mathcal{P}), \forall w \in \mathcal{U}^\Gamma, \tag{20}$$

$$\tau(w, \mathcal{P}) \le l, \forall w \in \mathcal{U}^\Gamma, \tag{21}$$

where $\tau(w, \mathcal{P})$ is the two-stage variable representing the arrival times at nodes starting from the depot at time zero and using the set of routes $\mathcal{P}$. Time $\tau(w, \mathcal{P})$ depends on the specific value of w since the vehicles may have to recharge their battery along their routes. $\mathcal{T}(\mathcal{P})$ is a polytope defining variable $\tau(w, \mathcal{P})$, described in what follows:

$$\mathcal{T}(\mathcal{P}) = \begin{cases} \tau_j(w,\mathcal{P}) \geq \\ \tau_i(w,\mathcal{P}) + (t_{ij}+s_i)x_{ij}^{\mathcal{P}} - M1(1-x_{ij}^{\mathcal{P}}), \\ \forall i \in \mathcal{N} \cup \{s\}, j \in \mathcal{V}^{\mathcal{P}} \setminus \{s\}, \\ \tau_j(w,\mathcal{P}) \geq \\ \tau_i(w,\mathcal{P}) + t_{ij}x_{ij}^{\mathcal{P}} + \frac{1}{\rho_i} g_{ij}^{\mathcal{P}} + w_i - M1(1-x_{ij}^{\mathcal{P}}), \\ \forall i \in \mathcal{R}^{\mathcal{P}}, j \in \mathcal{V}^{\mathcal{P}} \setminus \{s\}, \\ \tau_j(w,\mathcal{P}) \geq e_j, \forall j \in \mathcal{N}. \end{cases} \tag{22}$$

The set $\mathcal{V}^{\mathcal{P}} = \mathcal{N} \cup \mathcal{R}^{\mathcal{P}}$ is the set of nodes included in solution $\mathcal{P}$, i.e., it contains all nodes $i \in \mathcal{N}$ and possibly some nodes associated with recharge stations $i \in \mathcal{R}^{\mathcal{P}} = \bigcup_{p\in\mathcal{P}} R(p) \subseteq \mathcal{R}$.

Given a solution $\mathcal{P}$, the value of the arrival time $\tau(w,\mathcal{P})$ is adjusted based on the realization of w. The route-based two-stage model presents an infinite number of constraints (20)–(21) being $\mathcal{U}^\Gamma$ a polytope. However, one can readily see that, since $\tau(w,\mathcal{P})$ is defined via convex functions, actually linear (see (22)), we can consider the convex hull of $\mathcal{U}^\Gamma$, hence we can restrict our attention to extreme points of $\mathcal{U}^\Gamma$, i.e., $ext\left(\mathcal{U}^\Gamma\right) = \{w \in \mathbb{R}^{|\mathcal{R}|} : w_i = \bar{w}_i + \delta_i^+ \hat{w}_i, \delta_i^+ \in \{0,1\}, \forall i \in \mathcal{R}, \sum_{i\in\mathcal{R}} \delta_i^+ \leq \Gamma\}$ which contains a finite number of elements, named scenarios, since $\mathcal{U}^\Gamma$ is a polytope. However, even if there exist a finite number of scenarios, they grow exponentially with the dimension of the problem, i.e., $\left|ext\left(\mathcal{U}^\Gamma\right)\right| = \sum_{\gamma=1}^{\Gamma} \binom{|\mathcal{R}|}{\gamma}$. It can be computationally intractable enumerate all scenarios, thus a decomposition approach, where primal cuts are derived iteratively, is defined in the next section.

3 SOLUTION APPROACH

Model (18)–(21) induces a decomposition of the problem into two subproblems. The first one, referred to as the master problem, determines the routes $\mathcal{P} \in \bar{\mathcal{P}}$, i.e., (18) and (19); the second subproblem, referred to as separation problem, defines the schedule $\tau(w,\mathcal{P})$ and checks the feasibility, i.e., (20) and (21).

During the iterations of the decomposition approach, once a solution $\mathcal{P}^k$ is obtained at a given iteration k, the separation problem is solved. The latter, given the first stage solution, defines the second stage variables based on the realization of the uncertain parameter w. Hence, we obtain a scenario w^k where some recharge stations $i \in \mathcal{R}^{\mathcal{P}^k}$ experience a delay. If $\tau(w^k,\mathcal{P}^k) \leq l$, then the first and second stage variables design an optimal solution to (18)–(21). Otherwise, solution $\mathcal{P}^k$ must be excluded from $\bar{\mathcal{P}}$.

