Moving Solitons in Coupled Bragg Gratings with a Uniform and a Nonuniform Bragg Gratings

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Keywords: Soliton, Kerr Nonlinearity, Dispersive Reflectivity.

Abstract: We consider the dynamics of moving solitons in a dual-core nonlinear system which consists of a uniform Bragg grating coupled with a nonuniform Bragg grating where nonuniformity is provided by dispersive reflectivity. It is found that moving solitons fill the entire bandgap. We also consider the effect of the dispersive reflectivity on the stability of the moving solitons.

1 INTRODUCTION

Fiber Bragg grating (FBG) is a periodic optical medium which offers a strong dispersion that can be counterbalanced by the Kerr nonlinearity of optical fiber results a gap soliton (GS). Theoretically, it has been shown that GSs can travel at any velocity in the range of zero to the speed of light within the medium (Aceves and Wabnitz, 1989; De Sterke and Sipe, 1994; Christodoulides and Joseph, 1989). The existence of moving GSs have been confirmed experimentally (Eggleton et al., 1999; Mok et al., 2006; De Sterke et al., 1997; Taverner et al., 1998). So far, the moving GSswith the velocity of 23% of the speed of light speed in the medium have been observed (Mok et al., 2006). The existence and stability of GS have also been considered in other systems such as dual-core systems (Atai and Malomed, 2000; Mak et al., 1998a; Mak et al., 2004; Chowdhury and Atai, 2014), waveguide arrays (Dong et al., 2011; Mandelik et al., 2004), photonic crystals (Monat et al., 2010), nonuniform gratings (Atai and Malomed, 2005; Baratali and Atai, 2012; Neill et al., 2008), quadratic nonlinearity (Conti et al., 1997; Mak et al., 1998b), and cubic-quintic nonlinearity (Atai and Malomed, 2001; Dasanayaka and Atai, 2013). Slow moving Bragg solitons may be used to develop several optical devices such as optical delay lines, optical switches and logic gates (Krauss, 2008; Fraga et al., 2006).

The interest in dual-core and dual-mode nonlinear systems arises from their rich dynamical characteristics (Atai and Chen, 1992; Mak et al., 2004; Chen and Atai, 1998; Chen and Atai, 1995). Additionally, the nonlinear dual-core systems with non-identical cores are known to offer better switching characteristics than the ones with identical cores (Atai and Chen, 1992; Bertolotti et al., 1995). Also, it has been found that dispersive reflectivity in FBGs may significantly influence on the solitons' stability (Atai and Malomed, 2005; Neill et al., 2008; Baratali and Atai, 2012; Chowdhury and Atai, 2014). Hence, in this work, we consider the existence and stability of moving solitons in a dual-core system with the Kerr nonlinearity where one core has a uniform Bragg grating and the other one is equipped with a Bragg grating with dispersive reflectivity.

2 THE MODEL

Propagation of light in a dual-core system in the presence of Kerr nonlinearity where one core has a uniform Bragg grating and the other one has a Bragg grating with dispersive reflectivity is described by the following system of differential equations (Hossain and Atai., 2020):

$$
\begin{gathered}
i\left(u_{1t}+u_{1x}\right)+u_1\left(\tfrac{1}{2}\left|u_1\right|^2+\left|v_1\right|^2\right) \\
+\lambda u_2+v_1+mv_{1xx}=0, \\
i\left(v_{1t}-v_{1x}\right)+v_1\left(\tfrac{1}{2}\left|v_1\right|^2+\left|u_1\right|^2\right) \\
+\lambda v_2+u_1+mu_{1xx}=0, \\
i\left(u_{2t}+u_{2x}\right)+u_2\left(\tfrac{1}{2}\left|u_2\right|^2+\left|v_2\right|^2\right) \\
+\lambda u_1+v_2=0, \\
i\left(v_{2t}-v_{2x}\right)+v_2\left(\tfrac{1}{2}\left|v_2\right|^2+\left|u_2\right|^2\right) \\
+\lambda v_1+u_2=0.
\end{gathered} \tag{1}
$$

Hossain, M. and Atai, J.
Moving Solitons in Coupled Bragg Gratings with a Uniform and a Nonuniform Bragg Gratings.
DOI: 10.5220/0010229700320035
In *Proceedings of the 9th International Conference on Photonics, Optics and Laser Technology (PHOTOPTICS 2021)*, pages 32-35
ISBN: 978-989-758-492-3

In Eqs. (1), $u_{1,2}(x,t)$ and $v_{1,2}(x,t)$ represent the forward- and backward-propagating waves in core 1 and core 2, respectively. λ represents the coupling coefficient between two cores and m denotes the coefficient of dispersive reflectivity which accounts for nonuniformity in the Bragg grating. m varies from 0 to 0.5, since $m > 0.5$ doesn't have any physical importance (Atai and Malomed, 2005).

