Microfacet Distribution Function: To Change or Not to Change, That Is the Question

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Abstract: In computer graphics and multimedia, bidirectional reflectance distribution function (BRDF) is commonly used for modeling the reflection and refraction of light. In this study, one of the important components of the reflectance models, namely, the microfacet distribution function (MDF) has been considered. The analytical MDFs allow only approximating the real distribution of the surface. Modern graphic software gives the opportunity to select the MDF that fits the real reflection in the best way. The question arises: can we really replace one MDF with another in this situation? And if it is possible, how to convert parameters from one function to the other. The problem is topical, important and practical—for all users of graphic software. In this article, various examples of MDF have been discussed. After RMSE analysis the mathematical dependencies that allow for the exchange of one MDF with the other have been proposed. In this study, consequences of applying different MDFs have been also discussed and comparison of the visual effect has been presented.

1 INTRODUCTION

For many years, one of the most difficult tasks in computer graphics is modeling the reflection of light in the most consistent and realistic manner. The behavior of the light—object interaction depends on the material and surface properties of the object. Such phenomena are described in computer graphics by the BRDF (Dorsey et al., 2008). One of the important components of the reflectance models is the microfacet distribution function (MDF) (Hall, 1989). MDF is used in the category of BRDF whose form arises from the assumption that the surface has a microstructural character. Many interesting comparative studies about BRDF have been published (Hall, 1989, Kurt and Edwards, 2009, Ngan et al., 2004, Ngan et al., 2005, Rusinkiewicz, 1997, Schlick, 1994b). It seems that the topic is closed; however, recent publications show that the problem is still valid and worthy of further research. The anisotropic BRDF has been described in 1992 (Ward, 1992). In 2010, a new anisotropic BRDF was proposed with a discussion on the MDF (Kurt et al., 2010). In 2015, a new iridescent rendering method was proposed (Kang et al., 2015), based on modification (multi-peak) of anisotropic MDF. However, this method (and MDF) was designed to special kind of surfaces/reflection (different wavelengths reflection, diffraction effects) and cannot be compared to general purpose MDFs.

Modern graphic programs allow modeling the reflective properties of the material's surface in the best way with the appropriate BRDF. Advanced graphic programs allow not only changing BRDF but also modifying their components. It allows selecting MDF to the expectations related to the real reflection. On the other hand, it is known research on MDF shape to match the analytical character BRDF to real measurements of reflection. The authors of the work (Bagher et al., 2012) adjusted the form of MDF for Cook-Torrance BRDF, considering real examples of light reflection. The question arises: can we really replace one MDF with the other in this situation and what will be the consequences. There are many publications describing various BRDFs. Unfortunately, comparative analyses of MDF have

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been rarely reported in literature. There is only one book in which the properties of MDF are widely considered and several MDFs have been compared (Hall, 1989). The visual comparison on some MDFs can be found in publications from last years (Heitz 2014, Ribardière et al., 2019).

This article is aimed at analyzing the properties of various analytical MDFs. On the one hand, this analysis will allow for the conversion of the parameters value between MDF to get the most similar graphics. On the other hand, the analysis will allow to reveal the differences between MDF and will show the consequences of such a change. Today, a big challenge in practical applications is the attempt of fitting the MDF model to the real, measured (captured) reflection conditions of the surface (Bringier et al., 2020). And for this, knowledge of the properties of various known MDFs is needed.

2 MATERIAL AND METHODS

2.1 BRDF

A description of the BRDF itself does not seem to be necessary, because this function is known to all those dealing with computer graphics. However, a detailed description of basic characteristics and parameters is necessary for consistency with the description later in this article.

The BRDF $f(\vec{L},\vec{V})$, introduced by Nicodemus (Nicodemus, 1970, Nicodemus et al., 1977), can be defined in the form of a simple equation—the quotient of $dL(\vec{V})$ and $dE(\vec{L})$ (the outgoing radiance and the incoming irradiance, respectively).

Many different models of reflection exist because there is no universal mathematical description of light reflection for any surface and material. The best effects are obtained using the reflection model created based on the appropriate physical theory regarding the smoothness (roughness) of the surface (Pharr et al., 2016). In this case, the BRDF's description has the general form (1) with a set of specular components:

$$f(\vec{L},\vec{V}) = \frac{k \cdot F(\theta) \cdot G \cdot D}{M} \quad (1)$$

where $F(\theta)$ is the Fresnel factor of reflectivity (depends on the incidence angle θ); G is the geometric attenuation; D represents the MDF; M is the factor that describes the angle reflection properties, especially for surface of materials with good properties of specular reflection (e.g. metals) (Neumann et al., 1999); and k is the factor which allows fulfilling the energy conservation law.

There are many well-known BRDFs with form similar to (1): BRDFs, Cook-Torrance (Cook and Torrance, 1981), He (He, 1994, He et al., 1991, He et al., 1992), Embrechts (Embrechts, 1995, Embrechts, 1999), and Ashikhmin-Shirley (Ashikhmin and Shirley, 2000). A short analysis of the role of specular component from (1) can be found in (Mac Manus, 2009).

The crucial element of the discussed BRDF forms is the MDF represented as D in equation (1). Several different forms or approximations of the distribution function exist. It is worth analyzing the properties; similarities, and differences of these functions and their influence on the defined picture and the computational process.

2.2 MDF: Overview and Properties

To be able to replace one MDF with the other, it is necessary to analyze their properties. The MDF characterizes smoothness/roughness of the material's surface, and it determines the directional relation of reflection. The function is also called slope distribution (Schlick, 1994b) or roughness function (Hall, 1989). Sometimes it exists as normal distribution function (Akenine-Möller et al., 2008, Dong et al., 2015, Ribardière et al., 2019) as well. The first analysis of different MDFs can be found in Blinn's famous study (Blinn, 1977). The book (Hall, 1989) contains more information about MDF, some basic comparison of the functions, and C-source code examples. The MDF was also analyzed as a BRDF component in (Schlick, 1994c). It is noteworthy that the MDF can also be used in the description of the refraction phenomenon (Walter et al., 2007).

