Effect of Phase Mismatch between the Bragg Gratings on the Stability of Gap Solitons in Semilinear Dual-core System

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Abstract: The existence and stability of quiescent gap solitons are studied in a semilinear dual-core optical system, in which Bragg gratings (BGs) are written on the both cores with a phase shift and one core has the Kerr nonlinearity, while other one is linear. When the relative group velocity c in the linear core is zero, three separate band gaps are observed through the spectrum analysis, including one central band gap surrounded by upper and lower band gaps. Three band gaps are entirely filled with the stationary soliton solutions. However, in case of c is non-zero, only central band gap contains the stationary solution. Numerical techniques are used to find the stability of the quiescent gap solitons in terms of their frequency detuning.

1 INTRODUCTION

It is widely known that a strong effective dispersion induced from the cross-coupling between counter propagating waves on the fiber Bragg grating (FBG) and this dispersion can be up to six orders more than the standard fiber induce dispersion in magnitude. The grating originated dispersion can be compensated by the Kerr nonlinearity at sufficiently high intensity and that can generate a vast family of quiescent gap solitons (de Sterke and Sipe, 1994).

Solitons in FBG have been analyzed extensively by the researchers through theoretical analysis (Aceves and Wabnitz, 1989; Christodoulides and Joseph, 1989) and experimentally (Eggleton et al., 1996; Eggleton et al., 1999) in the last few decades due to their promising applications in novel optical devices, optical signal processing, filtering, switching, memory devices, sensing and pulse compression (Kashyap, 1999; Taverner et al., 1998). In the case of uniform Bragg grating, a two parameters family of gap solitons have been found from the theoretical studies. One of these parameters is the intrinsic frequency that determines the solitons' amplitude and width and the other parameter represents the soliton's velocity, which can range from zero to the speed of light in the medium (Aceves and Wabnitz, 1989; Christodoulides and Joseph, 1989; Barashenkov et al., 1998). The observation of quiescent or zero velocity soliton as well as slow gap soliton has been a subject of intensive experimental studies. Experimentally, gap solitons with a velocity as low as 23% of the speed of light in the medium have been reported (Mok et al., 2006).

Gap solitons have been investigated in different types of periodic structures and nonlinear systems, including grating assisted couplers (Atai and Malomed, 2005; Atai and Malomed, 2001; Mak et al., 1998), waveguide arrays (Mandelik et al., 2004), photonic crystals (Biancalana et al., 2008), cubic-quintic nonlinearity (Islam and Atai, 2014), and nonuniform gratings (Baratali and Atai, 2012; Chowdhury and Atai, 2014).

In Ref. (Tsofe and Malomed, 2007), gap solitons in gratings with phase mismatch in the dual-core system with identical cores were investigated. Since dual-core systems with non-identical cores (particularly semilinear dual-core fibers) have been shown to have superior switching characteristics, in this work we consider the existence and stability of quiescent gap solitons in a semilinear dual-core fiber where both cores are equipped with a grating and there is a phase mismatch between the gratings.

2 THE MODEL

The propagation of light in a linearly coupled Bragg grating with a phase shift between the gratings where one core has Kerr nonlinearity and the one is linear is governed by the following system of equations:

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$$
\begin{aligned}
iu_t + iu_x + \left[|v|^2 + \frac{1}{2}|u|^2\right] u + v + \kappa\phi = 0 \\
iv_t - iv_x + \left[|u|^2 + \frac{1}{2}|v|^2\right] v + u + \kappa\psi = 0 \\
i\phi_t + ic\phi_x + \psi e^{i\frac{\theta}{2}} + \kappa u = 0 \\
i\psi_t - ic\psi_x + \phi e^{-i\frac{\theta}{2}} + \kappa v = 0
\end{aligned} \qquad (1)
$$

where, $u(x,t)$ and $v(x,t)$ are the amplitudes of forward and backward traveling waves in the nonlinear core, $\phi(x,t)$ and $\psi(x,t)$ are their counterparts in the linear core, respectively. κ is a real positive parameter and accounts for the linear coupling coefficient between the cores. c defines the relative group velocity in the linear core, while group velocity term in the nonlinear core is set equal to 1. θ represents the phase mismatch between the two gratings . The range of θ is limited to the interval $0 \leq \theta \leq 2\pi$ (Tsofe and Malomed, 2007).

