Accelerating Homomorphic Encryption using Approximate Computing Techniques

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Abstract: This research proposes approximate computing techniques to accelerate homomorphic encryption (HE). In particular, the CKKS encryption scheme for approximate numbers is targeted. There is a requirement for HE in services dealing with confidential data, however current constructions are not efficient enough for real-time applications. A homomorphic encryption scheme which uses approximate arithmetic (showing faster results than previous HE schemes) already exists, the CKKS scheme, and this research applies a variation of the approximate computing techniques of task skipping and depth reduction (derived from loop perforation) to determine whether further approximating the functions evaluated using CKKS scheme can have a positive impact on performance of homomorphic evaluation. This is demonstrated via the evaluation of the logistic and exponential functions that this is possible, showing positive results. The speed up in running time for HE with task skipping is between 12.1% and 45.5%, depth reduction gives 35-45.5% speed-up with a small error difference than task skipping alone. The combination of both techniques corresponds to a halving of the running time, at the cost of increased error. This novel approach to further approximate homomorphic encryption shows that it is possible for certain functions, where running time is of paramount importance, that further approximations can be made with a lower-impacting greater error.

1 INTRODUCTION

As an increasing amount of data is stored in the cloud, data security and privacy is evermore important. Currently, to be able to compute on encrypted data stored on the cloud, data has to be downloaded and decrypted, using a key the provider has access to, before re-encrypting it and restoring on the cloud (Gentry, 2010). Homomorphic encryption (HE) removes the need for download and decryption, by allowing operations to be performed directly on encrypted data, thereby enabling safe computation and reducing the risk of data privacy-loss. Using HE, data on the cloud could be stored in ciphertext form, upon which analysis can be performed. Applications include machine learning algorithms to predict trends using sensitive data, electronic voting and secure database searches (Archer et al., 2017). The growing demand for analysis on encrypted data has motivated interest and activity towards standardisation of HE schemes and parameters (Albrecht et al., 2018).

Currently, HE schemes are effective in terms of evaluating functions in ciphertext form to a high level of accuracy, as shown by numerous applications, one being Secure Genome-Wide Association Studies (Blatt et al., 2019), but are not currently efficient enough for real-time usage. For example, in research which applied a logistic regression model homomorphically, the training took up to a few days (Chen et al., 2018). This research proposes the integration of approximate computing techniques with HE schemes to achieve improved HE evaluation runtimes.

One of the main motivations driving research into approximate computing is to design energy-efficient technology for the future (Barua and Mondal, 2019). One way to achieve this, is to trade-off between accuracy and the energy and running time required to compute algorithms. There are many applications where this trade-off is acceptable such as machine learning, signal processing, and big data analytics (Barua and Mondal, 2019). To the best of the authors' knowledge, evaluating the impact of approximate computing to advance the practicality of HE has not yet been conducted. This research provides an insight as to whether using approximate computing is feasible for approximate HE and the effect such techniques have when applied to the CKKS encryption scheme (Hee Cheon et al., 2017), currently the only HE scheme based on approximate arithmetic.

In this research, two approximate computing

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techniques, task skipping and loop perforation, are adapted to accelerate HE. To demonstrate the efficiency of the proposed approximate computing techniques, two target functions are considered for HE evaluation: the logistic and exponential functions. These are compared to the same operations without approximate computing optimisations. Both are implemented in Microsoft SEAL's HE library (SEAL, 2019), and results are given, measuring performance and average error with respect to the actual calculation.

2 RELATED WORKS

2.1 Homomorphic Encryption

Due to numerous data leaks and the increasing focus on the privacy of public data, including who has access to it and what is done with this data, being able to efficiently compute with encrypted data would be ideal for both the user and the company who wants to continue analysing data. An example application of HE, is e-voting (Chillotti et al., 2016). In 2009, the first fully homomorphic encryption (FHE) scheme was proposed (Gentry, 2009) after over 30 years research into finding the first FHE scheme. Over the past 10 years there has been a large advancement in the development of HE schemes. Further details on major HE schemes can be found in the following subsection.

2.1.1 Homomorphic Encryption (HE) Schemes

An elementary definition of homomorphic encryption is simply where computations can be performed on the ciphertexts, which, when decrypted, give the same result as if those same computations had been performed on the plaintexts (Halevi, 2017). This can be written as follows:

$$E(m_1) \star E(m_2) = E(m_1 \star m_2) \quad \forall m_1, m_2 \in M,$$

where E is an encryption algorithm, M is the set of all plaintexts and $\star$ is an arbitrary operation.

Definition 1. Homomorphic Encryption (Halevi, 2017), (Vaikuntanathan, 2011):
A homomorphic (public-key) encryption scheme $\mathcal{E}$ is defined as a quadruple of four probabilistic polynomial-time algorithms as follows:

- ***KeyGen:*** *An algorithm generating a public key pk and a private key sk from an input of a unary representation of the security parameter λ, 1^{λ}.*
- ***Encrypt:*** *An algorithm which takes as input the public key pk, and a single-bit message $\mu \in \{0,1\}$ and outputs a ciphertext c.*
- ***Decrypt:*** *An algorithm which takes as input the secret key sk, and a ciphertext c to output a message $\mu^{\star} \in \{0,1\}$.*
- ***Evaluate:*** *An algorithm which inputs the public key pk, a function represented using boolean circuits $BC \in \{BC\}$ (consisting of gates such as AND and XOR), the original ciphertext ($c =$ **Enc**(x)) and outputs a new ciphertext c'.*

A HE scheme is often denoted as
$\mathcal{E} =$ (***KeyGen, Encrypt, Decrypt, Evaluate***).

