A Multi-level Rank Correlation Measure for Image Retrieval

Nikolas Gomes de Sá, Lucas Pascotti Valem and Daniel Carlos Guimarães Pedronette
Department of Statistics, Applied Math. and Computing, São Paulo State University (UNESP), Rio Claro, Brazil

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Abstract: Accurately ranking the most relevant elements in a given scenario often represents a central challenge in many applications, composing the core of retrieval systems. Once ranking structures encode relevant similarity information, measuring how correlated are two rank results represents a fundamental task, with diversified applications. In this work, we propose a new rank correlation measure called Multi-Level Rank Correlation Measure (MLCM), which employs a novel approach based on a multi-level analysis for estimating the correlation between ranked lists. While traditional weighted measures assign more relevance to top positions, our proposed approach goes beyond by considering the position at different levels in the ranked lists. The effectiveness of the proposed measure was assessed in unsupervised and weakly supervised learning tasks for image retrieval. The experimental evaluation considered 6 correlation measures as baselines, 3 different image datasets, and multiple features. The results are competitive or, in most of the cases, superior to the baselines, achieving significant effectiveness gains.

1 INTRODUCTION

Ranking tasks are ubiquitous in many aspects of daily life: from arrangement of personal preferences to modelling tool of priorities in enterprise environments (Webber et al., 2010). Many institutions keep rankings of broad interest, aiming to guide decisions in diversified domains, including books, universities, artists, and many others. In fact, rankings represent a powerful organization instrument in many scenarios, allowing the definition of relationships among objects, according to a certain measure.

In artificial intelligence and information retrieval applications, rankings have been widely used to represent the preferences of agents (humans or systems) over a set of candidates (Xue et al., 2020). Due to desired properties, as data reduction, independence of scale and facilities for representation, rankings and other ordinal data structures have been attracting diverse applications (Farnoud Hassanzadeh and Milenkovic, 2014).

As a result of the widely possibilities of applications, rankings are often needed to be compared. Such comparisons often allow to infer the similarity of the processes or systems which have generated the rankings (Webber et al., 2010). Especially in information retrieval, where information representation is often supported by scores and ranked lists of items, the task of performing comparison between two ranked lists is of central importance (Yilmaz et al., 2008). This relevance arises from distinct applications, including comparison between rankings returned by different search engines, the lists of query recommendation given by different algorithms (Vigna, 2015), and complementarity between features in image retrieval (Valem and Pedronette, 2020).

In order to provide an objective and repeatable comparison of ranked lists, it is needed to define a rank correlation measure (Webber et al., 2010). In fact, correlation coefficients are well-known statistical tools, widely exploited in statistical analysis, pattern recognition, and image processing. One of the more traditional measures is the Pearson correlation coefficient, which only measures linear dependence relations (Couso et al., 2018). The *rank correlation measures* or distances between permutations have also a long and interdisciplinary history (Kumar and Vassilvitskii, 2010; Fagin et al., 2004; Webber et al., 2010).

The most popular rank correlation statistics are the Kendall's τ and Spearman correlation coefficient (Kumar and Vassilvitskii, 2010). While the Spearman correlation is equivalent to L1 distance between ranks, the Kendall's τ between two ranked lists is proportional to the number of pairwise inversions needed to convert one ranking into the other (Yilmaz et al., 2008; Kumar and Vassilvitskii, 2010). Both are originally non-weighted measures in the sense that they

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do not assign different weights to elements at top positions of ranked lists. With applications predominantly in information retrieval, various efforts have been made in extending traditional measures to generalized weighted models (Couso et al., 2018; Okada et al., 2015; Vigna, 2015).

In addition to weighted approaches of traditional measures, many other rank correlation measures have been proposed (Fagin et al., 2004; Tan and Clarke, 2015; Xue et al., 2020; Vigna, 2015). In a representative work (Fagin et al., 2004), the challenge of defining distance measures between top-k lists is addressed considering different aspects. Various rank correlation measures are presented under a unified framework proposed to catalog them. The intersection metric is firstly defined in this work, based on the size of intersection between ranked lists at different depths. This information is also exploited by the Rank-Biased Overlap (RBO) measure (Webber et al., 2010). RBO analyzes the overlap of two rankings at incrementally increasing depths, considering a parameter that models the user persistence in considering the overlap at the next level. The weight of the overlap measured at each depth is computed based on these probabilities.