The MDF is most often defined as a function of the β angle between normal vector $\vec{N}$ and $\vec{H}$ vector. Where $\vec{H}$ vector bisects the angle between vectors to observer and to source of light. The energy conservation law requires that D meets the normalization condition. There exist many descriptions of MDF normalization (Akenine-Möller et al., 2008, Pharr et al., 2016, Schlick, 1994c) depending on the assumed BRDF formula. For isotropic behavior and for MDF in formula dependent on the β angle, the normalization equation (Schlick, 1994c) is as follows (2):

$$\int_0^{\pi/2} D(\beta) \cdot 2 \cdot cos\beta \cdot sin\beta \cdot d\beta = 1 \quad (2)$$

The authors describing MDFs did not always care about meeting the normalization condition (2). In such a case, the calculations do not fulfill energy conservation law, and it was corrected within an independent study later. This was the case of the Phong reflection model—the expression considers an alteration (Lafortune and Willems, 1994, Lewis, 1994) to the original Phong formula.

In this study, the unit form (marked as $\overline{D}$) is considered. This facilitates comparison of different functions of distribution. Unit form means $\overline{D} = D/max(D)$, in most cases of distribution function, the maximum value occurs for $\beta=0$. It means $\overline{D} = D/D(0)$ (Table 1).

The MDF, as the special defined function, has been introduced in a book (Beckmann and Spizzichino, 1963). The authors have provided a theoretical analysis of the electromagnetic waves' reflection from random rough surface and proposed a statistical description of the surface roughness. They assumed the polyhedral character of the surface roughness. Authors justified the need to use the MDF and provided a comprehensive analysis of the proposed Beckmann formula (Table 1) as a function of the β angle. $m_B \in (0,1)$ is the parameter that characterizes the surface smoothness and reflectivity—the smaller the value is, the closer the reflection is to the perfect directional one.

The first (historically) microfacet distribution function used in the BRDF was the Gauss expression (Torrance and Sparrow, 1967). In this formula (Table 1) C_{TS} describes smoothness/roughness of the material—it determines the distribution of the faces' slope about the mean-surface plane in a polygonal

Table 1: Discussed microfacet distribution functions in normalized and unit form.

Author /used by/	Normalized form	Unit form
Beckmann Spizzichino (Beckmann and Spizzichino, 1963) /Cook Torrance (Cook and Torrance, 1981)/ /Embrechts (Embrechts, 1995, Embrechts, 1999)/	$D_B = \frac{1}{\pi \cdot m_B^2 \cdot cos^4\beta} \cdot e^{-\left[\frac{tan^2\beta}{m_B^2}\right]}$	$\overline{D_B} = \frac{1}{cos^4\beta} \cdot e^{-\left[\frac{tan^2\beta}{m_B^2}\right]}$
Gauss /Torrance Sparrow (Torrance and Sparrow, 1967)/	$D_G = \frac{2 \cdot ln(cos\beta_{half}) - ln2}{2 \cdot \pi \cdot ln(cos\beta_{half})} \cdot e^{-\left[\frac{\beta^2}{m_G^2}\right]}$ and $C_{TS} = 1/m_G$, $\beta_{half} = m_G \cdot \sqrt{ln(2)}$	$\overline{D_G} = e^{-\left[\frac{\beta^2}{m_G^2}\right]}$
Trowbridge Reitz (Trowbridge and Reitz, 1975) /GGX model (Burley, 2012)/	$D_{TR} = \frac{1}{\pi}\left[\frac{C_{TR}}{cos^2\beta \cdot (C_{TR}^2 - 1) + 1}\right]^2$	$\overline{D_{TR}} = \left[\frac{C_{TR}^2}{cos^2\beta \cdot (C_{TR}^2 - 1) + 1}\right]^2$
GTR model (Burley, 2012)	$D_{GTR} = \frac{1}{\pi}\frac{C_{TR}^2}{(cos^2\beta \cdot (C_{TR}^2 - 1) + 1)^\gamma}$ Practically in applications 1.5<γ<3, for γ>10 D_{GTR} is very similar to analysis in (Ribardière et al., 2017)	$\overline{D_{GTR}} = \left[\frac{C_{TR}^2}{cos^2\beta \cdot (C_{TR}^2 - 1) + 1}\right]^\gamma$
Blinn/Phong (Blinn, 1977, Lewis, 1994, Phong, 1975), /Strauss (Strauss, 1990)/ Anisotropic version: Ashikhmin-Shirley (Ashikhmin and Shirley, 2000)/modified in (Pharr et al., 2016)	$D_{BPh} = \frac{N+2}{2\pi} cos^N\beta \cdot$ $D_{AS} = \frac{\sqrt{(N_x + 2) \cdot (N_y + 2)}}{2\pi} cos^p\beta \cdot$ where $p = N_x \cdot cos^2\phi + N_y \cdot sin^2\phi$ ϕ – the angle of anisotropy	$\overline{D_{BPh}} = cos^N\beta$
Schlick (Schlick, 1994c)	$D_{Sch} = \frac{m_B^3 \cdot x}{\pi \cdot cos\beta \cdot (m_B \cdot x^2 - x^2 + m_B^2)^2}$ where $x = cos\beta + m_B - 1$	$\overline{D_{Sch}} = \frac{m_B^5 \cdot x}{cos\beta \cdot (m_B \cdot x^2 - x^2 + m_B^2)^2}$ where $x = cos\beta + m_B - 1$
Sawicki (Sawicki,2006)	$D_{DS} = \frac{96 \cdot (3 + N_{DS}) \cdot cos\beta}{\pi \cdot ((1 - N_{DS}) \cdot cos\beta + N_{DS} + 3)^4}$	$\overline{D_{DS}} = \frac{256 \cdot cos\beta}{((1 - N_{DS}) \cdot cos\beta + N_{DS} + 3)^4}$

model of the surface. According to the similarity between the Gauss and Beckmann formulas, the C_{TS} parameter in the Gauss formula is expressed as $1/m_G$ (Table 1).