To determine the bandgap structure, Eqs. (1) are first linearized and upon substitution of plane wave solutions $\{u, v, \phi, \psi\} \sim \exp(ikx - i\omega t)$ into the linearized equations followed by some algebraic manipulations we arrive at the following dispersion relation:

$$
\omega^4 - \left[2 + 2\kappa^2 + \left(1 + c^2\right)k^2\right]\omega^2 + \left(c^2 - 2c\kappa^2 + 1\right)k^2 + \left(\kappa^4 - 2\kappa^2 cos\left(\frac{\theta}{2}\right) + 1\right) + c^2k^4 = 0 \qquad (2)
$$

From the straightforward analysis of Eq. (2), it is found that in the case of $c = 0$ and $0 \leq \theta \leq 2\pi$, the spectrum contains three disjoint band gaps. It should be noted that the stationary soliton solutions fill with the entire three gaps. In the case of $c \neq 0$, only the central gap contains soliton solutions.

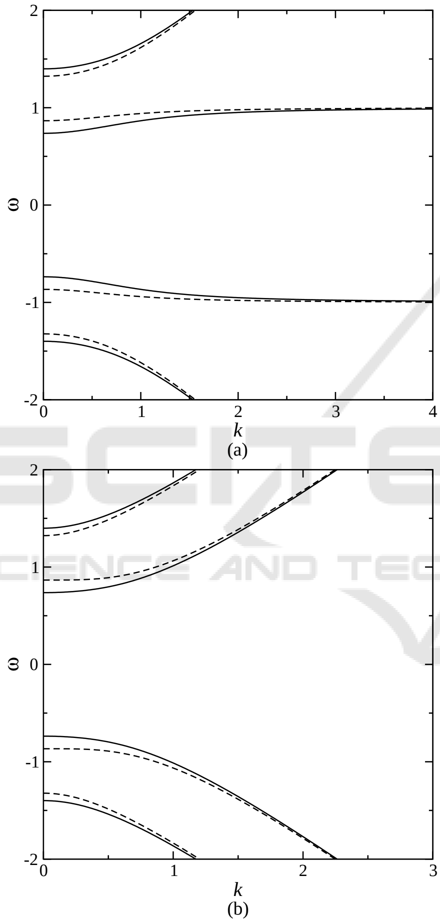

Figure 1: Dispersion relation diagrams obtained from Eq. (2) for $\kappa = 0.5$ and (a) $c = 0.0$; (b) $c = 1.0$. The solid and dashed lines correspond to $\theta = 0$ and $\theta = \pi$, respectively.

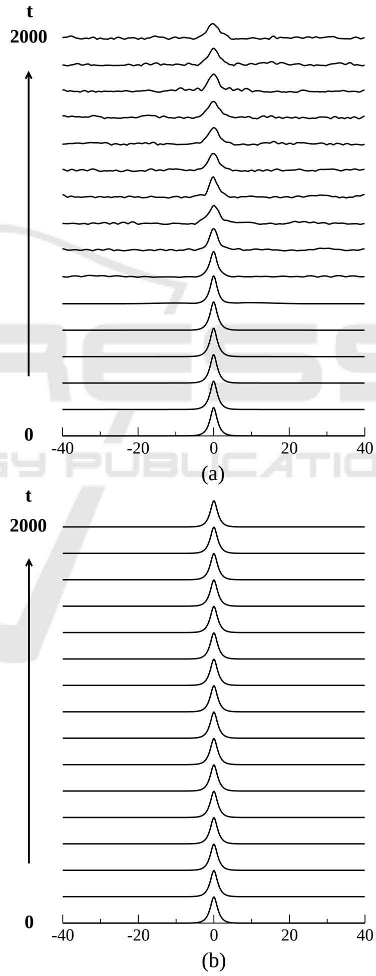

Figure 2: Evolution of quiescent gap solitons for (a) $\omega = 0.10$ and $\theta = 0.0$ (Unstable) ; (b) $\omega = 0.30$ and $\theta = 2\pi$ (Stable). The values of other parameters are $\kappa = 0.2$, $c = 0$. Only the $u-$component is shown here.