Other rank correlation measures were proposed by exploiting information retrieval measures formulations. In (Yilmaz et al., 2008), a rank correlation measure based on the average precision (AP) is proposed. In (Tan and Clarke, 2015), a family of rank measures based on effectiveness is proposed, considering some analogies with RBO. The interest of the research community on rank correlation measures keeps active and novel measures have been proposed. Recently, a novel framework (Xue et al., 2020) was proposed based on the analysis of the consensus of rankings by considering common patterns embedded in a ranking set.

Among the diversified scenarios of applications, image retrieval systems have been successfully employing rank-based analysis and rank correlation measures in the last years (Qin et al., 2011; Chen et al., 2014; Valem et al., 2018; Pedronette et al., 2019). The rank correlation measures have been mostly exploited in contextual distance/similarity learning tasks. In fact, ranked lists represent a relevant source of contextual information in retrieval tasks. Different from traditional distance/similarity measures, which perform only pairwise analysis, ranked lists establish relationships among sets of images. In these scenarios, unsupervised learning algorithms have been proposed to compute more effective distance/similarity measures based on comparisons of ranked lists (Chen et al., 2014). Diverse rank correlation measures have been used for this purpose and studies have shown that the measure drastically impacts the results (Okada et al., 2015).

This paper proposes a novel Multi-Level Correlation Measure (MLCM) for rank comparisons in image retrieval tasks. While weighted measures assign more relevance to top positions, our proposed approach goes beyond by considering the position at different levels in the ranked lists. A broad experimental evaluation was conducted in order to assess the effectiveness of the measure in image retrieval tasks. The experiments were performed on three public datasets considering different features and effectiveness evaluation. Comparisons with traditional and recent rank correlation measures were also conducted and the proposed approach achieved the higher results on most of the experiments.

The remaining of this paper is organized as follows. Section 2 describes the rank model used along the paper, and Section 3 presents the rank correlation measures proposed. Section 4 describes the experimental evaluation and Section 5 discusses conclusions and future work.

2 RANK MODEL DEFINITION

This section presents a formal definition of the ranking model considered along the paper. Let $\mathcal{C}$=$\{img_1, img_2, \ldots, img_n\}$ be an image collection, where n denotes the size of the collection.

A distance between two images img_i, img_j is defined as $\rho(i, j)$ and can be computed by different image features. Based on the distance function ρ, a ranking model can be derived. For a general image retrieval task, a ranked list τ_q can be computed in response to a query image img_q, according to the distance function ρ. The top positions of ranked lists are expected to contain the most relevant images with regard to the query image, such that only the top-L ranked images are considered, with $L \ll n$.

The ranked list τ_q can be formally defined as a permutation (img_1, img_2, $\ldots$, img_L) of the subset $\mathcal{C}_L \subset \mathcal{C}$, which contains the L most similar images to a query image img_q, such that $|\mathcal{C}_L| = L$. A permutation τ_q is a bijection from the set $\mathcal{C}_L$ onto the set $[n_L] = \{1, 2, \ldots, L\}$. The notation $\tau_q(i)$ defines the position (or rank) of image img_i in the ranked list τ_q. Therefore, if img_i is ranked before img_j in the ranked list of img_q, i.e., $\tau_q(i) < \tau_q(j)$, then $\rho(q, i) \leq \rho(q, j)$.

Considering every image in the collection as a query image, a set of ranked lists $\mathcal{T} = \{\tau_1, \tau_2, \ldots, \tau_n\}$ can be obtained. The ranked lists are used as input to the rank correlation measures. The set $\mathcal{T}$ represents a rich source of similarity information about the collection, which can be exploited through rank correlation measures in unsupervised learning tasks, as discussed in the experimental evaluation.

3 MULTI-LEVEL CORRELATION MEASURE

This section presents the proposed Multi-Level Correlation Measure (MLCM). The key ideas and motivations are introduced in Section 3.1. The formal definition of MLCM is presented in Section 3.2, while Section 3.3 discusses efficiency and complexity aspects.

3.1 Overview

In most of real-world information retrieval applications, both human and machines are interested in top-k lists (Fagin et al., 2004). The size constraint allows to handle the overhead of information and concentrate on the essential. However, the definition of k often offers a challenging trade-off. While small values can ignore relevant information, large values can include useless data or detrimental noise.