Blinn (Blinn, 1977) discussed the usage of Gauss distribution in a similar form. MDF in the form of the Gauss function also appears in contemporary literature (Ashikhmin et al., 2000). Cook and Torrance in their model of BRDF used the Beckmann distribution function. There is also a documented possibility (Lengyel, 2002) of adding anisotropic feature of reflection into Beckmann distribution by modifying this equation. Schlick (Schlick, 1994c) proposed the rational approximation (Table 1) as the answer to the computational complexity of Beckmann distribution. The apparent computational simplicity of the Shlick equation (Table 1) is connected with an additional condition: the distribution is defined only for $\cos\beta \in [1 - m_B, 1]$. An attempt to use this formula for a full range of the β angle leads to major errors (Sawicki, 2006). In some cases, it makes the calculations significantly difficult. Schlick also proposed (Schlick, 1994b) another MDF equation that approximated the Beckmann distribution. However, a significant difference of shape to original distribution resulted in such proposition never being used. Therefore, in this study, the term "Schlick distribution" denotes the original equation from (Schlick, 1994c).

The Torrance-Sparrow and Cook-Torrance models and Schlick approximation were developed with the assumption of the polygonal character of the surface smoothness (roughness). In this way, the MDF specifies the distribution of the microfacets of the material. The distribution proposed by in (Trowbridge and Reitz, 1975) (Table 1), is the next solution that is worth taking into consideration. It also represents a physically well-grounded model but with a different assumption that the surface has been built by microelements (micromirrors) with an elliptic shape. The basic advantage of this model is its computational simplicity. C_{TR} (Table 1) describes the roughness of surface with values ranging from 0 for ideal (mirror) smooth surfaces to 1 for the perfectly diffuse ones.

The Trowbridge-Reitz MDF was originally given in the unit form. Therefore, the proper normalization factor was calculated, and in this study, the normalized form is probably presented for the first time.

He's description (the so-called HTSG model) (He, 1994, He et al., 1991, He et al., 1992) belongs to the most complex BRDF models. Unfortunately, the complicated form of the He model's description, but first the need to solve a nonlinear equation during calculations, does not allow for effective use of this model and the MDF function in practice—even with the approximation which was suggested later (He et al., 1992).

The Phong function (Phong, 1975) especially in the Blinn version (Blinn, 1977), (modified by Lewis (Lewis, 1994) and verified in (Lafortune and Willems, 1994, Pharr et al., 2016)) is also treated as a distribution function (in the Blinn/Phong formula—Table 1. The $N \geq 1$ parameter characterizes the smoothness of the surface—the greater the value, the closer is the reflection to the perfect directional one (ideal mirror). The $\cos^N\beta$ function is used in many other MDF or BRDF descriptions, for example, in the Ashikhmin-Shirley model (Ashikhmin and Shirley, 2000), Lafortune (Lafortune and Willems, 1994), and Strauss model (Strauss, 1990). Table 1 presents the anisotropic Ashikhmin-Shirley MDF but in version that is improved in (Pharr et al., 2016). Phong proposed a very simple formula; the disadvantage of this solution is the inability to determine the analytical integral. There have been many attempts to improve the computational complexity of the $\cos^N\beta$ function by proper approximation (Bishop and Weimer, 1986, Kuijk and Blake, 1989, Poulin and Fournier 1990, Schlick, 1994a). In practice, approximations or decomposition into the Chebyshev series are used. Neither solution is computationally optimal. It should also be pointed out that the classical MDF function is a statistical function; however, the Blinn/Phong function simply describes the shape of specular reflection. To underline it, the parameter of the first function is sometimes called the Gaussian Roughness in the literature, and the parameter of the second function is called the Phong Specular Power (Ward, 1996). But due to a very close character of both functions, they are generally treated in the same way (Hall, 1989, Ward, 1996).

Another rational/polynomial model was proposed in 2006 (Sawicki, 2006) (Table 1). It is based on the modified Padé approximation. The N_{DS} coefficient has an analogous sense as N in the Phong function. However, it is worth treating this distribution as an entirely independent one; it means that the N_{DS} coefficient should be calculated or introduced independently and not as a value simply taken from the Blinn/Phong function.

It would seem, that the Beckmann MDF (with its various approximations) is completely enough for modern computer graphics. However, experiments have shown that this model does not provide realistic enough results in some practical applications (Burley, 2012, Walter et al., 2007). What is surprising is that

Trowbridge-Reitz MDF model (Trowbridge and Reitz, 1975) turned out to be the most useful one. In 2007, the authors of (Walter et al., 2007) described GGX—the new MDF. They concluded that "we developed a new microfacet distribution function ..." It is an interesting paper; however, the proposed formula of GGX MDF is mathematically identical to the earlier Trowbridge-Reitz model. However, the Trowbridge-Reitz publication is not cited in (Walter et al., 2007). This model has found more followers. In (Ribardière et al., 2017), the authors used both MDFs (Beckmann + GGX/Trowbridge-Reitz) in reflection description. The authors of (Chen et al., 2017) used GGX for analysis of reflection from highly specular surfaces. In (Barla et al., 2018) BRDF for hazy gloss reflection was built. In another study (Burley, 2012), we can find the generalization of Trowbridge-Reitz (GTR) model (Table 1), in addition to anisotropic properties. Today, many applications related to the movie and game development use the GTX/GTR model (Burley, 2012). The most spectacular examples confirming the presented tendency come from the simulation of reflection from the surface of the metal. The authors of (Burley, 2012, Dong et al., 2015, Heitz, 2014) have noticed that in some situation of high roughness parameter, Beckmann's distribution causes the surface to darken. GGX allows compensating these effects thanks to the "longer tail".

