

Determining the Required Size of a Military Training Pipeline

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Abstract: This paper addresses the problem of deciding how many positions to set aside, in a military establishment, for recruits undergoing training. We assume a cap on total strength, and thus must select a ratio between positions in the force's training pipeline versus its trained establishment. We develop a Markovian model of the training pipeline, with parameters derived from historical Human Resources data. Through Monte Carlo simulation we may then predict how often a given ratio will be sufficient to generate the required trained force, as well as how much surplus trained personnel it is expected to generate. Our modelling results have informed ongoing initiatives to optimize the force mix and structure of the Canadian Armed Forces.

1 INTRODUCTION

This paper describes the approach taken to solve a challenging Human Resource problem faced by the Canadian Armed Forces. This problem concerns how many positions must be set aside for Regular Force recruits undergoing their training. We present a solution based on a stochastic simulation of the training pipeline. Through simulation, we estimated how often a given number of positions reserved for trainees will be sufficient to generate the desired trained force. Simulation also gives us an estimate of the number of surplus trained personnel that is to be expected. Armed with these results, departmental decision makers will be able to select a training structure that balances the risk of not meeting operational requirements against the costs from pipeline overcapacity.

The scheme presented in this paper is simple and effective. It is presented in the hope that it will be appreciated as a practical application of military Operations Research. Our solution was developed in the context of designing a future fighting force, but is also relevant to examining the current force structure. Regularly revisiting the ratio of trainee-to-trained positions will be necessary to preserve a force where

all units can be sufficiently manned. Our model offers a way to inform this rebalancing.

2 BACKGROUND

In 2017, the Government of Canada issued Strong Secure and Engaged, the latest Canadian Defence Policy. Notably, this policy mandates the capability to conduct a defined set of concurrent operations, including both emergency responses and planned deployments, and ranging from limited to sustained commitments. To ensure that the Canadian Armed Forces have the right mix of military personnel to satisfy these requirements, the Force Mix and Structure Design initiative was launched. This initiative is in the process of designing, from the ground up, the required military establishment.

Operations Research analyses in support of the Force Mix and Structure Design initiative are not the first to be conducted in support of establishment reviews. For example, both Bender (2005) and Couillard et al (2015) present stochastic simulations that quantify to what extent given force mixes (mixes of military personnel from various occupation categories) meet contingent operational

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requirements. Filinkov et al (2011) similarly present a model designed to study the Australian Army.

A first phase of Force Mix and Structure Design strived to determine the Force Employment requirement – the required number of “boots on the ground” conducting operations. The second phase then looked at supporting elements involved in force generation and the institution of the Canadian Armed Forces. It is in this context that the required size of the combined Basic Training List (recruits in their initial phases of training) and Supplementary University Training List (recruits requiring training in higher education institutions) had to be determined. For simplicity, we will thereafter refer to the positions reserved for members on these lists as the *training pipeline*. Regular Force recruits remain in this pipeline until they reach an Operationally Functional Point – at which point they are considered trained and can occupy trained effective positions.

Straver and Christopher (2015) have conducted a study based on stochastic simulation to determine the sustainable composition of the Regular Force, including the size of the training pipeline. Nevertheless, it is now understood that the data underpinning that study were problematic. As such, our model is a successor to Straver and Christopher’s that now focuses exclusively on the training pipeline, and that was built to work with new and improved data.

3 MARKOV MANPOWER MODELS

Many approaches have been used to model personnel systems. For example, Wang (2005) categorizes these approaches into Markov Chain models, Computer Simulation models, Optimisation models and System Dynamics. The approach that we describe is a hybrid of the first two categories, being a discrete-time Markovian model serving as a basis for stochastic simulation.

The earliest described application of Markov Chains to a personnel systems is found in (Seal, 1945), whereas a general overview of their use in this context is found in (Guerry and De Feyter, 2009). Guerry and De Feyter define *Markov Manpower models* as satisfying four assumptions:

- They are memory-less (the usual Markovian assumption);
- Their flow rates are time-independent;

¹ We used 14 years of historical data. This goes beyond the maximum length of training, but excludes earlier years, when the system may have behaved differently.

