Quantum Control for Error Correction using Mother Tee Optimization

Wael Korani[a] and Malek Mouhoub[b]
Department of Computer Science, University of Regina, Regina, Saskatchewan, Canada

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Abstract: Quantum Control Problem (QCP) for Error Correction (EC) is a significant issue that helps in producing an efficient quantum computer. The QCP for EC can be tackled using Stochastic Local Search (SLS) methods. However, these techniques might produce low quality results for large dimensional quantum systems. Lately, Nature-Inspired (NI) algorithms including different variants of Particle Swarm Optimization (PSO) and Deferential Evolution (DE) were implemented in several studies to tackle the QCP for EC, but the results were not promising. In this paper, we propose a quantum model that is built on our NI algorithm, called Mother Tree Optimization for QCP (MTO-QCP), to overcome the stagnation issue that the other methods suffer from. In order to assess the performance of MTO-QCP, we conducted several preliminary experiments to adjust our MTO parameters. In this regard, our MTO-QCP achieves high-fidelity ($> 99.99\%$) for a Single-Shot (SS) three-qubit gate control at gate operation time of 26 ns. This recommended fidelity is an acceptable threshold fidelity for fault-tolerant Quantum Computing (QC) problems.

1 INTRODUCTION

Quantum mechanics research has been significantly growing as a corner stone in computation in different branches of science (Caves, 2013) such as, metrology (Giovannetti et al., 2011), secure communication (Scarani et al., 2009), femtosecond lasers (Assion et al., 1998), and nuclear magnetic reassurance (Hopkins et al., 2003). The heart of these applications is quantum dynamics, which is a desired state or operation that could be reached by controlling a quantum system.

Control theory is a process of guiding the system's dynamics to a desired state or to optimize the dynamics of a given objective function, as in QCP for EC. This theory is built on a mathematical model for a physical system. Quantum control as a part of control theory is a process of producing a feasible solution of control parameters for a given control problem to produce an efficient system. It is known that the quantum control process is a very complicated process, because the dynamics of quantum system is often nonlinear and/or noncontinuous. Analytical solution for these kind of models is very sophisticated and needs much computations and resources. An alternative solution is reinforcement learning that facilitates representing a physical system in an explicit mathematical model (Sutton et al., 1992). In the last couple of decades, several attempts were conducted to use optimization methods for quantum states problems to achieve efficient systems.

QC is growing significantly due to the availability of real quantum computers (QCos) that allow users to perform quantum experiments. QC helps in speeding up the computations using quantum bit (qubit) strings and quantum gates (QGs). QCos have been used in different quantum applications using specific number of qubits. For instance, in (Sisodia et al., 2017), discrimination of orthogonal entangled states has a significant role in quantum information processing. A SS Toffoli (controlled-controlled-NOT (CCNOT)) gate is recommended for quantum information processing and other classical quantum application (Zahedinejad et al., 2015). The noise of the output (fidelity) is one of the common challenges in the performance of quantum applications (Linke et al., 2017).

In order to achieve a scalable QC, a number of high fidelity QGs is required to construct a quantum circuit (CCr) (Nielsen and Chuang, 2002). The fragility of the state of a QCos is compensated using quantum EC (Cory et al., 1998). In the past, several experiments were conducted on single and two-qubit operations, but the quantum EC code requires at least three-qubit using the Toffoli gate (Cory et al., 1998). In (Zahedinejad et al., 2016), the fidelity was calcu-

[a] https://orcid.org/0000-0002-1419-1149
[b] https://orcid.org/0000-0001-7381-1064

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lated using a decomposition method that used lower order qubit gates to build a multi-qubit gates. However, this method is not recommended, as it takes long operation times.

The rest of the paper is structured as follows. Section 2 lists some related studies and our motivation to propose a quantum model. Section 3 discusses quantum control and quantum gate design model. Section 4 gives a brief introduction about our recently proposed MTO. Section 5 explains the optimization quantum control that our model will solve. Section 6 shows our proposed model combining our MTO algorithm and QCP for EC. Section 7 reports on our experiments. Finally, section 8 lists concluding remarks and ideas for future work.