The interesting version of MDF has been described in (Holzschuch and Pacanowski, 2017). The authors proposed two-scale microfacet reflectance model for complex surface (with micro-geometry and nano-geometry). It can be used also with multi-layer materials.

2.3 Conversion of the Distribution Function

Each MDF analyzed in this article is an entirely independent attempt to describe the phenomenon of reflection. However, the Beckmann function is used most often and in many studies it is treated as a reference function (Sawicki, 2006, Schlick, 1994b, Schlick, 1994c). For this, and only this reason, this assumption was also adopted in this study. In this way, in this study, a comparison between the different MDFs was made using the Beckmann function as a reference one. To compare the shape of different functions of distribution, the unit form (marked as $\overline{D}$) has been considered in this study (Table 1). Functions with practical importance were selected for the comparison—functions and approximations most often appearing in graphical applications.

It can be shown that the shapes of all considered functions are very similar. Blinn analyzed the properties of various MDF formulas and the influence of changes in the coefficients (Blinn, 1977). To obtain the correspondence of proper coefficients, he selected the case when the unit functions fall to a value of 1/2 at the same angle. Here, β_{half} denotes such an angle. Blinn introduced formulas to calculate proper coefficients. According to the notation used in this article, following are the respective formulas (3), (4), (5) for the Blinn/Phong, Gauss, and Trowbridge-Reitz MDFs:

$$N = \frac{-ln\,(2)}{ln\,(cos\beta_{half})} \tag{3}$$

$$m_G = \frac{1}{C_{TS}} = \frac{\beta_{half}}{\sqrt{ln(2)}} \tag{4}$$

$$C_{TR} = \sqrt{\frac{cos^2\beta_{half} - 1}{cos^2\beta_{half} - \sqrt{2}}} \tag{5}$$

For these conditions, $cos\beta_{half}$ can be determined respectively as (6), (7), (8):

$$cos\beta_{half} = exp\,(-ln\,(2)/N) \tag{6}$$

$$cos\beta_{half} = cos\left(m_G \cdot \sqrt{ln(2)}\right) \tag{7}$$

$$cos\beta_{half} = \sqrt{\frac{\sqrt{2} \cdot C_{TR}^2 - 1}{C_{TR}^2 - 1}} \tag{8}$$

Using the same rules, coefficients for Beckmann and Sawicki distribution can be calculated (9), (10), however, for these cases, determination of $cos\beta_{half}$ is not so simple.

$$m_B = \frac{tan\beta_{half}}{\sqrt{ln(2) - 4ln(cos\beta_{half})}} \tag{9}$$

$$N_{DS} = \frac{cos\beta_{half} + 3 - 4 \cdot \sqrt[4]{2cos\beta_{half}}}{cos\beta_{half} - 1} \tag{10}$$

The dependencies of coefficients for the discussed formulas as a function of β_{half} should be analyzed. Theoretically (Hall, 1989) the value of 1/2 could be used in any situation, but in practice the range of the angle is limited by the possible range of the proper coefficient. For example, for Beckmann MDF, $m_B \in (0,1)$ and β_{half} cannot be greater than 1.1.

The Phong MDF (or a function in a similar form) is frequently used in many different BRDFs. One of

Table 2: Comparison of microfacet distribution function coefficients of Beckmann, Gauss, Blinn/Phong, and Sawicki, and Trowbridge-Reitz for approximation with the smallest root mean square error (RMSE).

m_B	N (RMSE) /N based on (11)/	N (RMSE)	m_G (RMSE)	N_{DS} (RMSE)	C_{TR} (RMSE)
0.4473	10 (0.05929)	7.654 (0.02463)	0.4970 (0.03396)	9.218 (0.04690)	0.4865 (0.08765)
0.3163	20 (0.02722)	17.45 (0.009425)	0.3344 (0.01228)	20.80 (0.02967)	0.3624 (0.05726)
0.2	50 (0.009946)	47.33 (0.003024)	0.2046 (0.003992)	55.92 (0.02094)	0.2357 (0.04146)
0.1634	75 (0.006638)	72.29 (0.00195)	0.1658 (0.002583)	85.24 (0.01967)	0.1937 (0.03912)
0.1415	100 (0.004963)	97.29 (0.00144)	0.1431 (0.01910)	114.6 (0.01908)	0.1682 (0.03805)
0.1	200 (0.00246)	197.3 (0.0006967)	0.1006 (0.0009266)	232.0 (0.01806	0.1194 (0.03617)
0.06325	500 (0.0009687)	497.3 (0.0002703)	0.06339 (0.0003601)	584.2 (0.01728)	0.07573 (0.03472)
0.05164	750 (0.0006542)	747.2 (0.0001764)	0.05172 (0.0002350)	877.7 (0.01686)	0.06186 (0.03392)
0.04473	1000 (0.0005033)	997.3 (0.0001398)	0.04477 (0.0001863)	1171 (0.01778)	0.05360 (0.03572)
0.03163	2000 (0.0002518)	1997 ($6.975*10^{-5}$)	0.03164 ($9.298*10^{-5}$)	2345 (0.01771)	0.03791 (0.03558)
0.02	5000 ($9.515*10^{-5}$)	4997 ($2.632*10^{-5}$)	0.02 ($3.509*10^{-5}$)	5868 (0.01669)	0.02398 (0.03361)
0.01634	7500 ($6.826*10^{-5}$)	7497 ($1.747*10^{-5}$)	0.01633 ($2.329*10^{-5}$)	8804 (0.01661)	0.01958 (0.03345)
0.01415	10000 ($4.761*10^{-5}$)	9997 ($1.321*10^{-5}$)	0.01414 ($1.755*10^{-5}$)	11749 (0.01669)	0.01696 (0.03360)
0.01	20000 ($2.452*10^{-5}$)	19997 ($6.778*10^{-6}$)	0.01 ($9.035*10^{-6}$)	23480 (0.01718)	0.01199 (0.03456)
0.006325	50000 ($9.493*10^{-6}$)	49997 ($2.623*10^{-6}$)	0.006325 ($3.498*10^{-6}$)	58710 (0.01662)	0.007584 (0.03348)
0.005164	75000 ($8.546*10^{-6}$)	74996 ($1.793*10^{-6}$)	0.005164 ($2.389*10^{-6}$)	88060 (0.01703)	0.006193 (0.03427)
0.004473	100000 ($5.03*10^{-6}$)	99997 ($1.390*10^{-6}$)	0.004472 ($1.853*10^{-6}$)	117400 (0.01761)	0.005364 (0.03540)