We define a row-and-column generation algorithm (R&C) (Agra et al., 2013) where constraints (20) and (21) are iteratively included. The idea is to relax all constraints (20) and (21) but those in a finite subset $\mathcal{U}^*$. Then, the variables and constraints that correspond to scenarios in $ext\left(\mathcal{U}^\Gamma\right) \setminus \mathcal{U}^*$ are added on the fly. The new scenario w^k to be included into $\mathcal{U}^*$ at a given iteration k is determined by solving the separation problem (20)–(21) with $\mathcal{T}(\mathcal{P}^k)$. In particular, the scheduling is determined by solving problem (20) and then the feasibility of constraints (21) is checked. In the case problem (20)–(21) is infeasible, then the scenario w^k is included into $\mathcal{U}^*$, otherwise, $\mathcal{P}^k$ is an optimal solution. This type of strategy can be viewed as a Benders decomposition approach with primal cuts (Bruni et al., 2018). Indeed, introducing a new scenario into $\mathcal{U}^*$ means to introduce into the master problem second-stage variables and constraints. The steps of the proposed R&C approach are depicted in Algorithm 1.

Algorithm 1: R&C.

$k=0$, $\mathcal{U}^* = \{w^k\}$, $w_i^k = \bar{w}_i, \forall i \in \mathcal{R}$, stop=false
while !stop **do**
  $k++$
  stop=true
  Solve problem (18)–(21) with $\mathcal{U}^*$ obtaining $\mathcal{P}^k$
  Solve problem (20) with $\mathcal{T}(\mathcal{P}^k)$ obtaining a scenario w^k
  if $\tau_j(w^k,\mathcal{P}^k) > l_j$ for some $j \in \mathcal{N}$ **then**
    $\mathcal{U}^* \leftarrow \mathcal{U}^* \cup \{w^k\}$
    stop=false
  end if
end while
$\mathcal{P}^k$ is an optimal solution.

3.1 Solving the Separation Problem

Solving the separation problem (20)–(21) at iteration k means to determine the realization of w considering $ext(\mathcal{U}^\Gamma)$, i.e., the scenario w^k and consequently $\tau(w^k,\mathcal{P}^k)$. We can rewrite constraints (20) in the form

$$\max_{w\in ext(\mathcal{U}^\Gamma)} \mathcal{T}(\mathcal{P}) \tag{23}$$

Dualizing (23) and letting $f^1 \in \mathbb{R}_+^{\mathcal{N}\cup\{s\}\times\mathcal{V}^{\mathcal{P}}}$, $f^2 \in \mathbb{R}_+^{\mathcal{R}^{\mathcal{P}}\times\mathcal{V}^{\mathcal{P}}}$, and $f^3 \in \mathbb{R}_+^{\mathcal{N}}$ be the dual variables associated with the three inequalities, we obtain the following problem

$$\max_{w\in ext(\mathcal{U}^\Gamma)} \sum_{i\in\mathcal{N}\cup\{s\}} \sum_{j\in\mathcal{V}^{\mathcal{P}}} (t_{ij}+s_i) f_{ij}^1 + \tag{24}$$

$$+ \sum_{i\in\mathcal{R}^{\mathcal{P}}} \sum_{j\in\mathcal{V}^{\mathcal{P}}} \left(t_{ij} + \frac{1}{\rho_i} g_{ij}^p + w_i \right) f_{ij}^2 + \tag{25}$$

$$+\sum_{i\in\mathcal{N}} e_i f_i^3, \quad (26)$$

subject to classical flow-conservation constraints defined over variable f^1 and f^2, and $f^3 \geq 0$. The details of the dualization are reported in the Appendix. We remark that $\mathcal{P}^k$ is a set of routes p. Without loss of generality, we can duplicate nodes s and t for each route $p \in P^k$. Hence, $\mathcal{T}(\mathcal{P}^k)$ can be decomposed into $|\mathcal{P}^k|$ disjoint sets $\mathcal{T}(p)$, each associated with a route $p \in \mathcal{P}^k$. It follows that variables f^1 and f^2 assume value equal to 1 for each $(i,j) \in A(p), p \in \mathcal{P}^k$ and zero otherwise. We can rewrite (23) in the form

$$\max_{w\in ext(\mathcal{U}^\Gamma)} \bigcup_{p\in\mathcal{P}^k} \mathcal{T}(p). \quad (27)$$

and the associated dual becomes:

$$\max_{w\in ext(\mathcal{U}^\Gamma)} \sum_{p\in\mathcal{P}^k} \sum_{(i,j)\in A(p): i\in N(p)\cup\{s\}} (t_{ij}+s_i) + \quad (28)$$