2 RELATED WORK

NI techniques as a part of the approximate class of algorithms received more attention in the last of decades for solving different hard optimization problems including QCPs. While exact optimization techniques require a prior knowledge about system dynamics such as Bayesian and Markovian feedback (Wiseman et al., 2002), NI techniques solve QCPs as a black box (i.e. the systems characteristics are unknown). NI methods were successfully implemented to find a desired state for QCPs. In (Srinivas and Deb, 1994), Particle Swarm Optimization (PSO), Differential Evolution (DE) and Genetic Algorithms (GAs) were implemented to solve quantum control schemes. However, these algorithms could not find a satisfactory solution when the number of control parameters increases.

Optimization methods have been successfully implemented to tackle difficult and sophisticated nonlinear or/and non-continuous optimization problems including quantum problems. Several attempts were conducted to tackle the QCP for EC. In (Fletcher, 2013), Quasi-Newton (QN) based on the Broyden-Fletcher-Goldfarb-Shanno (BFGS) approximation of Hessian is implemented in a model to tackle QCP for EC. The Nelder-Mead simplex (NMS) method that is a direct method and does not require derivatives (Olsson and Nelson, 1975), Particle Swarm Optimization (PSO) that is built on the movement of birds (Clerc and Kennedy, 2002), and Differential Evolution (DE) that is built on Darwinian principle (Zahedinejad et al., 2014) were applied to tackle QCPs. In (Ghosh et al., 2013), the authors introduced a quantum-control procedure to produce an optimal pulse for a Toffoli gate. Quasi-Newton, simplex methods, variants of PSO, Greedy algorithms as well as DE techniques, all failed to solve the QCP for EC. Indeed, it has been reported in (Zahedinejad et al., 2016) that all listed algorithms (QN, NMS, PSO, and DE) fail to generate satisfactory fidelity to QCP for EC higher than 99.3% (with the best fidelity, 99.31%, obtained by DE). However, the recommended level of fidelity to QCP for EC is 99.99% (Ghosh et al., 2013).

However, the previous implemented optimization techniques suffer from several drawbacks that make them inappropriate to tackle a challenging problem such as QCP. The simplex technique may optimize a problem without computing its derivatives, which usually consume much computing power. Although the simplex technique is simple and robust in tackling small dimension optimization problems, it might easily fail for large dimension problems (Bűrmen et al., 2006) and (Kelley, 1999).

GAs and DE algorithms should not be used to solve a given problem where there is a traditional exact method that can solve it, because these algorithms cannot be better with less computational effort (Schwefel, 2000). In (Back, 1996), Back concluded that these algorithms are not appropriate methods to solve strongly convex problems. In (Leung et al., 1997), Leung et al. concluded that GAs suffer from premature convergence. The authors suggested that increasing the population size plays an important role to help overcome this problem. In (Hrstka and Kučerová, 2004), DE may offer easier convergence by increasing the number of parents and reducing the scaling factor; however, DE, may then suffer from high computational time. (Chen et al., 2004)and (Shi and Eberhart, 2001), PSO variants and other algorithms are growing fast, and they offer an alternative way for tackling complex problems, but they have some limitations: parameter tuning, stagnation, and time critical applications

In 2015, the QCP for EC was solved using a proposed model that is built on DE variant and decomposition multi-gates concept. In (Zahedinejad et al., 2015), the result shows that the proposed model achieved fast high-fidelity Toffoli gate. In 2016 (Zahedinejad et al., 2016), the same authors implemented other fast three-qubit gates that outperforms the decomposition of qubit gates to lower order (i.e, one-quibt or/and two-qubit). In (Palittapongarnpim et al., 2017), the authors developed an optimization variant of DE to solve the QCP, where PSO and other evolutionary algorithms failed to achieve satisfactory results.