- They are discrete-time, with fixed-increment intervals;
- Their stocks describe homogeneous populations.

Models developed to study the Canadian Armed Forces have generally obeyed the first two of these assumptions, but not always the last two. In fact, Discrete Event Simulation has been the most-employed paradigm (Okazawa, 2013), but is based on next-event time progression rather than fixed increments.

The homogeneous stocks assumption requires that populations be broken down into homogeneously behaving subsets. To do this, regression is often used to identify the variables that most-affect behaviour. In our case, this would pose a problem. The Regular Force employs members of various ranks, and who have completed different periods of service – factors that are closely tied to the propensity to release (to leave the forces). Also, they belong to many different occupations, and are recruited through a number of different entry plans, implying widely different durations of training. Given this wide spectrum of factors, and given the need to use data that remain representative of the current system,¹ we would not have sufficient data available to accurately estimate all the parameters that would be associated with the multiple resulting homogeneously behaving subsets (groups of members of similar ranks, years of service, occupations and entry plans).

We thus build a model that diverges from Markov Manpower models with respect to the homogeneous stock assumption. We group in our stocks cohorts of members who behave diversely, but who when taken collectively, have aggregate flow probabilities that are nevertheless fairly consistent from year to year. Therefore, the historically observed proportion of members from a stock who flow in a given direction cannot be interpreted as a probability applying to individuals, but it can still be interpreted as the expected proportion of individuals who will transition.

4 TRANSITION PROBABILITIES

For Markov Manpower models, Anderson and Goodman (1957) demonstrate that the maximum likelihood estimator for transition probabilities is given by the numbers of employees that underwent that transition divided by the total number of eligible employees in the relevant sub-periods (e.g. each year,

if looking for an annual probability). Under the homogeneous stock assumption, the probability for the total number of employees undergoing that transition is then binomial. Binomial distributions are thus commonly used in stochastic interpretations of Markov Manpower models, but this would be inappropriate in our case, as our stocks are not homogeneous.

In the absence of *a priori* knowledge of the shape of the flow probabilities in our model, we fit Gaussian distributions. For example, Figure 1 shows the distribution used for members graduating from the training pipeline within the year after they had first appeared in it. The 14 years of available annual observations are shown as a bar histogram. The Gaussian that was fit, with mean 45.2% and standard deviation 2.1% is shown as the dashed line.

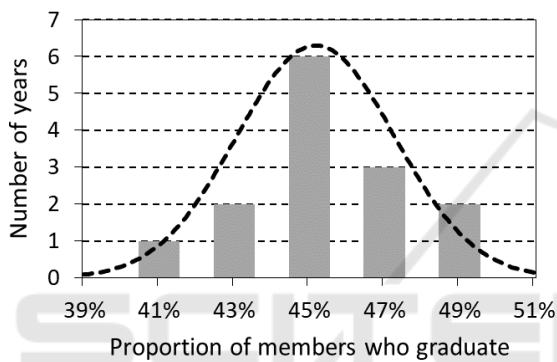


Figure 1: Distribution for the proportion of members who graduate within the next year, among members who first appeared in the training pipeline, 2006-2020. A Gaussian fit is also shown as the dashed line.

When we get to our simulation, the proportion of members who graduate from the training pipeline in a given year will thus be drawn from this and similar distributions in each annual iteration. In order to

avoid unrealistically extreme outcomes, we only draw within three standard deviations of the mean.