A natural way to reduce this problem is given by weighted rank correlation measures (Vigna, 2015). Assigning weights to top positions allows to consider more information, once the low weights are assigned to lower positions of ranked lists included in the analysis. Nevertheless, even for weighted measures, the k continues to represent a binary boundary which can exclude useful information right after the defined threshold.

With the objective of proposing a novel alternative to this problem, we propose a multi-level approach. Firstly, the elements at the top-k positions of a ranked list are considered. The co-occurrence of such elements are verified in the other ranked list, but considering a relaxed level, until the top-ck positions. In the following, the same analysis is reciprocally performed by inverting the ranked lists and the thresholds. In this way, relevant elements at top-k positions of one ranked list and just after k in the other ranked list can also contribute positively to the correlation analysis.

The proposed approach and its benefits are illustrated in Figure 1, representing the comparison between two ranked lists τ_i and τ_j. Analogous to typical real-world ranked lists, the top positions present high-effective results (in blue). In the following, a mixed zone contains both relevant and non-relevant elements (in gray). Both elements x and y are at top-k positions of one ranked list and at top-ck positions of the other ranked list.

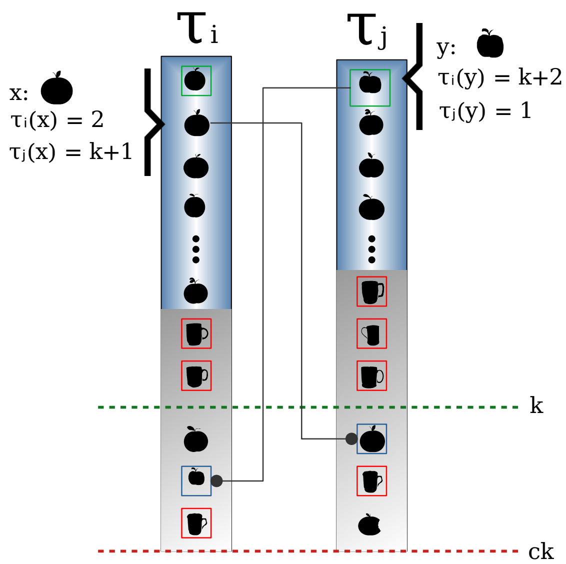


Figure 1: Multi-Level Correlation Measure (MLCM) applied to rank comparison between ranked lists τ_i and τ_j.

3.2 MLCM Formal Definition

This section formally defines the proposed MLCM measure. The measure is computed considering the top positions of ranked lists. Therefore, firstly we defined a k-neighborhood set $\mathcal{N}(\tau_i, k)$, which contains the k most similar images to img_i. The set can be formally defined according to our rank model as follows:

$$\begin{aligned} \mathcal{N}(\tau_i, k) = \{ & e : e \in \mathcal{S}, \mathcal{S} \subseteq \mathcal{C}, |\mathcal{S}| = k \wedge \\ & \forall x \in \mathcal{S}, y \in (\mathcal{C} - \mathcal{S}) : \tau_i(x) < \tau_i(y) \}. \end{aligned} \tag{1}$$

In order to characterize the multi-level behavior of the measure, an extended intersection set $\mathcal{E}(\tau_i, \tau_j, c, k)$ between ranked lists τ_i and τ_j is defined. The set takes the ranked list τ_i at a level of top-k positions, while takes the τ_j at a lower level, of top-ck. Formally, the set is defined as:

$$\mathcal{E}(\tau_i, \tau_j, c, k) = \mathcal{N}(\tau_i, k) \cap \mathcal{N}(\tau_j, c \times k). \tag{2}$$

The similarity between the ranked lists τ_i and τ_j is directly associated to the size of the extended intersection set, once similar ranked lists are expected to present co-occurrences at top positions. Beyond that, the proposed MLCM measure assigns a weight to each image in the set according to the position that it appears in each ranked list.

An one-directional MLCM measure is defined by the sum of products of weights assigned to each element in the extended intersection set. The function μ

is formally defined as:

$$\mu(\tau_i, \tau_j) = \sum_{x \in \mathcal{E}(\tau_i, \tau_j, c, k)} w_i(x) \times w_j(x), \quad (3)$$

where $w_i(x)$ denotes the weight of element img_x in the ranked list τ_i. Higher weights are assigned to first positions, with an exponential formulation according to the position of img_x in the ranked list τ_i. The function is formally defined as:

$$w_i(x) = p^{\tau_i(x)}, \quad (4)$$

where p is a parameter defined in the interval [0,1]. A low value of p reduces the weights of elements located at lower positions of ranked lists.