In the case of $\theta = 0$, maximum value of frequency detuning in the central band gap is limited to $|\omega_{max}| < (1 - \kappa)$. However, when $\theta \neq 0$, the central gap's edge

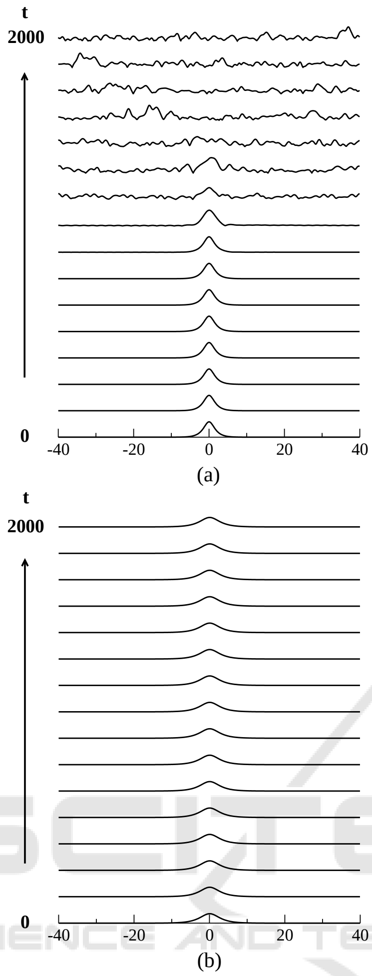

Figure 3: Evolution of quiescent gap solitons in the upper band gap for (a) $\theta = 0.0$ (Unstable) ; (b) $\theta = \pi$ (Stable). The values of other parameters are $\kappa = 0.2$, $c = 0$ and $\omega = 1.11$. u components only shown here.

change significantly. In a specific case, when the group velocity term of both cores is similar, i.e., $c = 1$, two different situations are possible. If $\kappa \leq cos\left(\frac{\theta}{4}\right)$, the maximum value of frequency detuning ω_{max} in the central band gap is obtained at $k = 0$.

3 STABILITY ANALYSIS

Since there are no exact analytical solutions for Eqs. (1), the soliton solutions have to be obtained numerically. This is done by substituting $\{u(x,t), v(x,t)\} = \{U(x), V(x)\} e^{-i\omega t}$ and $\{\phi(x,t), \psi(x,t)\} = \{\Phi(x), \Psi(x)\} e^{-i\omega t}$ into Eqs. (1) which results in a set of ordinary differential equations that can be solved by means of a relaxation algorithm. In the case of $c = 0$, quiescent solitons exist in the upper, lower and the central bandgaps. On the other hand, for $c \neq 0$, soliton solutions exist only in the central bandgap.

We have investigated the stability of the numerically obtained gap soliton solutions using the split-step Fourier method. It is found that there exist stable and unstable solitons in the system. Figs. 2 and 3 show the examples of stable and unstable quiescent gap solitons for different values of c, κ , ω and θ. It is noteworthy that unstable solitons may either evolve to another quiescent soliton (see Fig. 2 (a)) or be completely destroyed.

Fig. 4 summarizes the results of the stability for $c = 0.0$ and $\kappa = 0.2$ in the (θ, ω) plane. A notable feature shown in this figure is there exist a vast stable region in both the central and upper bandgaps. However, no stable solitons are observed in the lower band gap.

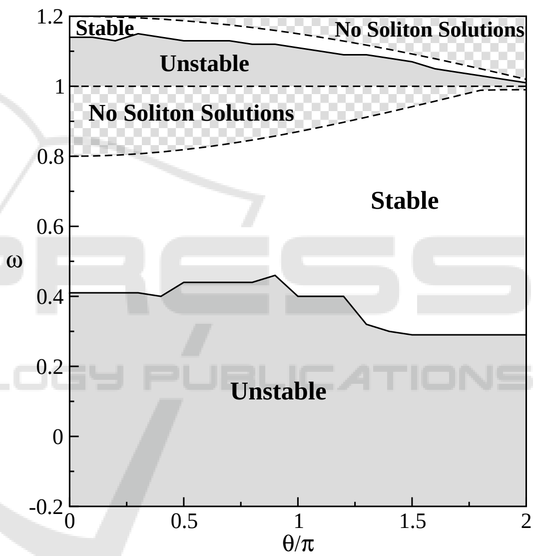

Figure 4: Stability diagram of the quiescent gap solitons in the (θ, ω) plane for $\kappa = 0.2$ and $c = 0.0$.

4 CONCLUSIONS

We have introduced a model of semilinear dual-core system, where Bragg gratings with a phase shift θ between them are written on both cores, and one core has Kerr nonlinearity, while the other one is linear. When the group velocity mismatch is zero, three disjoint band gaps are found including one central gap and two lower and upper gaps. In this case, quiescent solitons exist throughout the three band gaps. However, in case of $c \neq 0$, only the central gap contains the quiescent soliton solutions and no solitons are found in the lower and upper gaps. Stability of gap solitons is investigated numerically. For $c = 0$, stable solitons are found only in the upper and central bandgaps.

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