Previous efforts to model the Regular Force training pipeline, such as (Bender, 2005) and (Straver and Christopher, 2015) have sought to explicitly model the main specific processes of the human resources system. For example, graduation from the training pipeline would be modelled as the result of separately considering graduations of Officers and Non-Commissioned Members, further split according to their entry plan, as various categories of trained, semi-trained and untrained recruits. Instead, our approach considers no more than a single flow between each model stock, consolidating a number of sub-flows. This greatly simplified our task of historical data analysis for estimating model parameters, as we did not need to categorize the data associated with past recruits. We also expect the resulting model to be more reliable, as covariance between sub-flows would have been impossible to determine from our limited data, but is avoided by directly estimating the aggregate flows.

5 THE MODEL

Figure 2 depicts our model. The percentages shown on arrows correspond to the mean proportions for certain modelled flows. The model has two parts: A stock for the Trained Effective Strength (TES), and a set of stocks for the training pipeline. The TES encompasses all members who have completed their training up to the Occupationally Functional Point, and are not in certain operationally unavailable categories. Members who change occupation generally require re-training – these correspond to the 0.8% arrow flowing toward the training pipeline.

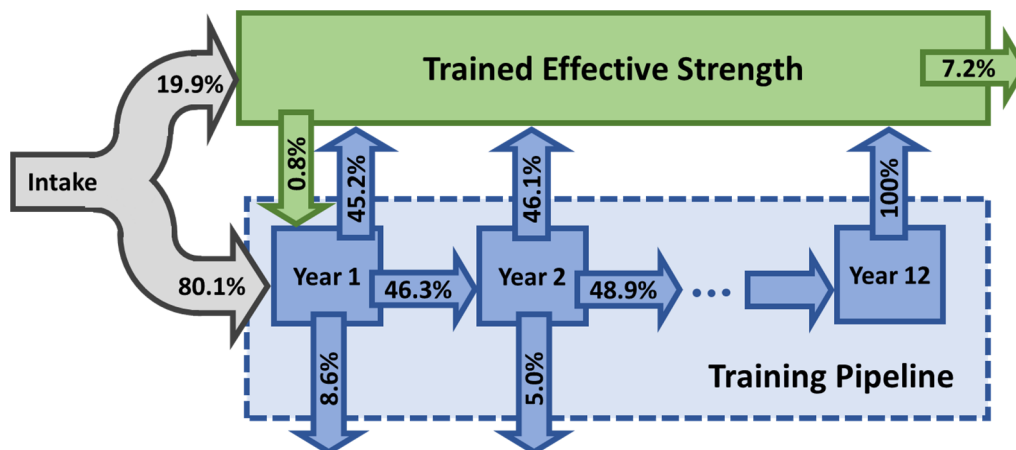


Figure 2: Illustration of our model of the Canadian Armed Forces training pipeline.

Others leave the system, corresponding to the 7.2% arrow. These leaving members may be leaving the forces completely through release or death, but could also be moving to the Reserve Force, or to a non-effective status (ill, injured or pre-release).

The training pipeline is divided into cohorts. After having first appeared in the pipeline, members may graduate (45.2%), remain for at least another year (46.3%), or leave the system (8.6%). A total of 12 such similar stocks are modelled, with any remaining trainees graduating after the twelfth year. In our historical record, only one member was in the training pipeline for 12 consecutive years. No occupation requires this much training, but delays can result from changes in occupation, or pauses in training. The most frequent type of pause is parental leave – an entitlement for new parents.

The remaining important flow is intake, which here includes recruitment, but also return from ill or injured status. On average, 19.9% of intake go straight to the TES. This includes trained recruits (re-hires or transfers from the Reserve Force), but also recruits requiring less than a year of training, who joined the Regular Force and move on to the TES within the year (our model being based on annual iterations).

Markov Chains can be treated as deterministic or stochastic. Davies (1982) introduced a partially stochastic Markov model. In that model, attrition is considered an uncontrollable flow, and treated as stochastic, whereas promotions are decided by management, and thus treated as deterministic. Our model does not consider promotions, but does treat the magnitude of total intake deterministically, and is thus also partially stochastic. Intake is set to re-fill the training pipeline each year (with a hard cap on total strength, i.e. the total Regular Force population) rather than varying stochastically. This intake is also the only pull flow in our model. It is generated by vacancies in the destination (pull), rather than arising spontaneously from the source (push), as defined by Bartholomew et al (1991).