However, in (Zahedinejad et al., 2016), the authors proposed a variant of DE called SuSSADE to solve the QCP for EC and achieved high fidelity. The proposed variant SuSSADE was not well described and how the author implemented to solve QCP. The lack of description of SuSSADE and the drawbacks of other algorithms motivate us to propose a quantum control model that is built on our NI algorithm called Mother Tree Optimization (MTO) (Korani et al., 2019) algorithm, and using SS multi-QGs (Lanyon et al., 2009). The MTO has the capability to escape from local solutions using our fixed off-spring topology, and it is appropriate to solve high dimension problems compared to other PSO variants (Korani et al., 2019) and (Korani and Mouhoub, 2020). Here, a SS is represented a continuous time evolution that is also controlled to realize the QG. The decomposition of multi-QGs to lower order of qubit gates affects the performance and makes it slower. Thus, we use fast SS multi-QG as a basic component for achieving high fidelity, because it works significantly faster than the decomposition technique. In our model, the optimization problem is constructed based on Unitary Evaluation (UE) function that includes a Hamiltonian description of the quantum physical system, and it delivers high fidelity QG.

Our model, based on our MTO, is implemented to tackle the QCP for EC. MTO is a NI optimization technique that is simple and can be implemented and programmed in a few lines of code. MTO is applied on one instance (three-qubit gate control using a gate operation time of 26 ns) as an example, because QCP experiments take long time that might reach a month. MTO achieves promising results that reaches a high fidelity value ($> 99.99\%$). We expect that our quantum model can be implemented for different number of qubit gate control at different operating gate times to achieve the optimal parameters for piecewise-constant pulse.

3 QUANTUM CONTROL

The framework of quantum control procedure applications, that its dynamics are governed by quantum mechanics, is called quantum control (Dong and Petersen, 2010). The QG design is an example of quantum control procedure that uses the control theory to apply logic gates on different quantum bits (Zahedinejad et al., 2014). This QG design is used in QC to speed up computation using quantum bit (qubit). The mathematical model of QG design is discussed in this section.

3.1 Quantum Gate Design Model

QC is built on quantum control theory to perform computation in an efficient way, and it helps in speeding up the computation of different operations such as factorization (Shor, 1999) and searching process in database (Grover, 1996). The interaction between quantum system and environment in QC implementation may introduce noise called errors. The resultant errors will reduce the efficiency and invalidate the benefits of using quantum resources (Shor, 1996). Therefore, achieving protected quantum information could be obtained by decreasing the error rate to a specific threshold. Our aim is to propose a quantum model to increase the system fidelity to a certain level as shown in our case study for QCP of EC.

Qubit is the basic unit in QC, and is used to perform computations. This qubit has characteristics different than the classical bit. It may exist in any superposition state, unlike the classical bit that has only two states (0 or 1). In decomposition case, QC operations can be reduced to a set of lower order qubit gates (Barenco et al., 1995). However, the decomposition process will lead to increase the processing time and decrease the fidelity. Toffoli gates that work on more than two qubits is one of the recommended ways to increase system efficiency and fidelity. The Toffoli gate is a controlled/controlled-not gate working on three-qubits.

We implement the same mathematical model as in (Ghosh et al., 2013) and (Spiteri et al., 2018). We consider z coupled super conducting artificial atoms with parameters appropriate for a transmon system (TS). In addition, TS can also be capacitively coupled (CC). The TS includes individual transmon that has specific number of energy states (m), and $j \in [z] := [1,2,\ldots,z]$ are denoted their positions. The CC between transmons prodcues an interaction between all adjacent transmons in XY plane. The produced coupling strength is evaluated to be $g = 30MHz$. The applied pulse in ideal gate has a shifted frequency that is located in range of $-2.5\ GHz \leq \varepsilon_j(t) \leq 2.5\ GHz$. The Hamilton for z CC-TS is represented as the m^z-dimensional block diagonal matrix (Ghosh et al., 2013).

$$\frac{\boldsymbol{H}(t)}{h} := \hat{\boldsymbol{H}}(t) = \sum_{j=1}^{z} \mathcal{P}_{\mathbf{I}}^{\otimes z}(\operatorname{diag}\left(0,\varepsilon_k(t),2\varepsilon_k(t)-\eta,3\varepsilon_k(t)-\eta'\right)) + \frac{g}{2}\sum_{j=1}^{z-1} \mathcal{P}_{\mathbf{I}}^{\otimes(z-1)}\left(\boldsymbol{X}_j \otimes \boldsymbol{X}_{j+1} + \boldsymbol{Y}_j \otimes \boldsymbol{Y}_{j+1}\right) \quad (1)$$

$$\boldsymbol{X}_j = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & \sqrt{2} & 0 \\ 0 & \sqrt{2} & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}_j \quad (2)$$