$$+\sum_{p\in\mathcal{P}^k} \sum_{(i,j)\in A(p): i\in R(p)} \left(t_{ij} + \frac{1}{\rho_i} g_{ij}^p + w_i\right) + \quad (29)$$

$$+\sum_{p\in\mathcal{P}^k} \sum_{i\in N(p)} e_i f_i^3, \quad (30)$$

with $f_i^3 \geq 0, \forall i \in \mathcal{N}$. We note that only the term (29) depends on the uncertain parameter w. Thus, we can rewrite (28)–(30) as

$$\sum_{p\in\mathcal{P}^k} \sum_{(i,j)\in A(p): i\in N(p)\cup\{s\}} (t_{ij}+s_i) + \quad (31)$$

$$+\max_{w\in ext(\mathcal{U}^\Gamma)} \sum_{p\in\mathcal{P}^k} \sum_{i\in R(p)} w_i +$$

$$+\sum_{p\in\mathcal{P}^k} \sum_{(i,j)\in A(p): i\in R(p)} \left(t_{ij} + \frac{1}{\rho_i} g_{ij}^p\right) + \quad (32)$$

$$+\sum_{p\in\mathcal{P}^k} \sum_{i\in N(p)} e_i f_i^3. \quad (33)$$

It follows that the scenario associated with solution $\mathcal{P}^k$ is determined by solving the following problem

$$\max_{w\in ext(\mathcal{U}^\Gamma)} \sum_{i\in\mathcal{R}^{\mathcal{P}^k}} w_i. \quad (34)$$

Problem (34) can be easily solved by determining the set $\mathcal{R}_{max}$ containing $\max\{|\mathcal{R}^{\mathcal{P}^k}|, \Gamma\}$ recharge stations associated with the highest value of $\bar{w} + \hat{w}$. Hence, the scenario w^k is determined by setting $w_i^k = \bar{w}_i + \hat{w}_i, \forall i \in \mathcal{R}_{max}$ and $w_i^k = \bar{w}_i, \forall i \in \mathcal{R}^{\mathcal{P}^k} \setminus \mathcal{R}_{max}$.

Given that each node $i \in \mathcal{N} \cup \mathcal{R}^{\mathcal{P}^k}$ belongs to a single route $p \in \mathcal{P}^k$, we can define $\tau_i(w^k, p), \forall i \in N(p) \cup R(p), p \in \mathcal{P}^k$ as the arrival time to node i in route p. Thus, once w^k is available, $\tau(w^k, \mathcal{P}^k) = \bigcup_{p\in\mathcal{P}^k} \tau(w^k, p)$ are computed as follows

$$\tau_s(w^k, p) = 0, \forall p \in P^k, \quad (35)$$

$$\tau_j(w^k, p) = \max\{e_j, \tau_i(w^k, p) + t_{ij} + s_i\},$$
$$\forall (i,j) \in A(p) : i \in N(p) \cup \{s\}, p \in \mathcal{P}^k, \quad (36)$$

$$\tau_j(w^k, p) = \max\{e_j, \tau_i(w^k, p) + t_{ij} +$$
$$+\frac{1}{\rho_i} g_{ij} + w_i^k\},$$
$$\forall (i,j) \in A(p) : i \in R(p), p \in \mathcal{P}^k. \quad (37)$$

4 COMPUTATIONAL RESULTS

The aim of this Section is to evaluate the effect of the uncertain waiting time on the optimal solution. The decomposition approach has been implemented in Java and the tests have been carrier out on an Intel(R) Core(TM) i7-4720HQ CPU, 2.60 GHz, 8GB RAM machine under Microsoft Windows 10 operating system. In the next Section we describe the considered instances, whereas we present the numerical results in Section 4.2.

4.1 Instances Generation

We consider the benchmark instances proposed in (Schneider et al., 2014) for the E-VRP with time windows with 5 and 10 customers, i.e., $\mathcal{N} \in \{5, 10\}$. These instances are created based on the benchmark instances for the VRP with time windows proposed by (Solomon, 1987) where recharge stations are included at random. For more details on the characteristics of the considered instances, the reader is referred to (Schneider et al., 2014). We modify such instances by introducing the parameters w and Γ. In particular, we generate the mean value of the waiting time w as $\bar{w}_i = \tilde{w} * (1+\alpha), \forall i \in \mathcal{R}$ and α chosen in the uniform interval $(0,1)$. Whereas, the pick value $\hat{w}$ is computed as $\hat{w}_i = \bar{w}_i * (1+\beta), \forall i \in \mathcal{R}$ with $\beta \in (0,1)$. Starting from the original instances, we generate a set of test problems by considering $\tilde{w} \in \{10, 15, 30, 45\}$ and $\beta \in \{0.5, 0.7, 0.9\}$. We consider several degree of the risk aversion of the decision maker, by letting $\Gamma = \lceil \rho * |\mathcal{R}| \rceil$ with $\rho \in \{0.25, 0.50, 0.75\}$.