Although we set the magnitude of intake deterministically, we vary the proportion going to the TES versus the training pipeline stochastically. This treatment of TES intake as a direct proportion of total intake resembles the proportionality constraint introduced by Nilakantan and Raghavendra (2005). Their constraint requires that a fixed proportion of vacancies in a given grade be filled externally. Our model is however different in that our proportion varies according to the observed historical distribution.

6 MODEL LIMITATIONS

We will now highlight three limitations of our model. We do not believe that these limitations invalidate our results, but they should be kept in mind when interpreting them. A first limitation has to do with using historical data to estimate the rates of flow out of the training pipeline. Currently, delays result from limitations on training institution capacity or from their sub-optimal organisation. However, our results are meant to be applicable to future force structures, where sources of delay will hopefully have been reduced. Historical observation could therefore overestimate future training durations, and consequently, over-estimate the number of required training pipeline positions.

A second limitation of our model is that it is based on annual-duration iterations (taken at fiscal year-end: 31 March). However, that day does not correspond to the annual peak for the training pipeline. Typically, the peak will be in summer, when more recruits begin their training. As such, slightly more training pipeline positions are likely to be required than is determined by our model. It would however be possible to introduce a correction factor for our results based on the historical differences between end fiscal year and annual peaks. Finally, our model's last important limitation is that it only considers an overall TES target, ignoring its composition in terms of ranks and occupations. This will mask specific gaps in trained personnel. In the normal course of business, retention encounters ups and downs at various ranks and occupations, leading to local gaps. Certain positions can be filled from a range of different ranks and occupations, but others cannot, and a larger training pipeline cannot address gaps in senior or specialized positions in the short term. It should therefore be understood that some vacancies in the establishment are to be expected, even when the pipeline trains enough members to counter the raw number of departures.

Fully addressing these three limitations with an enhanced model is likely impossible, given data constraints. For example, given that there are only so many members in each occupation, and that it is only relevant to look back so many years in the data record, accurately estimating occupation-specific training and attrition model parameters would not be feasible. However, decision makers can appreciate the constraints' impact on our modelling results, and consider them in developing policy. Overall, our model outputs remain informative, especially if interpreted as slightly under-estimating true training pipeline requirements.

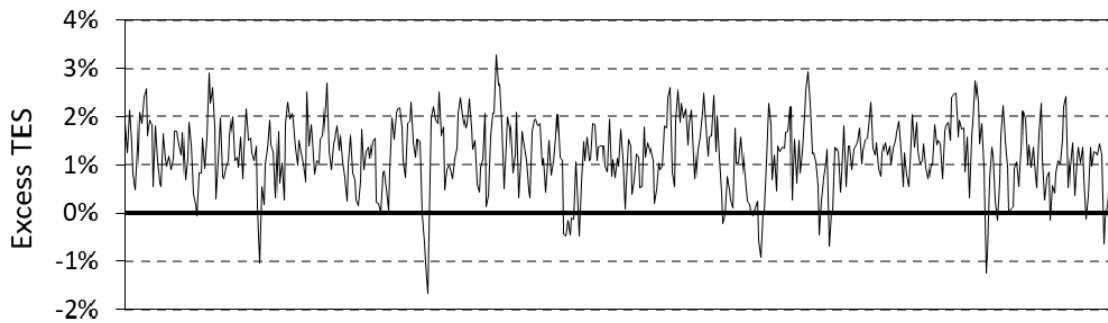


Figure 3: Five hundred years of simulation result showing excess TES – trained effective personnel generated above the set requirement.

7 SIMULATION

We derived Monte Carlo simulation results using a spreadsheet. Each line of the spreadsheet recorded the state of the stocks in a given year. Subsequent years were then computed from the line above, with the annual flows drawn from specified Gaussian distributions. The number of lines in the spreadsheet then corresponds to the number of simulated years.