To obtain the dispersion relation for moving solitons, the system of Eqs. (1) are first transformed into the moving frame using the transformation $\{X,T\} = \{x - st, t\}$ where s stands for the moving solitons' velocity. After some straiaghtforward algebraic manipulations, the following dispersion relation for the moving solitons is obtained:

$$\begin{aligned}\Omega = \pm\big(1 + k^2 + \lambda^2 \mp \tfrac{1}{2}\big(m^4k^8 - 4m^3k^6 \\ +4m^2\lambda^2k^4 + 4m^2k^4 - 16m\lambda^2k^2 + 16\lambda^2k^2 \\ +16\lambda^2\big)^{\frac{1}{2}} - mk^2 + \tfrac{1}{2}m^2k^4\big)^{\frac{1}{2}} - sk.\end{aligned} \tag{2}$$

Here, Ω denotes the frequency in the moving frame. Figure 1 shows the linear spectrum in (k,Ω) plane for different values of m.

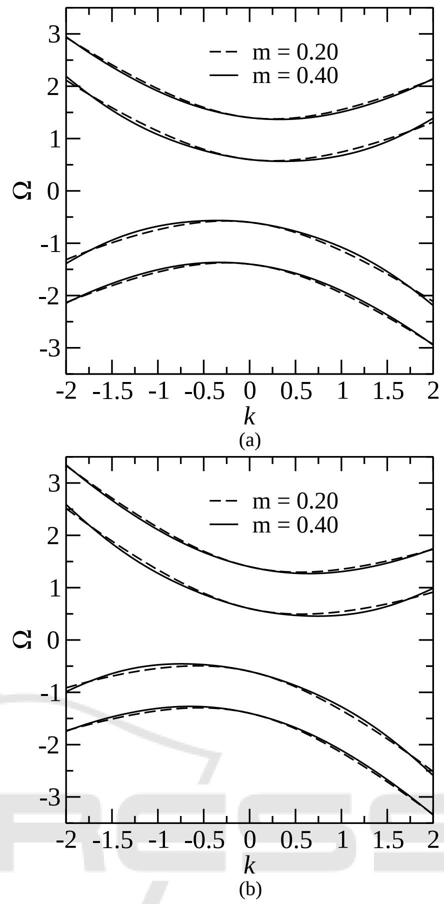

Figure 1: Examples of dispersion diagrams for moving solitons for different values of dispersive reflectivity at $\lambda = 0.40$. (a) $s = 0.20$ and (b) $s = 0.40$.

3 SOLITON SOLUTIONS

To find the solutions for moving solitons, we substitute $U(X,t) = U(X)\exp(-i\Omega t)$ and $V(X,t) = V(X)\exp(-i\Omega t)$ into the system of Eqs. (1) which leads to the following system of equations:

$$\begin{aligned}\Omega U_1 + i(1-s)U_{1X} + U_1\left(\tfrac{1}{2}|U_1|^2 + |V_1|^2\right) \\ +\lambda U_2 + V_1 + mV_{1XX} = 0, \\ \Omega V_1 - i(1+s)V_{1X} + V_1\left(\tfrac{1}{2}|V_1|^2 + |U_1|^2\right) \\ +\lambda V_2 + U_1 + mU_{1XX} = 0, \\ \Omega U_2 + i(1-s)U_{2X} + U_2\left(\tfrac{1}{2}|U_2|^2 + |V_2|^2\right) \\ +\lambda U_1 + V_2 = 0, \\ \Omega V_2 - i(1+s)V_{2X} + V_2\left(\tfrac{1}{2}|V_2|^2 + |U_2|^2\right) \\ +\lambda V_1 + U_2 = 0.\end{aligned} \tag{3}$$

Since there is no exact analytical solution for Eqs. (3), the equations need to be solved numerically. Figure 2 shows examples of the moving solitons' profile (the real component, imaginary component, and amplitude of u_1 and v_1).

4 STABILITY ANALYSIS

We have investigated the stability of solitons by numerically simulating their propagation. We have found both stable and unstable solitons in the bandgap. Fig. 3 shows some examples of the stable and unstable moving solitons. As is shown in Fig. 3(c), unstable solitons radiate some energy after some period of time and subsequently are destroyed. Fig. 4 shows the stability diagram which summarizes the stability of solitons in (m,Ω) plane for $\lambda = 0.2$ and $s = 0.1$. A notable feature shown in this figure is that for moderate values of dispersive reflectivity (i.e. between 0.25 and 0.35), the stable region is enlarged. The effect and interplay of λ and the velcoity of solitons on the stability of solitons are very complex and are currently being investigated.