4.2 Experimental Results

Before showing the numerical results, we analyze the characteristics of the generated instances starting from the benchmarks (Schneider et al., 2014).

4.2.1 Instances Analysis

The instances generated are not all feasible. Table 1 reports the percentage of feasible instances.

Table 1: Percentage of feasible instances.

		$\lvert\mathcal{N}\rvert = 5$			$\lvert\mathcal{N}\rvert = 10$		
		ρ			ρ		
$\tilde{w}$	β	0.25	0.50	0.75	0.25	0.50	0.75
10	0.5	100%	100%	100%	100%	100%	100%
	0.7	100%	100%	100%	100%	100%	92%
	0.9	100%	100%	100%	100%	92%	92%
15	0.5	100%	100%	100%	83%	83%	83%
	0.7	100%	100%	100%	83%	83%	83%
	0.9	100%	100%	100%	83%	83%	83%
30	0.5	100%	83%	83%	83%	83%	83%
	0.7	83%	83%	83%	83%	83%	83%
	0.9	83%	83%	83%	83%	83%	83%
45	0.5	75%	75%	75%	67%	67%	67%
	0.7	75%	75%	75%	67%	67%	67%
	0.9	75%	75%	75%	67%	67%	67%

The results reported in Table 1 highlight that adding the waiting time to the recharge stations make some benchmark instances infeasible. In particular, the higher the values of $\tilde{w}$, β, and ρ, the lower the percentage of feasible instances. In the sequel, we refer only to the instances that are feasible for each value of $\tilde{w}$, β, and ρ.

Referring to the risk aversion of the decision maker, Table 2 reports the average value of Γ and the average number of recharge stations (#RS) included in the solution by varying the value of ρ.

Table 2: Average value of Γ and average number of recharge stations included in the optimal solution at varying ρ.

	$\lvert\mathcal{N}\rvert = 5$			$\lvert\mathcal{N}\rvert = 10$		
	ρ			ρ		
	0.25	0.50	0.75	0.25	0.50	0.75
Γ	1.00	2.44	3.33	1.38	2.88	4.25
#RS	2.06	2.04	2.04	2.68	2.70	2.70

Analyzing the results reported in Table 2, we observe that Γ computed with $\rho = 0.75$ presents the same risk aversion of Γ computed by considering $\rho = 0.50$. Indeed, #RS is lower than Γ when $\rho = 0.50$. It means that, on average, in all recharge stations the vehicles experience a delay. Thus, increasing the value of ρ to 0.75 does not produce any modification on the realization of the uncertain parameter w. Hence, on average, the solutions obtained with Γ computed by setting $\rho = 0.50$ are the same as those determined by considering $\rho = 0.75$.

4.2.2 Numerical Results

The aim of this Section is twofold. Firstly, we analyze the transportation system behaviour by varying the values of the parameters $\tilde{w}$, β, and ρ, and we highlight some insight on how the uncertain waiting time influences the optimal solutions. Secondly, we analyze the characteristics of the robust solutions in terms of both cost and reliability in comparison with the deterministic ones.

Behavior of the System under Uncertainty. Table 3 reports the average results in term of objective function under column obj, number of recharge stations included in the solutions under column #RS, number of iteration under column #iter, and execution time (in seconds) under column time, at varying the values of the parameters $\tilde{w}$, β, and ρ.

Table 3: Average numerical results.

			obj	#RS	#iter	time
$\lvert\mathcal{N}\rvert = 5$	$\tilde{w}$	10	216.99	2.00	0.11	0.32
		15	220.44	2.06	0.12	0.36
		30	222.04	2.00	0.02	0.38
		45	231.56	2.11	0.07	0.35
	β	0.5	222.34	2.02	0.06	0.36
		0.7	222.82	2.06	0.08	0.34
		0.9	223.11	2.06	0.10	0.36
	ρ	0.25	221.21	2.06	0.03	0.33
		0.50	223.53	2.04	0.11	0.36
		0.75	223.53	2.04	0.11	0.37
$\lvert\mathcal{N}\rvert = 10$	$\tilde{w}$	10	302.91	2.63	0.00	7.86
		15	309.27	2.75	0.13	14.40
		30	311.63	2.63	0.00	25.63
		45	313.89	2.76	0.31	35.49
	β	0.5	309.29	2.70	0.10	20.62
		0.7	309.29	2.68	0.10	20.33
		0.9	309.69	2.70	0.11	21.59
	ρ	0.25	308.89	2.68	0.07	20.49
		0.50	309.69	2.70	0.13	21.18
		0.75	309.69	2.70	0.13	20.87