Our goal was to assess the adequacy of various force structures by considering ratios of the number of positions allocated to the TES versus the training pipeline. From year to year, the simulated TES varied with the simulated stochastic flows, and could either be driven above or below the set objective. Each year, the intake was set to re-fill the training pipeline, but subject to a cap of total strength, which came into play when the number of trained personnel exceeded the target TES.

Each simulation began with a fully manned TES, and a training pipeline full of fresh recruits. We would then simulate 200 years, in order to allow the composition of the pipeline to stabilize. Each subsequent year was then captured toward the output. As an example, Figure 3 shows 500 years of simulation, with a ratio of 1,547 training pipeline positions to 10,000 required TES. The graph shows the resulting annual variation in excess TES. We see that the excess TES tends to vary within a set range. Whenever the excess is too great, the cap on total strength means that the training pipeline cannot be filled to capacity, eventually leading to fewer graduations into the TES, and thereby eventually reducing the excess. Conversely, when the excess is negative, the training pipeline is filled to capacity, allowing for eventual growth of the TES. For the ratio of training pipeline positions to required TES used in this example, we see that the TES meets (or exceeds) the requirement approximately 95% of the time.

All other results presented in this paper are based on 100,000 simulated years, which empirically proved to be enough for very stable results.

8 RESULTS

The main consideration, in fixing the capacity of the Regular Force’s training pipeline is ensuring that it is sufficient to generate the required TES. Figure 4 was obtained by simulating various ratios of training pipeline positions to required TES. It shows how often each ratio is expected to fail to yield the required TES. The failure rate decreases as the capacity of the training pipeline increases.

Because a small number of unfilled TES positions might be a tolerable outcome, Figure 4 also shows an alternative measure of effectiveness. This alternative measure records how often the shortage is above 1% of the required TES.

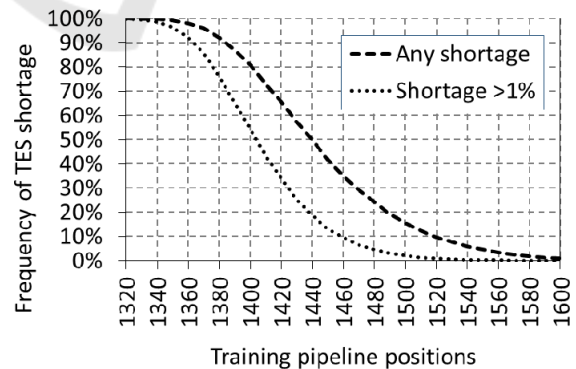


Figure 4: Fraction of simulated years where a TES shortage is observed, as a function of the number of training pipeline positions, given a 10,000 TES requirement.

However, there is a trade-off in increasing the number of training pipeline positions. The larger

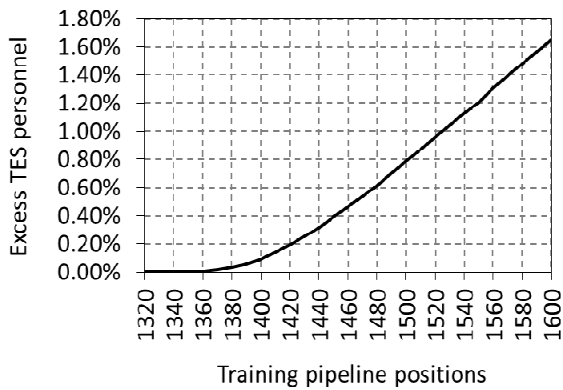


Figure 5: Mean TES above the requirement in those simulated years when the TES requirement is met, as a function of the number of training pipeline positions, given a 10,000 TES requirement.

pipeline risks generating more TES than required. This over-generation would be costly in and of itself, but would also burden the forces with more trained personnel than required, leading to persistent excess costs. Figure 5 shows how this expected over-generation increases with the size of the pipeline.