$$\frac{\boldsymbol{Y}_j}{\mathrm{i}} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & -\sqrt{2} & 0 \\ 0 & \sqrt{2} & 0 & -\sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}_j \tag{3}$$

where $\mathbf{I}$ is represented the identity matrix and each block works on $\mathscr{H}_4^{\otimes z}$, where $\mathscr{H}_4^{\otimes z}$ is denoted the Hilbert space. The sum of all $\mathbf{B}$ with $(z-1)$ copies of $\mathbf{A}$ is the $\mathcal{P}_{\mathbf{I}}^{\otimes z}$ operator, where $\mathbf{B}$ is represented the Kronecker products. In addition, the coupling $\boldsymbol{X}_j$ is the generalized Pauli operators, and $\frac{\boldsymbol{Y}_j}{\mathrm{i}}$ is a simulation for uniform coupling, giving that there is no relationship between coupling operators and k (Ghosh et al., 2013).

Equation 1 is reduced to subspace due to the block-diagonal property of Hamiltonian:

$$\hat{\boldsymbol{H}}_p(t) = \mathscr{O}_z \hat{\boldsymbol{H}}(t) \mathscr{O}_z^{\dagger}, \tag{4}$$

where $\mathscr{O}_z^{\dagger}$ is denoted the operator that truncate $\hat{\boldsymbol{H}}(t)$. $\hat{\boldsymbol{H}}_p(t)$ is calculated over θ, where θ is denoted the gate time. The ***unitary operator*** is calculated as follows:

$$\boldsymbol{U}(\Theta) = \mathcal{T} \exp\left\{ \int_0^{\Theta} \mathrm{d}t \, \frac{\boldsymbol{H}_p(t)}{\mathrm{i}\hbar} \right\} \tag{5}$$

$$= \mathcal{T} \exp\left\{ -2\pi \mathrm{i} \int_0^{\Theta} \mathrm{d}t \, \hat{\boldsymbol{H}}_p(t) \right\},$$

where $\mathcal{T}$ operator is denoted to the time-ordering. $\boldsymbol{U}(\Theta)$ of the most three excitations are used to define Transmon states, where is denoted $\boldsymbol{U}(\Theta)$ computation space. The projection is calculated as follows:

$$\boldsymbol{U}_{\mathrm{cs}}(\Theta) = \mathscr{P} \boldsymbol{U}(\Theta) \mathscr{P}^{\dagger}, \tag{6}$$

The ***projected unitary*** is an operator that is used to calculate the *fidelity* of the system. The fidelity is evaluated by measuring the difference between the actual result and target gate U_{target}. The ***fidelity*** of the system is calculated as follows:

$$\mathcal{F}(\Theta, n; \varepsilon(t)) = \frac{1}{2^n} \left| \mathrm{tr}\left(\boldsymbol{U}_{\mathrm{target}}^{\dagger} \boldsymbol{U}_{\mathrm{cs}}(\Theta) \right) \right| \in [0,1] \tag{7}$$

and it represents the QG operation for given shifted frequencies. The value of desired fidelity is ($>$ 99.99%).

$$\mathcal{F}(\Theta, n; \varepsilon(t)) \geq 99.99\% \tag{8}$$

4 MOTHER TREE OPTIMIZATION (MTO) ALGORITHM

MTO and its variant, MTO-CL, are built on a co-operative system of Douglas fir trees (Korani et al., 2019). The basic idea of the MTO algorithm is built on our agent communication topology called Fixed-Offspring (FO) (Korani et al., 2019). The number of agents in the population is represented by group of Active Food Sources (AFSs). The size of the population is denoted as $\mathrm{N_T}$. Figure 1 shows that the population is divided into four different partitions. Position of each agent is updated based on the group that agent belongs to (Korani et al., 2019). Top Mother Tree (TMT) updates its position randomly as shown in Equation 9, 10, 11, and 12 (Korani et al., 2019). Figure 1 shows the Partially Connected Trees (PCTs) and Fully Connected Trees (FCTs). In addition, the PCTs is divided into FPCTs and LPCTs as shown in Figure 1.