The results collected in Table 3 highlight that obj increases by increasing the value of all parameters, i.e., $\tilde{w}$, β, and ρ. This is an expected behaviour. Indeed, when the mean value of the waiting time $\bar{w}$ increases, the number of feasible solutions, with respect to the time windows, decreases. We observe a similar trend for variations of the pick value $\hat{w}$ expressed in terms of β. However, in this case, the variations of obj are less evident than that observed when the mean waiting time $\bar{w}$ changes. In addition, the higher the risk aversion of the decision maker, i.e., the higher the value of Γ, the higher the value of obj.

The number of recharge stations in the optimal solution remains almost the same by varying the parameters values (see columns #RS). In addition, a clearly trend is not observed. These results suggest that the uncertain parameter w does not influence the vehicles recharging. Indeed, they have to serve all customers avoiding the out-of-battery. Thus, the solutions adjust themselves in terms of sequence of visiting of both customers and recharge stations, in order to guarantee the satisfaction of the time windows.

As expected, the higher the deviation $\hat{w}$, the higher the #iter. Hence, a higher number of scenarios have to be included in the master problem in order to obtain a feasible solution. The same trend is observed at varying the value of ρ. This behaviour is due to the fact that the higher the risk aversion of the decision maker, the lower the solution space. Thus, since our approach starts with a relaxation of the polytope defining the realization of w, a higher number of scenarios have to be included in order to construct a feasible solution space.

The execution time is limited for the instances with $\mathcal{N} = 5$. For these problems, the solution approach requires the same computational time, by varying all the parameters values (see column time). For the instances with 10 customers, an increase in the computational overhead is observed by increasing the value of $\tilde{w}$. On the other hand, the execution time remains almost unchanged by varying the values of both β and ρ.

Effect of the Robustness. In order to show how the robust solution behaves under uncertain waiting time, we perform a sampling analysis. In particular, we build 1 hundred samples generated as described in what follows. The mean waiting time of sample s is calculate as $\bar{w}_i^s = \tilde{w} * (1 + r_1)$ with r_1 randomly chosen in the interval $(0,1)$, the deviation is calculated as $\hat{w}_i^s = \bar{w}_i^s * (1 + r_2\beta)$, with r_2 randomly chosen in the interval $(0,1)$. Hence, we set either $w_i^s = \bar{w}_i^s + \hat{w}_i^s$ or $w_i^s = \max\{0, \bar{w}_i^s - \hat{w}_i^s\}$, $\forall i \in \mathcal{R}$. It follows that $w_i^s \in (0, 2\tilde{w}(1+\beta)), \forall i \in \mathcal{R}$, for each sample s. Thus, we recalculate the scheduling considering all generated samples and we check the feasibility with respect to the time window constraints for each optimal solution, both robust and deterministic.

Table 4 reports the average percentage of infeasible solutions when considering the robust (see column %inf R) and the deterministic ones (see column %inf D). Column PoR reports the price of robustness calculated as $\frac{\text{obj}-\text{obj}_D}{\text{obj}_D} \times 100$, where obj is the cost of the robust solution, whereas obj_D is the cost of the deterministic one.

As expected, the robust solutions are more reliable than the deterministic ones. Indeed, the average percentage of infeasibility over the generated samples is 0.93% and 1.29% for the robust solutions against the 3.06% and the 3.39% observed for the deterministic ones, considering $\mathcal{N} = 5$ and $\mathcal{N} = 10$, respectively.

Table 4: Average results showing the effect of the robustness.

		$\lvert\mathcal{N}\rvert = 5$			$\lvert\mathcal{N}\rvert = 10$		
		PoR	%inf R	%inf D	PoR	%inf R	%inf D
$\tilde{w}$	10	1.22%	0.06%	2.78%	0.00%	0.36%	0.93%
	15	1.59%	1.58%	3.93%	0.73%	0.11%	1.19%
	30	0.02%	0.40%	0.59%	0.00%	0.22%	0.22%
	45	0.51%	1.69%	4.41%	0.73%	4.46%	9.56%
β	0.5	0.61%	0.73%	1.72%	0.32%	0.70%	2.00%
	0.7	0.82%	0.84%	2.81%	0.32%	1.39%	2.89%
	0.9	0.96%	1.22%	4.25%	0.45%	1.77%	4.03%
ρ	0.25	0.10%	2.28%	2.93%	0.19%	2.79%	2.97%
	0.50	1.15%	0.26%	2.93%	0.45%	0.54%	2.97%
	0.75	1.15%	0.26%	2.93%	0.45%	0.54%	2.97%
AVG		0.80%	0.93%	3.06%	0.37%	1.29%	3.39%

In addition, the price of robustness is quite limited with an average cost for the robust solutions 0.80% and 0.37% higher than that of the deterministic ones, on average, for the instances with 5 and 10 customers, respectively. As expected, the higher the risk aversion of the decision maker, the higher the PoR and the lower the %inf R for all the considered instances.