The one-directional MLCM measure definition given by the function $\mu(\tau_i, \tau_j,)$ is not symmetric, due to different levels defined in the ranked lists. Therefore, we can verify that $\mu(\tau_i, \tau_j) \neq \mu(\tau_j, \tau_i)$. In order to solve this problem, making the function symmetric, a bi-directional MLCM measure is defined as follows:

$$MLCM(\tau_i, \tau_j) = (1 - p) \times \mu(\tau_i, \tau_j) \times \mu(\tau_j, \tau_i). \quad (5)$$

In addition to effectiveness, efficiency aspects are crucial in real-world scenarios. The MLCM measure can be also efficiently computed as discussed in the next section.

3.3 Efficiency and Complexity Aspects

All the analysis computed by the MLCM measure are constrained to the top-ck positions of ranked lists, and therefore, independent of the collection size (n) or the size of ranked lists (L). Such characteristic is crucial to allow an efficient computation of the measure.

We consider a hash table data structure, for representing each ranked list. The structure allows to insert and find elements in $O(1)$ time complexity. Therefore the construction of hash tables presents a complexity of $O(ck)$. In order to compute the extended intersection set, for each element at top-k positions of one ranked list, its presence in the other hash table should be verified. Once each verification requires $O(1)$ complexity, the whole set can be computed in $O(k)$.

The computation of function μ (Equation 3) requires to retrieve the position of each element in the intersection set in both ranked lists. Again, using the hash structure, the position can be computed in $O(1)$ for each element, totaling $O(ck)$ for the whole set. In this way, the MLCM can be fully computed in $O(ck)$, i.e, in linear time according to the extended ck-neighborhood.

4 EXPERIMENTAL EVALUATION

This section presents a broad experimental evaluation conducted to assess the effectiveness of the proposed measure. Section 4.1 discusses aspects of the experimental protocol, describing the datasets, features, and the effectiveness measures considered in the experiments. Section 4.2 evaluates the proposed MLCM measure in weakly supervised scenarios, on the task of identifying similarity relationships among images of the same class. Section 4.3 evaluates MLCM in unsupervised re-ranking tasks for image retrieval, in comparison with other rank correlation measures.

4.1 Experimental Protocol

This subsection describes the experimental protocol adopted in this work, including information about the datasets, features, effectiveness measures, and parameter settings.

The experimental evaluation considered three public datasets, with different characteristics and sizes ranging from 1,360 to 5,000 images. The datasets used were MPEG-7 (Latecki et al., 2000), Flowers (Nilsback and Zisserman, 2008) and Corel5k (Liu and Yang, 2013).

Multiple features were considered[1], including global, local, and deep learning ones. In the first set of experiments, two features were used per dataset. For the re-ranking evaluation, all the features were considered. The employed Convolutional Neural Networks (CNN) were all trained on the ImageNet dataset. The implementations of these CNNs are publicly available on the PyTorch framework [2].

In the experimental evaluation, different well-established effectiveness measures are considered, such as Precision, Recall, F-Measure and Mean Average Precision (MAP).

4.1.1 Parameters Settings

The MLCM measure used the multi-level parameter as $c = 2$ for all experiments. Regarding the similarity relationships identification tasks (Section 4.2), the MLCM measure used the parameter value as p=0.93. The size of neighborhood k was used as k=10 for MPEG-7 and k=50 for Corel5k and Flowers datasets. The baseline measures used k=20 for MPEG-7 and k=50 for the Corel5k and Flowers datasets.

For the unsupervised re-ranking tasks (Section 4.3), the parameter p=0.96 was used in all ex-

[1] For the MPEG-7 dataset, we have used the distances to other images as features.

[2] https://github.com/Cadene/pretrained-models.pytorch

periments[3]. The neighborhood size k was defined as k=20 for MPEG-7 and k=50 for Corel5k and Flowers datasets. The re-ranking algorithm used T=2 for MPEG7 and T=3 for Corel5k and Flowers datasets.

4.2 Similarity Relationships Identification

This section presents the experimental evaluation of MLCM measure on weakly supervised scenarios, with the objective of identifying similar elements. Section 4.2.1 describes the task while Section 4.2.2 presents the baselines. Section 4.2.3 discusses the results.