the first analyses of replacing these BRDF functions was conducted in (Ward, 1996). The author suggested the approximate relation between m_B and N in a simple equation (11), which allows obtaining a value similar to the Blinn analysis:

$$m_B^2 \cdot N = 2 \tag{11}$$

Preliminary analysis shows that MDF in Blinn/Phong and Gauss versions extremely well approximate the Beckmann function. One can consider whether it is worth using the simplification proposed in (Hall, 1989) suggesting the use of $cos\beta_{half}$ during conversion or using equation (11). To obtain better approximations, I used the MATLAB curve fittings tool. It was assumed that the aim is to approximate Beckmann MDF by Blinn/Phong and Gauss functions. The N and m_B coefficients are selected in such a way as to obtain the smallest root mean square error (RMSE). The m_B values were analyzed in the range of 0.0044–0.44. This corresponds to the variability of N in the range of (10, 10^5). It represents a very wide range of parameters for specular reflective surfaces. A set of various forms of conversion equations between MDFs parameters have been tested in the curve fitting tool.

3 RESULTS

Table 2 presents a summary of the analysis performed in MATLAB environment. RMSE (root mean square error) values reported there show that approximations obtained here were significantly better than those determined based on $cos\beta_{half}$ (using equation (11)). In addition, it is noteworthy that RMSE is clearly decreasing for smooth surfaces (well reflective), that is, for $N \geq 100$. Considering the appropriate m_B, m_G, and N values, we can use the same MATLAB tools, and determine functions that approximate the relationship between these parameters. This will allow converting the values to replace one MDF with another one. Of course, the conversion functions determined in this way will be different from those using $cos\beta_{half}$, and equation (11), but the approximations will be better (smaller RMSE).

MDF in versions Trowbridge-Reitz, Sawicki, and Schlick does not give the possibility of approximating Beckmann MDF with such a small RMSE as Blinn/Phong and Gauss. However, it is worth conducting a similar RMSE analysis, to propose the conversion of values between MDFs. It was done for Sawicki MDF and Trowbridge-Reitz MDF — the last columns in Table 2.

According to the results of this study, the method of replacing one MDF by another can be introduced for all MDFs discussed in this article. However, according to this analysis, there are two groups of MDFs treated differently. In the first group, MDFs will be in versions of Beckmann, Blinn/Phong, and Gauss. In the second group, MDFs will be in versions of Trowbridge-Reitz, Sawicki, and Schlick. Sets of equations (12) and (13), and (14) describe conversion of the coefficients in the first group respectively between functions Beckmann—Blinn/Phong, Beckmann—Gauss, and Blinn/Phong—Gauss.

$$N = \frac{2}{m_B^2} - 2.5 \quad \text{and} \quad m_B = \frac{\sqrt{2}}{\sqrt{N+2.5}} \tag{12}$$

$$m_G = 0.5591 \cdot m_B^3 + m_B \quad \text{and} \quad m_B = m_G - 0.4134 \cdot m_G^3 \tag{13}$$

$$N = \frac{2}{m_G^2} - 0.5 \quad \text{and} \quad m_G = \frac{\sqrt{2}}{\sqrt{N+0.5}} \tag{14}$$

Additionally, similar analysis based on minimum of RMSE allows for simple conversion of the coefficients between functions Blinn/Phong—Sawicki: (15).

$$N_{DS} = 1.174 \cdot N + 0.3 \quad \text{and} \quad N = \frac{N_{DS} - 0.3}{1.174} \tag{15}$$

In the second group, the conversion of the coefficients' value for the Trowbridge-Reitz and Sawicki MDFs require proposed approximations and usage of equations (12) and (14). Tables 3 and 4 summarizes the procedures for cases. The conversion of coefficients for Schlick MDF is not necessary because the Schlick function approximates Beckmann MDF for the same value of m_B.

Table 3: Procedures of coefficients conversion for Trowbridge-Reitz and Sawicki MDFs. Part 1.

FROM / TO	C_{TR} (Trowbridge-Reitz)	N_{DS} (Sawicki)
m_G (Gauss)	$m_G = 0.8222 \cdot C_{TR} + 0.82 \cdot C_{TR}^3$	$N = \frac{N_{DS} - 0.3}{1.174}$ then $m_G = \frac{\sqrt{2}}{\sqrt{N+0.5}}$
m_B (Beckmann)	$m_B = 0.8388 \cdot C_{TR} + 0.7 \cdot C_{TR}^4$	$N = \frac{N_{DS} - 0.3}{1.174}$ then $m_B = \frac{\sqrt{2}}{\sqrt{N+2.5}}$
N (Blinn/Phong)	$N = \frac{1.696^2}{C_{TR}^2} - 4.5$	$N = \frac{N_{DS} - 0.3}{1.174}$
C_{TR} (Trowbridge-Reitz)	X	$N = \frac{N_{DS} - 0.3}{1.174}$ then $C_{TR} = \frac{1.697}{\sqrt{N+4.5}}$
N_{DS} (Sawicki)	$N = \frac{1.696^2}{C_{TR}^2} - 4.5$ then $N_{DS} = 1.174 \cdot N + 0.3$	X

Table 4: Procedures of coefficients conversion for Trowbridge-Reitz and Sawicki MDFs. Part 2.