5 CONCLUSION

We have studied the existence and stability of moving solitons in a dual-core system with Kerr nonlinearity where one core possesses a uniform Bragg grating and another core possesses a nonuniform Bragg

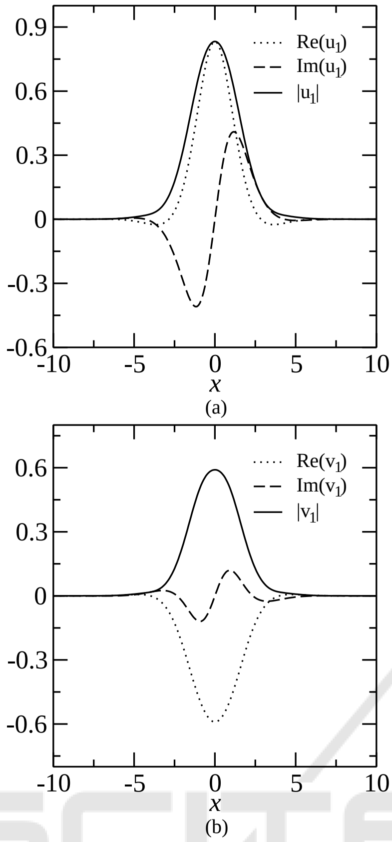

Figure 2: Moving solitons' profiles for $s = 0.20$, $\lambda = 0.20$, $m = 0.40$, and $\Omega = 0.40$. (a) u_1 and (b) v_1.

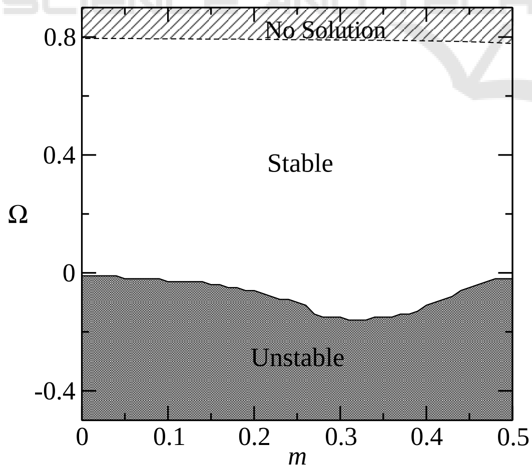

Figure 4: Stability diagram for $\lambda = 0.20$ and $s = 0.10$.

grating. The analysis shows that moving solitons exist throughout the bandgap. The stability characteristics of moving solitons have been investigated numerically. It is found that both stable and unstable solitons exist in the bandgap. We have determined nontrivial stability borders in the plane of dispersive

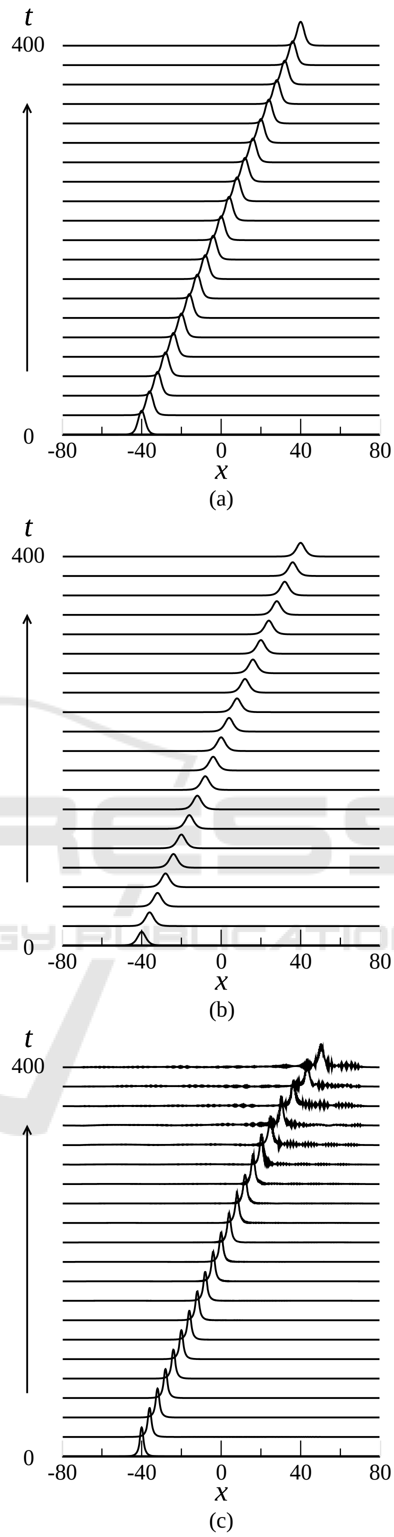

Figure 3: Examples of moving solitons' propagation of (a) stable u_1, (b) stable u_2 for $m = 0.32$, $\Omega = 0.64$, and (c) unstable u_1 for $m = 0.12$, $\Omega = -0.64$ while $s = 0.20$, $\lambda = 0.20$.

reflectivity and frequency. Another finding is that the presence of dispersive reflectivity in one core can lead to enhancement of the stability of solitons in certain parameter ranges.

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