4.2.1 Task Description

In a weakly supervised scenario, where only a small set of labeled images are available, identifying similar images is of crucial relevance. In this way, we evaluate the capacity of MLCM measure on identifying similar elements (of the same class) in the dataset. If two images have their ranked lists very correlated, i.e, with a rank correlation measure greater than a certain threshold, we assume that they belong to the same class. This assumption may be more or less accurate depending on the effectiveness of each measure.

With the objective of analyzing the behavior of the proposed measure, an experiment was conducted by varying the threshold to evaluate the impact in the effectiveness measures. High values of threshold lead to small or insignificant expansions. On the other hand, as the values decrease, the number of images contained in the expanded set also increases. However, it tends to incorporate incorrect images in this set as well, which can be especially harmful to the accuracy results. This trade-off can be analyzed through Precision and Recall measures.

4.2.2 Compared Rank Correlation Measures

Six correlation measures often used in the literature were considered as baselines and have their results compared to MLCM measure. The measures used were Intersection (Fagin et al., 2003), Jaccard (Levandowsky and Winter, 1971), Jaccard$_k$ (Okada et al., 2015), Kendallτ (Fagin et al., 2003), Spearman (Fagin et al., 2003) and RBO (Webber et al., 2010).

[3]Except for AIR features on the MPEG-7 dataset, which used p=0.81

4.2.3 Results and Discussion

Firstly, we evaluate the impact of threshold variation on Precision, Recall, and F-Measure. The curves were reported in according to the threshold variation in the interval [0, 1]. Figures 2 and 3 report the results for the datasets MPEG-7, Flowers and Corel5K. The features used were ASC for MPEG-7 and RESNET for Flowers, and Corel5K. For comparison purposes, we also report the results obtained by RBO measures considering the same scenario. We can observe that MLCM results are more stable to threshold variations when considering F-Measure. RBO often achieves higher precision scores, but with smaller recall scores. In opposite, MLCM combine better both measures, which leads to higher F-Measure scores.

The threshold that achieved the highest F-measure for each measure/feature/dataset is reported in Table 1. The results with the two best F-Measure values are highlighted in bold for each feature and dataset with the corresponding threshold. As we can observe, the results obtained by MLCM are very significant, since F-Measure is practically always between the two best results, which does not occur for any other metric. The average F-measure is presented for each measure and it is noticeable that MLCM presented the highest mean as well. We can also observe that, in comparison to the RBO measure, MLCM achieved superior or comparable results in all the cases.

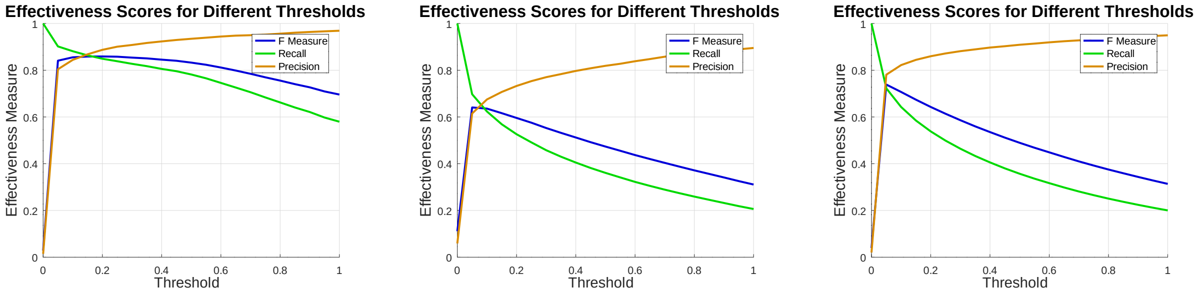
4.3 Unsupervised Re-ranking on Image Retrieval Tasks

This section discusses the evaluation of MLCM measure on unsupervised re-ranking tasks of image retrieval. Section 4.3.1 provides more details about the task. Section 4.3.2 discusses the results and 4.3.3 present some visual results.