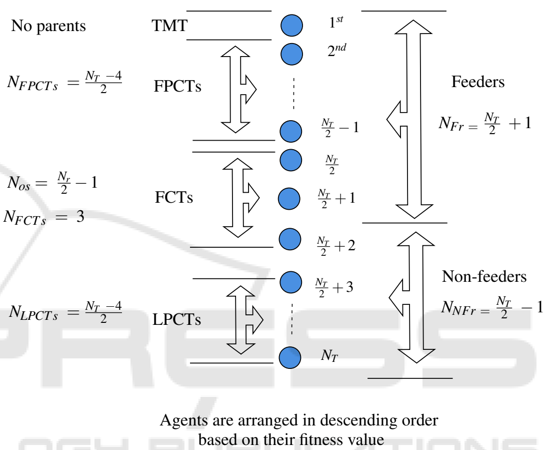

Figure 1: MTO Topology.

The TMT has two searching levels: At the first level, it updates its position randomly as follows:

$$P_1(x_{k+1}) = P_1(x_k) + \delta R(d) \tag{9}$$

$$R(d) = \frac{R}{\sqrt{R \cdot R^{\intercal}}} \tag{10}$$

$$\vec{R} = 2\,(round\,(rand\,(d,1)) - 1)\,rand\,(d,1) \tag{11}$$

where δ is the root signal step size, $R(d)$ is a random vector, and d is the dimension of the problem (in our case 26*3). At the second level, the TMT updates its position randomly with smaller step size Δ as follows.

$$P_1(x_{k+1}) = P_1(x_k) + \Delta R(d) \tag{12}$$

where Δ is the Mycorrhizal Fungi Network (MFN). Values of δ and Δ are adapted based on several preliminary experiments.

The FPCTs group updates its agents' positions as follows:

$$P_n(x_{k+1}) = P_n(x_k) + \sum_{i=1}^{n-1} \frac{1}{n-i+1}(P_i(x_k) - P_n(x_k)), \quad (13)$$

where $P_n(x_k)$ is denoted the current position of any member in the group, $P_i(x_k)$ is denoted the current position of a feeder agent, and $P_n(x_{k+1})$ is denoted the updated position of the current member (Korani et al., 2019). **The FCTs group** updates its members' positions as follows (Korani et al., 2019):

$$P_n(x_{k+1}) = P_n(x_k) + \sum_{i=n-N_{os}}^{n-1} \frac{1}{n-i+1}(P_i(x_k) - P_n(x_k)). \quad (14)$$

The LPCTs group updates its agents' positions as follows (Korani et al., 2019):

$$P_n(x_{k+1}) = P_n(x_k) + \sum_{i=n-N_{os}}^{N_T-N_{os}} \frac{1}{n-i+1}(P_i(x_k) - P_n(x_k)). \quad (15)$$

Algorithm 1 is the pseudo-code of the MTO method. More details about this algorithm can be found in (Korani et al., 2019).

5 THE QUANTUM CONTROL FOR ERROR CORRECTION PROBLEM

The QCP for EC is formulated as an optimization problem that could be tackled using our MTO algorithm. Our objective is to find a solution for a three-qubit optimization problem. In addition, our optimization problem should meet a significant constraint such that the fidelity meets a threshold (99.99%). The transmon-shifted frequencies are considered control pulses. These pulses are discretized using piecewise-constant. More formally, our quantum optimization problem is formalized as follows:

$$\text{find } \varepsilon(t), t \in [0, \Theta], \quad (16a)$$

$$\text{subject} -2.5 \leq \varepsilon_k(t) \leq 2.5, n = 1, 2, \ldots, k, \quad (16b)$$

$$\mathcal{F}(\Theta, n; \varepsilon(t)) \geq 99.99\%. \quad (16c)$$

The frequency components in Equation 16b are represented as piecewise-constant step function that has a time step duration $\Delta = 1$ ns. The gate time is an integer number of Δ t for a given simulation. Equation 16c has $n\theta/\Delta t$ degrees of freedom, so that increasing the gate time θ or decreasing the time step Δt, results in increasing the probability of finding solution. The degree of freedom represents the piecewise-constant values.