To highlight the interplay between the two metrics shown in Figures 4 and 5, the results were combine into Figure 6. This depiction of the results was highlighted to departmental decision makers, as it quantifies the trade-off involved in right-sizing the training pipeline.

9 DISCUSSION OF MODELLING ERROR

Guerry and De Feyter (2009) distinguish three types of error that apply to Markov Manpower models: statistical, estimation and specification. Statistical error results from the stochastic nature of models and may be reduced by increasing the number of simulation iterations. In our case, we expect this error to be small, given the 100,000 simulated years per scenario, and because that increasing that number minimally changes our results.

Estimation error is related to the accuracy to which parameters are estimated. In our case, we relied on 14 years of available Human Resources data to estimate parameters. Older data, or data from other

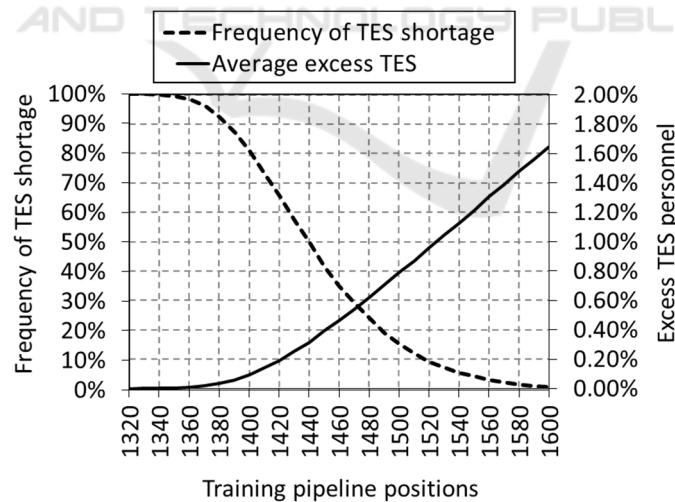


Figure 6: Combination of our two metrics into a single chart.

Table 1: Four potential choices of training pipeline to TES ratios, with corresponding metrics obtained from simulation.

Training pipeline to TES requirement	1,489:10,000	1,519:10,000	1,547:10,000	1,598:10,000
Frequency of TES shortage	80%	90%	95%	99%
Excess TES personnel	0.69%	0.95%	1.19%	1.64%

organisations would likely have been less representative of the current system, and therefore of little use in further reducing estimation error. Therefore, although we expect our estimation error to be substantial, there is no viable path to reducing it.

Lastly, specification error results from the model being an inaccurate representation of reality. Reducing specification error eventually involves increasing the complexity of the model, thereby increasing the number of parameters. This introduces a trade-off with estimation error, since estimation error increases with the number of parameters that must be estimated. In our case, it would be unadvisable to attempt reducing specification error by introducing more parameters. We believe that our model design strikes an appropriate balance between estimation and specification error.

10 CONCLUSIONS

This paper described an application of Monte Carlo simulation on a straightforward Markovian model of the Canadian Armed Forces' Regular Force training pipeline. The results of this simulation were provided to military staff tasked with redefining the organization's force mix. An advantage of the method employed in deriving our results is that it was straightforward and easy to follow. Despite the complexity of the Canadian Armed Forces' personnel system, our modelling results were readily trusted and appreciated.

Although the Regular Force training pipeline has been undersized in recent years, efforts have been directed to bringing awareness to the issue and to improving the sustainability of the establishment. It is hoped that our model will support this continuing effort.

Under existing total strength caps, sustainability can be improved by converting some TES positions into positions for trainees, but this comes at the expense of military capability. Alternatively, the current TES requirement can be maintained if the training pipeline (and by extension the total strength) are increased.

Our model was nevertheless developed to support the currently ongoing Force Mix and Structure Design initiative. In that context, we have informed the future force structure. Subject to the limitations presented in this paper, our simulation results have allowed initiative staff to foresee the range of possible outcomes from different proposed force structures.

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