4.3.1 Task Description

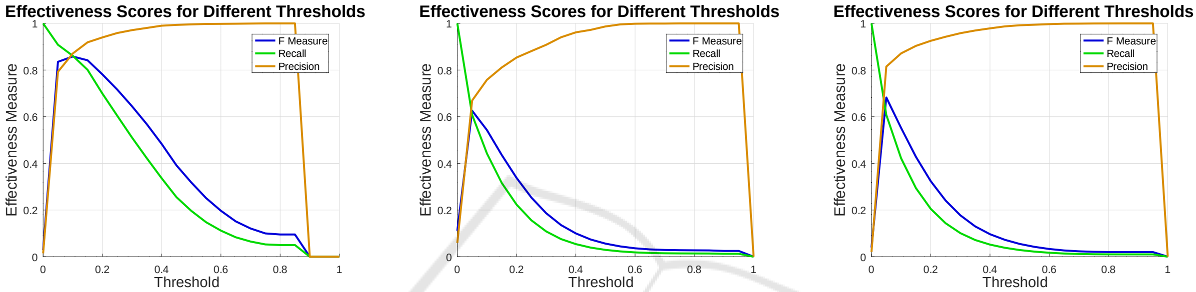
Despite the huge advances on image retrieval achieved in last decades, mainly supported by deep learning technologies, computing effective similarity measures remains a challenging tasks. In this scenario, various approaches have been proposed for post-processing image similarities through more global a contextual analysis. Such unsupervised re-ranking approaches provides an attractive solution, capable of significantly improving the retrieval results without the use of any labeled data.

The RL-Sim* (Okada et al., 2015) method is an unsupervised re-ranking algorithm that relies on a



(a) MPEG-7 (b) Flowers (c) Corel5k

Figure 2: Precision, Recall and F-Measure obtained for MLCM measure considering different thresholds. Results for **MPEG-7 - ASC, Flowers - RESNET**, and **Corel5k - RESNET**, respectively.



(a) MPEG-7 (b) Flowers (c) Corel5k

Figure 3: Precision, Recall and F-Measure obtained for RBO measure considering different thresholds. Results for **MPEG-7 - ASC, Flowers - RESNET**, and **Corel5k - RESNET**, respectively.

Table 1: F-Measure results: maximum F-Measure achieved by each rank correlation measure on different datasets.

		MPEG-7		Flowers		Corel5k		Mean
		ASC	CFD	ACC	RESNET	ACC	RESNET	
MLCM	**F-Measure**	**0.859**	**0.849**	**0.247**	**0.641**	**0.287**	0.739	**0.6037**
	Threshold	0.2	0.15	0.05	0.05	0.05	0.05	—
Intersection	**F-Measure**	0.850	0.838	0.226	**0.633**	0.286	**0.759**	0.5987
	Threshold	0.3	0.25	0.05	0.05	0.15	0.15	—
Jaccard	**F-Measure**	0.825	0.810	0.243	0.616	0.281	0.756	0.5885
	Threshold	0.3	0.25	0.1	0.15	0.1	0.1	—
Jaccard$_k$	**F-Measure**	0.853	0.842	**0.248**	0.626	**0.289**	**0.759**	0.6028
	Threshold	0.15	0.1	0.05	0.05	0.05	0.05	—
Kendalτ	**F-Measure**	0.809	0.802	0.241	0.618	0.27	0.697	0.5728
	Threshold	0.4	0.4	0.3	0.35	0.3	0.35	—
RBO	**F-Measure**	**0.858**	**0.849**	0.232	0.626	0.268	0.682	0.5858
	Threshold	0.1	0.1	0.05	0.05	0.05	0.05	—
Spearman	**F-Measure**	0.851	0.838	0.245	**0.633**	0.286	**0.759**	0.6020
	Threshold	0.3	0.25	0.15	0.15	0.15	0.15	—

correlation measure in order to compute a new similarity score among images by comparing their kNN sets. In this section, the proposed MLCM measure is evaluated on re-ranking tasks through the RL-Sim* algorithm. The evaluation is conducted considering several recent deep learning features. We used the RL-Sim* implementation available on the Unsupervised Distance Learning Framework (UDLF) (Valem and Pedronette, 2017). The RBO measure was also considered as a baseline for this evaluation.

4.3.2 Results and Discussion

Tables 2, 3, and 4 present the results for re-ranking tasks on MPEG-7, Corel5k and Flowers datasets. Different effectiveness measures are

Table 2: Effectiveness evaluation of MLCM compared to RBO, considering different measures on MPEG-7 dataset.