5 CONCLUSIONS

We have studied a vehicle routing problem variants in which a fleet of electrical vehicles must serve a set of customers within their own time windows. Partial battery recharging of the vehicles to any available recharge station is allowed. We consider a particular case in which a charger can be busy when a driver visits a station, thus he must wait. We address the problem under the robust optimization framework, by modelling the realization of the uncertain parameter with the budget of uncertainty polytope. We propose a robust formulation and a row-and-column generation method, based on a two-stage reformulation, to solve our problem.

The proposed approach is tested on benchmark instances for the E-VRP with time windows. We consider only small-size instances. The collected computational results highlight that the robust solutions are more reliable than the deterministic ones with a limited increase of the total cost. Solving to optimality large-size instances is not viable. We remark that the proposed decomposition approach solves instances of the E-VRP with time windows and partial recharges, at each iteration. The corresponding MILP cannot be solved with commercial solver to optimality, within reasonable computational time for larger instances. Future work should consider heuristic approaches for solving larger problems. It is possible to derive a heuristic based on the proposed decomposition approach by imposing a time limit to the solver for solving the master problems. Hence, a feasible rather than an optimal solution is considered, at each iteration.

It is worth noting that we consider a simplified discharging model of the battery, in which the discharging is a linear function depending on the distance travelled. Actually, the discharge is influenced by several factors, such as the speed, the load of the vehicle, the gradient, and so on. Hence, the discharging function is non-linear. As future work, investigating how a more realistic discharging function influences the transportation system under uncertain waiting time, should be worthy.

An interesting version of the problem is to consider uncertain discharge rate. This assumption allows to avoid to take into account complicating nonlinear discharging function and to prevent energy disruption.

REFERENCES

Agra, A., Christiansen, M., Figueiredo, R., Hvattum, L. M., Poss, M., and Requejo, C. (2013). The robust vehicle routing problem with time windows. *Computers & operations research*, 40(3):856–866.

Basso, R., Kulcsár, B., Egardt, B., Lindroth, P., and Sanchez-Diaz, I. (2019). Energy consumption estimation integrated into the electric vehicle routing problem. *Transportation Research Part D: Transport and Environment*, 69:141–167.

Bertsimas, D. and Sim, M. (2003). Robust discrete optimization and network flows. *Mathematical programming*, 98(1-3):49–71.

Bruni, M., Di Puglia Pugliese, L., Beraldi, P., and Guerriero, F. (2017). An adjustable robust optimization model for the resource-constrained project scheduling problem with uncertain activity durations. *Omega*, 71:66–84.

Bruni, M., Di Puglia Pugliese, L., Beraldi, P., and Guerriero, F. (2018). A computational study of exact approaches for the adjustable robust resource-constrained project scheduling problem. *Computers & Operations Research*, 99:178–190.

Desaulniers, G., Errico, F., Irnich, S., and Schneider, M. (2016). Exact algorithms for electric vehicle-routing problems with time windows. *Operations Research*, 64:1388–1405.

Di Puglia Pugliese, L., Guerriero, F., and Poss, M. (2019). The resource constrained shortest path problem with uncertain data: A robust formulation and optimal solution approach. *Computers & Operations Research*, 107:140–155.

Ding, N., Battay, R., and Kwon, C. (2015). Conflict-free electric vehicle routing problem with capacitated charging stations and partial recharge. https://www.chkwon.net/papers.

Erdoğan, S. and Miller-Hooks, E. (2012). A green vehicle routing problem. *Transportation Research Part E: Logistics and Trasportation Review*, 48(1):100–114.

Felipe, A., Ortuño, M. T., Righini, G., and Tirado, G. (2014). A heuristic approach for the green vehicle routing problem with multiple technologies and partial recharges. *Transportation Research Part E*, 71:111–128.

Froger, A., Mendoza, J., Jabali, O., and Laporte, G. (2019). Improved formulations and algorithmic components for the electric vehicle routing problem with nonlinear charging functions. *Computers & Operations Research*, 104:256–294.