TO / FROM	C_{TR} (Trowbridge-Reitz)	N_{DS} (Sawicki)
m_G (Gauss)	$C_{TR} = 1.186 \cdot m_G - 0.835 \cdot m_G^3$	$N = \frac{2}{m_G^2} - 0.5$ then $N_{DS} = 1.174 \cdot N + 0.3$
m_B (Beckmann)	$C_{TR} = 1.2 \cdot m_B - 0.56 \cdot m_B^3$	$N = \frac{2}{m_B^2} - 2.5$ then $N_{DS} = 1.174 \cdot N + 0.3$
N (Blinn/Phong)	$C_{TR} = \frac{1.697}{\sqrt{N + 4.5}}$	$N_{DS} = 1.174 \cdot N + 0.3$

4 DISCUSSION

4.1 Comparison of Formulas

The Beckmann distribution (Beckmann and Spizzichino, 1963) has been assumed as the reference function. The unit form of the MDF was used and proper values of coefficients were calculated based on the Beckmann m_B parameter.

Figs. 1a and 2a show the graphs of the MDFs discussed for different sets of parameters. Figs. 1b and 2b show the differences—the relative error concerned to the maximum value of the function in relation to Beckmann distribution for the same values of parameters as in Figs. 1a and 2a.

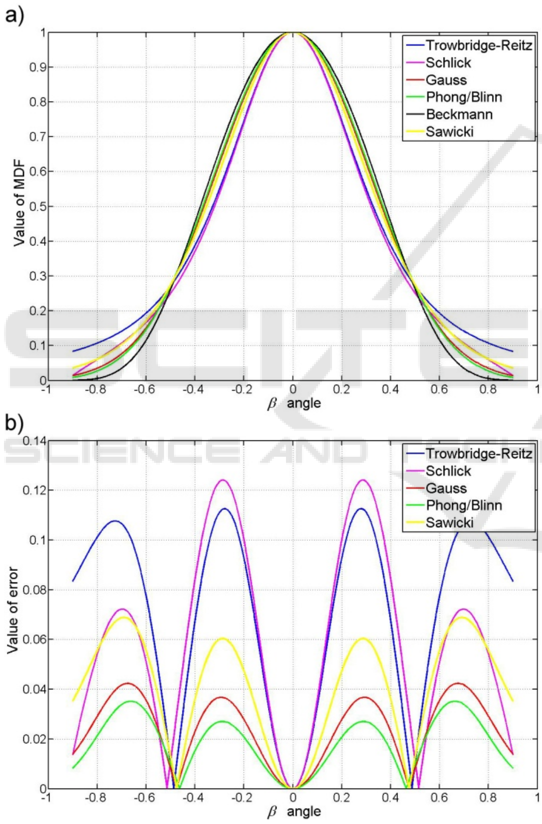

Figure 1: Graphs of different distribution functions for parameters calculated based on m_B=0.4 (N=10, m_G=0.4364, C_{TR}=0.4456, N_{DS}=12.04). a) Values of MDF as a function of the β angle. b) The relative error (as a function of the β angle) pertaining to the maximum value of the function in relation to Beckmann distribution.

Phong's proposal is a surprisingly good approximation of the Beckmann distribution. The max relative error decreases in this case when the N Phong parameter increases: for N=10, the maximum relative error is on the level of 3.5%, for N=40 it is 1%, for N=100 is 0.4% and for N=1000 it decreases to 0.03%. At the same time, it is worth remembering that MDF is used in modeling of specular reflection and in practice there are usually values from the range of N>100.

However, it is noteworthy that a very small relative error is obtained after replacing the Beckmann function with the Gauss function (Figs. 1b and 2b). Many effective approximations of the Gauss function are well-known, for example, Lee's polynomial cubic function (Lee, 2000), the tricube function (Cleveland and Loader, 1995), and the Wendland solution (Wendland, 1995). However, practically, none of these approximations are useful for the description of the MDF function.

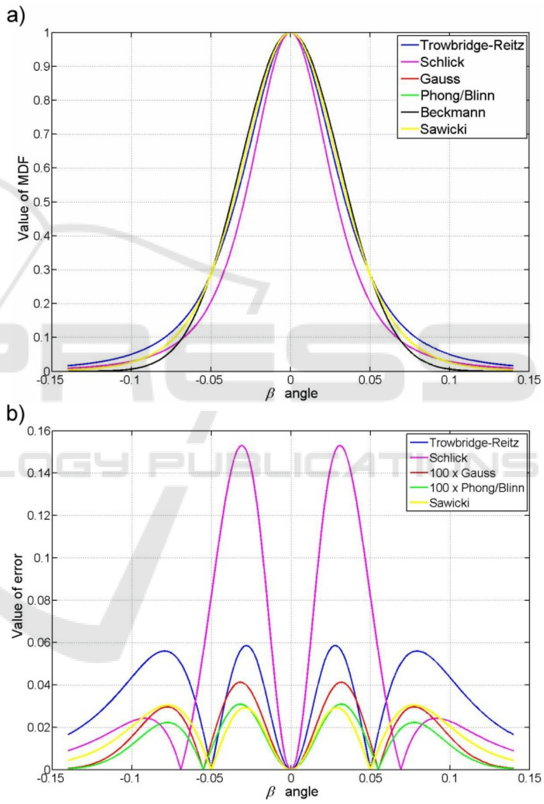

Figure 2: Graphs of different distribution functions for parameters calculated based on m_B=0.04464 (N=1000, m_G=0.04471, C_{TR}=0.05354, N_{DS}=1174.3). a) Values of MDF as a function of the β angle. b) The relative error (as a function of the β angle) pertaining to the maximum value of the function in relation to Beckmann distribution.