MPEG-7		**Effectiveness**							
Descriptors	**Measures**	**P@10**	**P@15**	**P@20**	**P@30**	**P@50**	**P@100**	**Recall@40**	**MAP**
AIR	**MLCM**	**0.96**	**0.953**	**0.939**	**0.657**	**0.4**	**0.2**	**1.0**	**0.969**
(Gopalan et al., 2010)	**RBO**	0.951	0.949	0.939	0.656	**0.4**	**0.2**	0.999	0.961
ASC	**MLCM**	**0.927**	**0.904**	0.874	**0.619**	**0.382**	**0.194**	**0.946**	**0.908**
(Ling et al., 2010)	**RBO**	0.924	0.903	**0.878**	0.617	0.381	**0.194**	**0.946**	0.907
BAS	**MLCM**	**0.842**	**0.786**	0.734	**0.536**	**0.341**	**0.179**	**0.834**	**0.778**
(Arica and Vural, 2003)	**RBO**	0.831	0.781	**0.735**	0.534	0.34	0.178	**0.834**	0.774
CFD	**MLCM**	**0.934**	**0.907**	0.878	**0.621**	**0.384**	**0.195**	**0.949**	**0.912**
(Pedronette and da S. Torres, 2010)	**RBO**	0.929	0.906	**0.879**	0.617	0.382	0.194	0.944	0.909
IDSC	**MLCM**	**0.91**	**0.875**	0.845	**0.605**	**0.373**	**0.191**	**0.925**	**0.882**
(Ling and Jacobs, 2007)	**RBO**	0.907	**0.875**	**0.851**	0.603	**0.373**	**0.191**	**0.925**	**0.882**
SS	**MLCM**	**0.55**	**0.478**	**0.424**	**0.332**	**0.227**	**0.13**	**0.539**	**0.458**
(da S. Torres and Falcão, 2007)	**RBO**	0.538	0.465	0.417	0.328	0.224	0.128	0.531	0.451

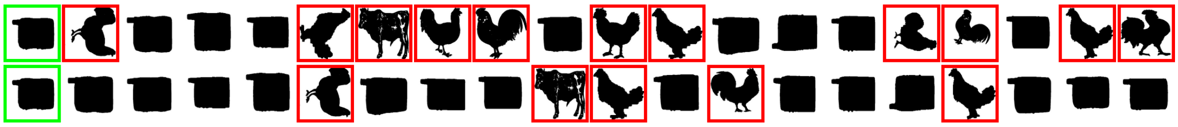
Table 3: Effectiveness evaluation of MLCM compared to RBO, considering different measures on Corel5k dataset.

Corel5k		**Effectiveness**							
Descriptors	**Measure**	**P@10**	**P@15**	**P@20**	**P@30**	**P@50**	**P@100**	**Recall@40**	**MAP**
CNN-BnInception	**MLCM**	**0.895**	**0.882**	**0.872**	**0.855**	**0.823**	**0.716**	**0.336**	**0.739**
(Ioffe and Szegedy, 2015)	**RBO**	0.887	0.871	0.858	0.837	0.801	0.691	0.328	0.712
CNN-DPNet	**MLCM**	**0.905**	**0.893**	**0.885**	**0.87**	**0.846**	**0.776**	**0.343**	**0.807**
(Chen et al., 2017)	**RBO**	0.899	0.886	0.876	0.859	0.831	0.754	0.338	0.785
CNN-FBResNet	**MLCM**	**0.924**	**0.914**	**0.906**	**0.895**	**0.872**	**0.804**	**0.354**	**0.836**
(He et al., 2016)	**RBO**	0.913	0.9	0.891	0.878	0.855	0.776	0.347	0.809
CNN-ResNet	**MLCM**	**0.923**	**0.912**	**0.904**	**0.891**	**0.867**	**0.794**	**0.352**	**0.829**
(He et al., 2016)	**RBO**	0.919	0.905	0.895	0.879	0.854	0.771	0.347	0.808
CNN-ResNeXt	**MLCM**	**0.921**	**0.911**	**0.904**	**0.891**	**0.869**	**0.795**	**0.352**	**0.827**
(Xie et al., 2017)	**RBO**	0.915	0.903	0.894	0.878	0.852	0.771	0.346	0.804
CNN-VGGNet	**MLCM**	**0.874**	**0.858**	**0.846**	**0.824**	**0.788**	**0.678**	**0.322**	**0.705**
(Liu and Deng, 2015)	**RBO**	0.863	0.844	0.83	0.806	0.765	0.657	0.314	0.679
CNN-Xception	**MLCM**	**0.891**	**0.877**	**0.867**	**0.851**	**0.82**	**0.723**	**0.335**	**0.737**
(Chollet, 2017)	**RBO**	0.883	0.866	0.853	0.834	0.8	0.7	0.327	0.714