Goeke, D. (2019). Granular tabu search for the pickup and delivery problem with time windows and electric vehicles. *European Journal of Operational Research*, 278:821–836.

Gonçalves, F., Cardoso, S. R., Relvas, S., and Barbosa-Póvoa, A. P. F. D. (2011). Optimization of a distribution network using electric vehicles: A vrp problem. 15ˆCongresso Nacional da Associação Portuguesa de Investigação Operacional.

Hiermann, G., Hartl, R. F. Puchinger, J., and Vidal, T. (2019). Routing a mix of conventional, plug-in hybrid, and electric vehicles. *European Journal of Operational Research*, 272:235–248.

Hiermann, G., Puchinger, J., Ropke, S., and Hartl, R. F. (2016). The electric fleet size and mix vehicle routing problem with time windows and recharging stations. *European Journal of Operational Research*, 252:995–1018.

Joo, H. and Lim, Y. (2018). Ant colony optimized routing strategy for electric vehicles. *Journal of Advanced Transportation*, 2018:9.

Keskin, M. and Çatay, B. (2016). Partial recharge strategies for the electric vehicle routing problem with time windows. *Transportation Research Part C*, 65:111–127.

Keskin, M., Çatay, B., and Laporte, G. (2021). A simulation-based heuristic for the electric vehicle routing problem with time windows and stochastic waiting times at recharging stations. *Computers and Operations Research*, 125:105060.

Keskin, M., Laporte, G., and Çatay, B. (2019). Electric vehicle routing problem with time-dependent waiting times at recharging stations. *Computers & Operations Research*, 107:77–94.

Löffler, M., Desaulniers, G., Irnich, S., and Schneider, M. (2020). Routing electric vehicles with a single recharge per route. *Networks*, 76(2):187–205.

Lin, J., Zhou, W., and Wolfson, O. (2009). Electric vehicle routing problem. In *Transportation Research Procedia*, pages 508–521, Tenerife, Canary Islands (Spain). The 9th International Conference on City Logistics.

Lu, D. and Gzara, F. (2015). The robust crew pairing problem: model and solution methodology. *Journal of Global Optimization*, 62(1):29–54.

Macrina, G., Di Puglia Pugliese, L., Guerriero, F., and Laporte, G. (2019a). The green mixed fleet vehicle routing problem with partial battery recharging and time windows. *Computers & Operations Research*, 101:183–199.

Macrina, G., Laporte, G., Guerriero, F., and Di Puglia Pugliese, L. (2019b). An energy-efficient green-vehicle routing problem with mixed vehicle fleet, partial battery recharging and time windows. *European Journal of Operational Research*, 276(3):971–982.

Montoya, A., Guéret, C., Mendoza, J. E., and Villegas, J. G. (2017). The electric vehicle routing problem with non-linear charging function. *Trasportation Research Part B*, 103:87–110.

Paz, J. C., Granada-Echeverri, M., and Escobar, J. W. (2018). The multi-depot electric vehicle location routing problem with time windows. *International Journal of Industrial Engineering Computations*, 9:123–136.

Pessoa, A. A., Di Puglia Pugliese, L., Guerriero, F., and Poss, M. (2015). Robust constrained shortest path problems under budgeted uncertainty. *Networks*, 66(2):98–111.

Poss, M. (2014). Robust combinatorial optimization with variable cost uncertainty. *European Journal of Operational Research*, 237(3):836–845.

Sassi, O., Cherif, W. R., and Oulamara, A. (2014). Vehicle routing problem with mixed feet of conventional and heterogenous electric vehicles and time dependent charging costs. Technical report. https://hal.archives-ouvertes.fr/hal-01083966.

Schneider, M., Stenger, A., and Goeke, D. (2014). The electric vehicle routing problem with time windows and recharging stations. *Transportation Science*, 48(4):500–520.

Solomon, M. M. (1987). Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations Research*, 35(2):254–265.

Yang, J. and Sun, H. (2015). Battery swap station location-routing problem with capacitated electric vehicles. *Computers & Operations Research*, 55:217–232.

Zhang, S., Chen, M., and Zhang, W. (2019). A novel location-routing problem in electric vehicle transportation with stochastic demands. *Journal of Cleaner Production*, 221:567–581.