In all cases, the functions of Beckmann, Blinn/Phong, and Gauss are indeed very similar. In particular, it is noticed for the smooth surface ($N \geq 1000$), when the relative error for Gauss and Blinn/Phong MDFs are less than 0.05%. Comparing the graphs (Figs. 1 and 2) and value of RMSE presented in Table 2, it is noteworthy that the approximation by the Blinn/Phong function is always better than by the Gauss function. Although both approximations are very good.

The behavior of the Trowbridge-Reitz distribution is noteworthy. Its function graph differs considerably from the Beckmann function. As it shows, the assumed models of roughness differ from one another. The unquestionable advantage of the Trowbridge-Reitz distribution is gentle change in value for larger angles—“long tail” (Burley, 2012). It was used in GGX/GTR MDFs. In addition, advantage is the simplicity of the calculation and the fact that the integral of the distribution can be analytically calculated in a simple way, which is sometimes quite useful in the computational application.

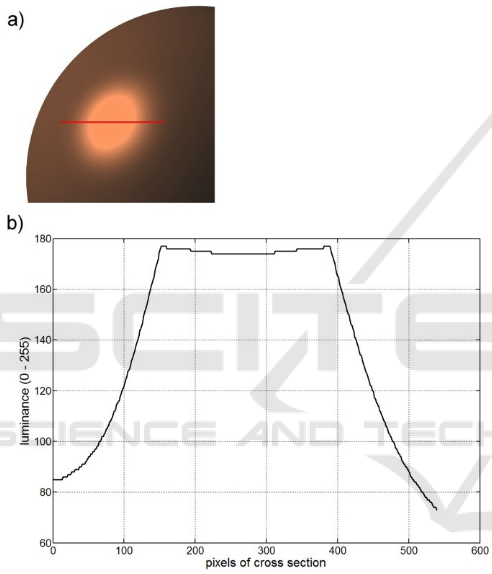

Figure 3: The light reflection from the sphere surface and graph of luminance on the line of “cross section” through the spot of light.

In this study, two other MDF representations in a polynomial/rational form were considered: Schlick and Sawicki MFD. Both are computationally attractive. However, both do not give a good approximation of Beckmann function. The Schlick function differs significantly from the other MDFs in all cases. It is the worst solution especially for smooth surfaces: the maximum relative error of Schlick MDF is at a level of 13%–15% (in cases presented in Figs. 1b and 2b). A much better result is achieved by Sawicki MDF, with a relative error of about 3%–7% (Figs. 1b and 2b).

The evaluation of the implementation of speed of the MDF functions is worthy of discussion. Blinn (Blinn, 1977) suggested the Trowbridge-Reitz polynomial/rational function because of the improvement in the effectiveness of calculations; however, in the book (Akenine-Möller et al., 2008), we can read that such an approximation was important and relevant while the article was being written (1977). In my opinion, today this will not be the main factor determining the choice of MDFs for the general usage.

4.2 Comparison of Formulas Graphical Experiments

Experiments with the Ashikhmin-Shirley reflection model (Ashikhmin and Shirley, 2000) have been conducted, where different MDFs were used. To show the differences between the MDFs, the simplest object has been chosen to make the visual effects and their interpretation dependent only on the distribution function used.

A comparison of the visual properties of applying different MDFs was conducted using an example in which the light reflection from the sphere surface was simulated (Fig. 3). To reduce the influence of the subjective perceptual assessment, the graph of brightness changes on the line of “cross section” through the spot of light.

In Fig. 4, the implementation of different MDFs and graphs of the luminance on the cross section is shown with the assumption that there was a less smooth surface (rough), whereas in Fig. 5, different MDFs are shown with the assumption of very smooth character. The graph of luminance has been presented similar to the cross section in Fig. 3, but in order not to cover stains of light, the line segment is not marked in Figs. 4 and 5.

As can be seen, according to the expectations, differences in the appearance of light reflection for the function of Gauss, Beckmann, and Blinn/Phong are very small. It is, practically, unnoticeable in the picture. There are visible changes of colors at applying the Schlick and Trowbridge-Reitz function: Schlick because of approximation, Trowbridge-Reitz because of different model of MDF. The result of the comparison of the pictures is not surprising if the differences between shapes of functions are analyzed (Figs. 1b and 2b). However, these differences do not change the character of reflection but insert subtle changes in the reflective properties.

The impact of the Trowbridge-Reitz MDF is noteworthy, especially for very smooth surface (Fig. 5). Despite correct conversion of coefficients, the reflection drawn with the use of the Trowbridge-Reitz MDF has gentler edges. This fact is used as a more realistic reflection in the GGX/GTR models.

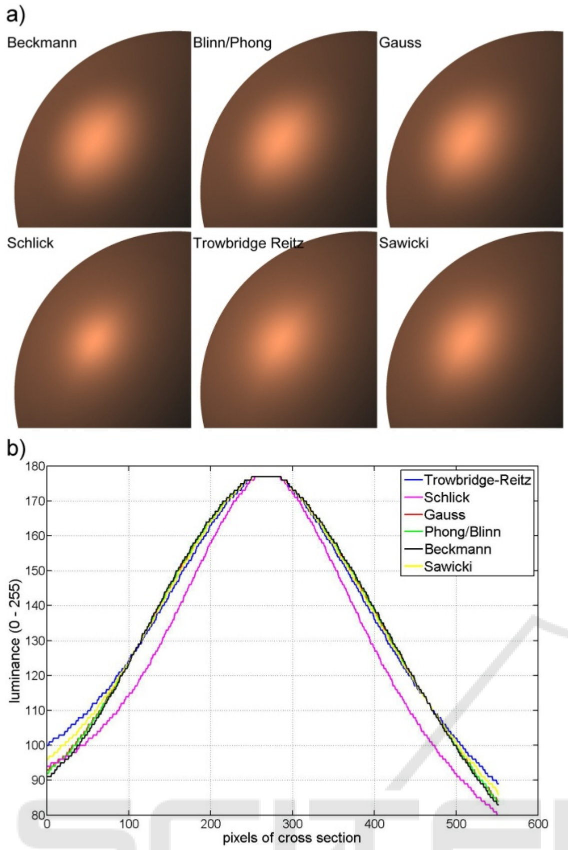

Figure 4: Light reflection from the sphere surface with the assumption that the surface is rough (less smooth)—according to Figure 1. a) The view for different microfacet distribution functions (MDFs). b) The graphs of luminance on the cross section for used MDFs. Cross section is made similar to Fig. 3. Graphs for Gauss, Blinn/Phong, and Beckmann MDFs are so similar that only one line (black) is visible.