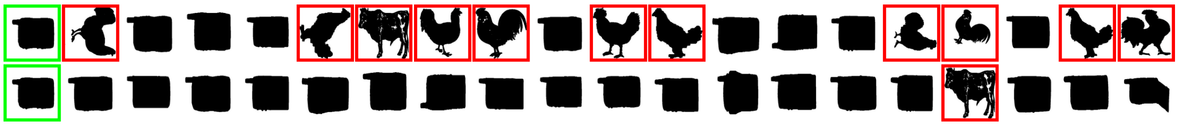
Table 4: Effectiveness evaluation of MLCM compared to RBO, considering different measures on Flowers dataset.

Flowers		**Effectiveness**							
Descriptors	**Measure**	**P@10**	**P@15**	**P@20**	**P@30**	**P@50**	**P@100**	**Recall@40**	**MAP**
CNN-BnInception	**MLCM**	**0.863**	**0.845**	**0.829**	**0.801**	**0.749**	0.578	**0.387**	**0.71**
(Ioffe and Szegedy, 2015)	**RBO**	0.853	0.834	0.817	0.791	0.748	**0.58**	0.386	0.704
CNN-DPNet	**MLCM**	**0.85**	**0.832**	**0.817**	**0.791**	**0.747**	0.577	**0.385**	**0.702**
(Chen et al., 2017)	**RBO**	0.842	0.818	0.805	0.779	0.735	**0.58**	0.379	0.69
CNN-FBResNet	**MLCM**	**0.871**	**0.854**	**0.841**	**0.819**	**0.774**	0.591	**0.398**	**0.734**
(He et al., 2016)	**RBO**	0.857	0.838	0.824	0.802	0.76	**0.594**	0.391	0.72
CNN-ResNet	**MLCM**	**0.857**	**0.838**	**0.825**	**0.802**	**0.759**	0.587	**0.391**	**0.723**
(He et al., 2016)	**RBO**	0.851	0.831	0.818	0.795	0.753	**0.592**	0.387	0.715
CNN-ResNeXt	**MLCM**	**0.852**	**0.839**	**0.825**	**0.804**	**0.766**	**0.593**	**0.392**	**0.727**
(Xie et al., 2017)	**RBO**	0.844	0.827	0.812	0.789	0.748	0.589	0.384	0.709
CNN-VGGNet	**MLCM**	**0.779**	**0.755**	**0.735**	**0.702**	**0.646**	**0.487**	**0.338**	**0.591**
(Liu and Deng, 2015)	**RBO**	0.775	0.749	0.728	0.695	0.639	**0.487**	0.333	0.583
CNN-Xception	**MLCM**	**0.826**	**0.803**	**0.788**	**0.761**	**0.713**	**0.559**	**0.368**	**0.677**
(Chollet, 2017)	**RBO**	0.819	0.796	0.777	0.748	0.7	0.555	0.361	0.665

considered: Precision, Recall and MAP. It can be observed that MLCM achieved the best results in most of the evaluated features and effectiveness measures. The MLCM measure also reaches the highest MAP scores for the three datasets.



(a) Re-Ranking by RBO measure.



(b) Re-Ranking by MLCM measure.

Figure 4: Visual results before and after RL-Sim* Re-Ranking computed by RBO and MLCM measures on MPEG-7 dataset.

4.3.3 Visual Results

Visual retrieval results for MPEG-7 dataset are illustrated on Figure 4. The figure shows the results before and after the application RL-Sim* re-ranking algorithm considering both the RBO and MLCM rank correlation measures. The query image is illustrated in a green board. The ranked lists obtained are presented on the right, with incorrect images in red borders. Remarkable effectiveness gains can be observed.

5 CONCLUSIONS

In this work, a novel rank correlation measure is proposed, capable of exploiting multi-level information of ranked lists. A diversified experimental evaluation showed that the proposed MLCM measure achieves results comparable or superior to other relevant measures. As future work, we intend to evaluate other retrieval scenarios (e.g. video, sound, and text retrieval for example).

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