APPENDIX

The problem defined by constraints (23) can be recast as follows:

$$\min \ 0 \tag{38}$$

$$s.t.$$

$$\max_{w \in ext(\mathcal{U}^\Gamma)} \mathcal{T}(\mathcal{P}). \tag{39}$$

The polytope $\mathcal{T}(\mathcal{P})$ can be rewritten as:

$$\tau_j(w,\mathcal{P}) - \tau_i(w,\mathcal{P}) \geq (t_{ij}+s_i)x_{ij}^{\mathcal{P}} - M1(1-x_{ij}^{\mathcal{P}}),$$

$$\forall i \in \mathcal{N} \cup \{s\}, j \in \mathcal{V}^{\mathcal{P}} \setminus \{s\}, \tag{40}$$

$$\tau_j(w,\mathcal{P}) - \tau_i(w,\mathcal{P}) \geq t_{ij}x_{ij}^{\mathcal{P}} + \frac{1}{\rho_i} g_{ij}^{\mathcal{P}} + w_i +$$

$$-M1(1-x_{ij}^{\mathcal{P}}), \forall i \in \mathcal{R}^{\mathcal{P}}, j \in \mathcal{V}^{\mathcal{P}} \setminus \{s\}, \tag{41}$$

$$\tau_j(w,\mathcal{P}) \geq e_j, \forall j \in \mathcal{N}. \tag{42}$$

We remark that $x^{\mathcal{P}}$ and $g^{\mathcal{P}}$ represent the first stage solution. In particular, $x_{ij}^{\mathcal{P}}$ assume value equal to 1 for each arc (i,j) included in some route, i.e., $x_{ij}^{\mathcal{P}} = 1, \forall (i,j) \in A(\mathcal{P})$ where $A(\mathcal{P}) = \{(i,j) \in \bigcup_{p \in \mathcal{P}} A(p)\}$ is the set of all arcs included in the solution $\mathcal{P}$.

Variables $g_{ij}^{\mathcal{P}} > 0$ for each $i \in \mathcal{R}^{\mathcal{P}}$ and $j \in \mathcal{V}^{\mathcal{P}}$ such that $(i,j) \in A(\mathcal{P})$ since a recharge station is visited only if the vehicle recharges the battery. We can consider two disjoint subsets, i.e., $A_1(\mathcal{P}) = \{(i,j) \in A(\mathcal{P}) : i \in \mathcal{N} \cup \{s\}\}$ and $A_2(\mathcal{P}) = \{(i,j) \in A(\mathcal{P}) : i \in \mathcal{R}^{\mathcal{P}}\}$. Hence, constraints (40)–(42) become

$$\tau_j(w,\mathcal{P}) - \tau_i(w,\mathcal{P}) \geq t_{ij} + s_i,$$

$$\forall (i,j) \in A_1(\mathcal{P}), \tag{43}$$

$$\tau_j(w,\mathcal{P}) - \tau_i(w,\mathcal{P}) \geq t_{ij} + \frac{1}{\rho_i} g_{ij}^{\mathcal{P}} + w_i,$$

$$\forall (i,j) \in A_2(\mathcal{P}), \tag{44}$$

$$\tau_j(w,\mathcal{P}) \geq e_j, \forall j \in \mathcal{N}. \tag{45}$$

Let $\mathcal{A}_1^{|A_1(\mathcal{P})| \times |\mathcal{V}^{\mathcal{P}}|}$ and $\mathcal{A}_2^{|A_2(\mathcal{P})| \times |\mathcal{V}^{\mathcal{P}}|}$ be the coefficient matrix of constraints (43) and (44), respectively, and $\mathcal{A}_3^{|\mathcal{N}| \times |\mathcal{N}|}$ be the coefficient matrix of constraints (45). Let t_1, t_2, and e be the right-hand-side terms of constraints (43), (44), and (45), respectively. The dual of problem (38), (39) is the following:

$$\min \max_{w \in ext(\mathcal{U}^\Gamma)} t_1 f_1 + t_2 f_2 + e f_3 \tag{46}$$

$$s.t.$$

$$\mathcal{A}_1^T f_1 = 0, \tag{47}$$

$$\mathcal{A}_2^T f_2 = 0, \tag{48}$$

$$\mathcal{A}_3^T f_3 = 0, \tag{49}$$

$$f_1, f_2, f_3 \geq 0, \tag{50}$$

where $f_1 \in \mathbb{R}_+^{|A_1(\mathcal{P})|}, f_2 \in \mathbb{R}_+^{|A_2(\mathcal{P})|}$ and $f_3 \in \mathbb{R}_+^{|\mathcal{N}|}$ are the variables dual to constraints (43), (44), and (45), respectively. We note that $\mathcal{A}^T = \mathcal{A}_1^T \cup \mathcal{A}_2^T$ represents the incident matrix related to the first stage routing solution. Hence, $f = f_1 \cup f_2$ are flow variables and constraints (47) and (48) represent the flow-conservation constraints. The constraints $\mathcal{A}^T f = 0$ is a circulation, indeed, all nodes are of transit type. This because each route starts and ends at the same depot.