Algorithm 1: MTO (Korani et al., 2019).

```
Require: : N_T, P_T, d, K_rs, Cl, and El
  N_T: The population size (AFSs)
  P_T: The position of the active food sources
  d: The dimension of the problem
  K_rs: The number of kin recognition signals
  Cl: The number of climate change events (0 for MTO)
  El: The elimination percentage
  Distribute T agents uniformly over the search space (P_1, ..., P_T)
  Evaluate the fitness value of T agents (S_1 ... S_T)
  Sort solutions in descending order based on the fitness and store them in S
  S = Sort(S_1 ... S_T)
  The sorted positions with the same rank of S stored in array A
  A = (P_1 ... P_T)
  loop
    for k_rs = 1 to K_rs do
      Use equations (9)–(15) to update the position of each agent in A
      Evaluate the fitness of the updated positions
      Sort solutions in descending order and store them in S
      Update A
    end for
    if Cl = 0 then
      BREAK;
    else
      Select the best agents in S ((1 - El) S)
      Store the best selected position in Abest
      Distort Abest (mulitply by random vector)
      Distort(Abest) = Abest * R(d)
      Remove the rest of the population (El)S
      Generate agents equal to the the number of removed agents
      Cl = Cl - 1
    end if
  end loop (Cl > 0)
  S = Sort(S_1 ... S_T)
  Global Solution = Min(S)
  return Global Solution.
```

We implement our mathematical model for QCP for EC to design SS high fidelity QGs (Toffoli gates). In our model, the quantum dynamics is represented using Hamiltonian evolution. However, the complexity of our quantum optimization problem becomes very difficult with increasing the number of transmons n. In addition, there are some limitation of the selected θ and Δt to meet the practical requirements of quantum computers.

6 PROPOSED MODEL

Our proposed model includes two major parts: the optimization component (MTO) and the mathematical model for QCP for EC. The optimizer is shown on

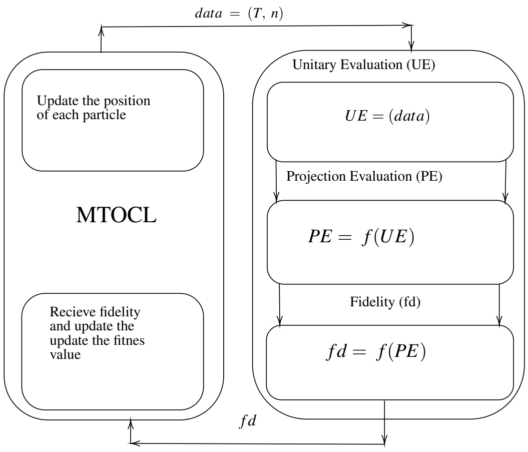

Figure 2: SS high-fidelity three-qubit Toffoli gate.

the left hand side while the quantum model is located on the right hand side, as shown in Figure 2. The QCP for EC represents SS high-fidelity three-qubit Toffoli gate.

The data exchange between our optimizer (MTO) and our quantum model (QCP for EC) runs for a predefined number of iterations until stopping criteria is reached. In the first iteration, the optimizer generates random agents (equal to the population size), and each agent has a vector of all required parameters' values to the quantum model, so that each agent in the population represents a solution for our quantum problem. The number of parameters is equal to $(\theta * n)$, where θ is the gate operation time (in our case 26 *ns*), and n is the number of transmonsn (in our case 3 qubit). In the quantum model side, the computations is performed in several stages. In the first stage, the UE function receives the $(\theta * n)$ (26*3) values for each agent in the population from our MTO optimizer and computes the most three excitation. In the second stage, the output of UE function feeds the Projection Evaluation (PE) function to calculates the projection of these excitations as explained in Section 3. Finally, the fidelity function receives the output of the PE function and compares to the target gate U_{target}. The fidelity is calculated for each agent and then the results are sent back to the MTO optimizer as updated fitness values for their associated agents. In the optimizer side, in the next iteration, each agent in the population will update its position (quantum model solution) according to its updated fitness value (the received fidelity). The system keeps running and improves the fidelity record until it reaches the stopping criteria.