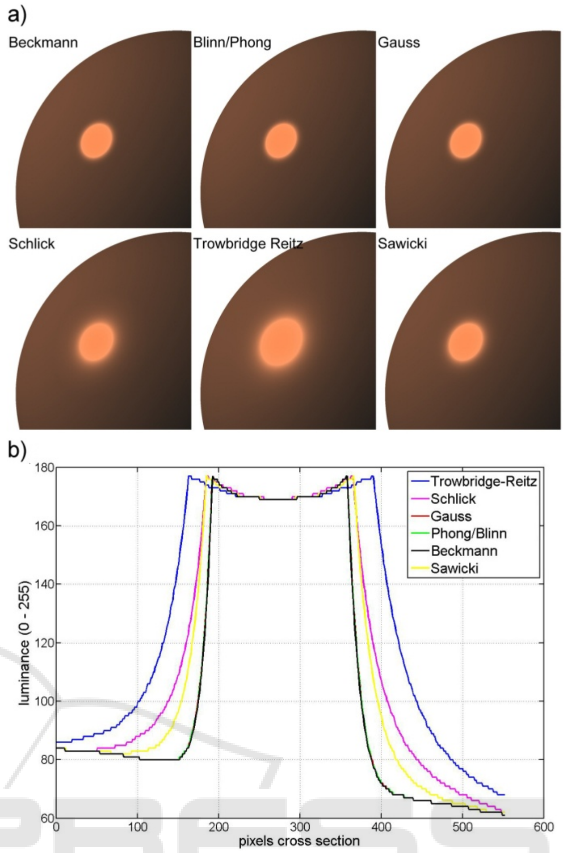

Figure 5: Light reflection from the sphere surface with the assumption that the surface is smooth—according to Figure 1. a) The view for different microfacet distribution functions (MDFs). b) The graphs of luminance on the cross section for used MDFs. Cross section is made similar to Fig. 3. Graphs for Gauss, Blinn/Phong and Beckmann MDFs are so similar that only one line (black) is visible.

5 CONCLUSIONS

In this article, review of the most important properties of the MDF that is applied in the BRDF and reflection models has been presented. Furthermore, the advantages and disadvantages of those different MDFs have been considered. The normalized form for Gauss and Trowbridge-Reitz distribution has been proposed. Various versions of the rational MDF form were also analyzed. After RMSE analysis the mathematical dependencies, that allow for the exchange of one MDF with the other, have been proposed.

The answer to the question posed in the title of this article (to change or not to change) is not so simple.

A comparison of different functions shows the possibility of exchanging one distribution function by another without the loss of the image quality; however, it is not always a trivial task. An examination of Figs. 4 and 5 reveals that reflections modeling using different MDFs shows very close effects. Proper conversion of functions parameters is significant in this case. The introduced and presented equations and relationship between the parameters of different MDFs help in this task. However, a deeper analysis shows a certain small change—subtle differences. It is particularly visible if the cross section of the light spot is analyzed (Figs. 4 and 5).

Differences between functions of Beckmann, Gauss, and Blinn/Phong are unnoticeable, and these three functions can be used interchangeably in practically all situations—which is a very important conclusion from presented here analysis. However, for these MDFs, it is worth paying attention to the more important problem. Equations (12) – (14) describe the relationships between the parameters of these MDFs. For very smooth surfaces, N takes values from a very wide range from about 1000 to infinity. This corresponds to changes of m_B (m_G) in a very small range. At the same time, for a less smooth surface (rough surface), we have a relatively larger range of changes m_B (m_G) than N. This is due to the nature of the rational function. This determines a very practical proposal for its application. For very smooth surfaces (well reflective), it is worth to use Blinn/Phong MDF because it is easier to control reflective properties (subtle changes) with a parameter in a wider range. In contrast, the Beckmann (Gauss) MDF is worth using for less smooth surfaces (poorly reflective).

However, replacing one MDF with another one can be intentional—to get the proper visual effect. The application of the Trowbridge-Reitz distributions causes significant differences in the created pictures—the visible effect of "long tail" for smooth surfaces (Fig. 5). This is a significant difference compared to Beckmann (Gauss, Blinn/Phong) MDF assuming a similar general nature of changes—resulting from the conversion of coefficients. This is a very important advantage of this MDF for modern applications where GGX/GTR is used. This has also been confirmed in the publications discussed. A similar effect to Beckmann, but with subtle "long tail" for smooth surfaces (Fig. 5) can be obtained with Sawicki MDF. However, it does not seem that this MDF can compete with GGX/GTR applications. Especially if we consider the development of GGX toward GTR in contemporary studies (Burley, 2012).

Not all MDFs are easy to implement to the same extent. The Schlick MDF can cause significant problem because of the range of approximation. The conversion of Beckmann MDF to Gauss MDF seems to be justified only in specific situations, if it could speed up the calculation (which could result from the use of appropriate similar functions to describe the material properties). The computational complexity and the visual properties of both these functions are practically identical. However, if a function similar to Beckmann would be needed, but in a polynomial/rational form, none of the discussed here two functions make a good approximation.

About MDF, Hall wrote in his book (Hall, 1989) that "no comparative study has been performed with these distribution functions." After approximately 30 years, there is a hope that this article will fill this gap.

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