7 EXPERIMENTATION

We have two objectives in this experiments: find a solution for our proposed quantum model and evaluate the performance of our MTO algorithm in solving hard optimization problem. Our experiment is designed to achieve high output fidelity of our proposed model (QCP for EC) using our recently proposed MTO method. In addition, we test the reliability, efficiency, and validity of using MTO optimizer as a recently proposed NI algorithm in solving QCP for EC optimization problem. In (Korani et al., 2019), the authors recommended two different population sizes 40 or 60. In our experiments, we set the population size to be 60 active food sources, because it achieves better results than using other population sizes. In this regard, when we increase the population size the fidelity improves more quickly. The QCP for EC is considered one of the recent challenging optimization problems that may consume much resources and time that may reach to months. We set a limit of 10^7 iterations as the stopping criterion, which could takes months to finish. The time of the function evaluation totally depends on the computing resources that the user might use. In our experiment, our proposed model has been implemented using Python language and all experiments have been performed on a MacBook Pro with 2.3 GHz Intel Core i9 and 16GB 2400 MHz DDR4.

The performance of MTO is evaluated in tackling QCP for EC with respect to other well-known optimization algorithms. In (Ghosh et al., 2013), Ghosh et. al. have implemented and evaluated other basic optimization algorithms to solve the same quatum optimization problem, and the results are reported in Table 2. We compare these results to those we obtained for MTO. The authors in (Ghosh et al., 2013) conducted experiments comparing the performance of different optimization methods to their proposed DE variant. The parameters of MTO are adjusted based on extensive preliminary experiments that took long time as shown in Table 1.

Table 1: Parameter Settings.

Algorithm	Parameters settings
MTO or MTO-CL	Root signal $\delta = 50.0$ MFN signal $\Delta = 40.03$ Small deviation $\phi = 7.0$ Population size = 60 CL = 10 EL = 20%

Our quantum control model proposed in Section 6 is solved using MTO optimizer. Pulses are generated using MTO for a simulated SS high-fidelity three-

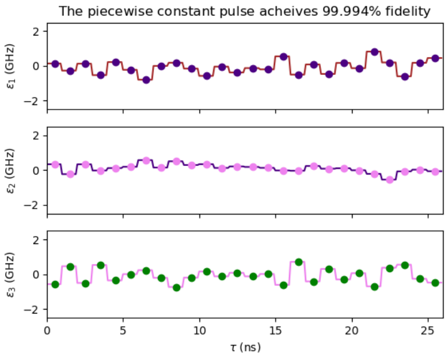

Figure 3: Optimal pulses for designing a Toffoli gate.

qubit Toffoli gate as described in Figure 2. In our experiment, the gate time is adjusted to 26 ns, and the model produces feasible solution in each iteration. At the end of the experiments, the optimal solution is recorded, which represents optimal pulses for our quantum model. The optimal pulses (piecewise-constant pulses) is depicted in Figure 3 for designing a Toffoli gate that achieves high fidelity (>99.994%). The dots circles in the figure denote the learning parameters for MTO. Our results show that our MTO optimizer could be considered as an alternative solution for QCP for EC compared to that results reported in (Ghosh et al., 2013) using different optimization algorithms: Quasi-Newton, Simplex, and DE as shown in Table 2. The main issue of using MTO in solving the QCP for EC is that it takes long time to produce the desired solution. In addition, solving the QCP for EC requires a steady system that could work for long time without interruption.

Table 2: The fidelity of 3-qubit Toffoli gate using different optimization algorithms (Ghosh et al., 2013).

$\theta = 26$ and $n = 3$	
Optimization method	Fidelity
Quasi-Newton	0.9912
Simplex	0.9221
DE	0.9931
SuSSADE	0.9999
MTO or MTOCL	**0.99994**

8 CONCLUSION AND FUTURE WORKS

We have designed a quantum model for solving QCP for EC using our recently proposed nature-inspired algorithm called MTO. Our model has been evaluated to solve one quantum instance for 3-qubit and gate time 26 ns. The results show the capability of MTO optimizer to tackle the QCP for EC and achieve the desired fidelity. MTO can be used as an alternative method for solving different quantum instances. We expect that MTO could be used for solving higher qubit order 4-qubit and 5-qubit of QCP of EC. Indeed, in the near future, we plan to implement more instances for 3-qubit and 4-qubit QCP for EC using more computing resources. The number of qubit n will be set to increase the difficulty of the problem. The frequency will be tuned until we get the best frequency along with gate operation time θ to produce the desired high fidelity.

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