The difference between a standard encryption scheme and a HE scheme is that the HE scheme contains an *Evaluation* stage, where multiple ciphertexts (usually represented as a vector of ciphertexts) are inputs into the evaluation algorithm (which evaluates a given function) and outputs a new ciphertext. The choice of function to be evaluated depends on the application requirements.

Since Gentry's first FHE scheme (Gentry, 2009), a variety of HE schemes have been introduced. Several are based on the Learning with Errors (LWE) problem proposed by Regev (Regev, 2005), or its Ring variant (RLWE) (Lyubashevsky et al., 2013), for example, the BV Scheme (Brakerski and Vaikuntanathan, 2011), the LTV scheme (Lyubashevsky et al., 2013), the BGV scheme (Brakerski et al., 2012), the GSW scheme (Gentry et al., 2013) and the CKKS encryption scheme (Hee Cheon et al., 2017). There are other schemes which derive from those named above and more details (including the implementation libraries) can be found at (Albrecht et al., 2018). Since the aim of this research is to accelerate HE evaluation using approximate computing techniques, the CKKS scheme is best suited, since it enables approximate arithmetic (Hee Cheon et al., 2017).

2.1.2 The CKKS Encryption Scheme

The CKKS HE scheme was introduced in 2017 (Hee Cheon et al., 2017), and uses the RLWE assumption. However, unlike previous RLWE-based HE schemes, it does not use exact arithmetic, instead enabling approximate arithmetic. This approach enables the calculation of the approximate value of the plaintext after decryption, to a predetermined accuracy.

Before a text can be encrypted, it is scaled (by multiplying a scaling factor to the message), ensuring the size of the error (often considered as noise) does not grow out of hand during evaluation. It must also be encoded (and decoded after evaluation), where the input vector is converted into a polynomial using isometric ring isomorphisms, to ensure the size of the error and plaintext remain the same. The encryption of a message m, giving the ciphertext c satisfies

$< c, sk >= m + e(modq)$ where e is a small error, c is the ciphertext and sk is the secret key, stating that when the ciphertext, c, is decrypted using a secret key, sk, the result is the message, m with a small error, e. The scheme has a batching technique used to pack the plaintext messages into a number of slots, which can be run simultaneously, improving efficiency. Key switching techniques (similar to the BGV scheme) are required for ciphertext operations and whenever a ciphertext multiplication has occurred, the new ciphertext must be rescaled to the chosen scale before any further calculations are calculated. Once all operations are evaluated the final ciphertexts must be decrypted and decoded. A detailed explanation of the CKKS scheme is given in (Hee Cheon et al., 2017).

The Residual Number System (RNS) variant (Cheon et al., 2019) has fast polynomial approximation and can perform speedy multiplicative inverses and Discrete Fourier Transforms (DFT), when compared with the original CKKS scheme (Hee Cheon et al., 2017). It allows for larger integers to be implemented and for techniques based on RNS decomposition and the Number Theoretic Transform (NTT). Further information regarding the RNS-variant of the CKKS scheme can be found in (Cheon et al., 2019).

The CKKS scheme has been proposed for use in specific applications, e.g. logistic regression (Blatt et al., 2019) and a multi-party version for use in Neural Networks (Chen et al., 2019b). The HEAAN library (HEAAN, 2018) and Microsoft's HE library SEAL (SEAL, 2019) are two of the many libraries which implement the RNS variant of the CKKS scheme.

2.2 Approximate Computing

Due to the increasing demand of computing power outpacing supply (Mittal, 2016), and the increasing need for more 'real-time' computations, this has motivated the research into approximate computing, where a compromise is made between the quality and efficiency of calculations. Mittal (2016) provides a comprehensive survey of approximate computing techniques, summarised in this section. Software techniques include precision scaling, load value approximation, loop perforation and task skipping, memoization, multiple inexact program versions, voltage scaling, reducing branch divergence in SIMD architectures and specific neural network-focussed approximate computing techniques. Hardware techniques include using inexact or fault hardware, approximating SRAM/eDRAM/DRAM/non-volatile memories, using approximate computing techniques for GPUs/FPGAs, using scalable effort design for approximate computing and reducing error-correction overhead using approximate computing. The applications of these techniques has been shown in areas such as machine learning, signal processing and financial analysis, to name a few (Mittal, 2016).

Since, to the best of the authors' knowledge, this is the first research applying approximate computing to HE, the techniques which the authors believe could have a substantial impact on performance are targeted. This research proposes and adapts definitions of task skipping and loop perforation for acceleration of the CKKS HE scheme.

2.2.1 Task Skipping

Task skipping is an approximate computing technique, whereby a particular task, input or memory reference is skipped according to a user-set condition (Mittal, 2016). This can be any condition, for example when a particular variable hits a certain threshold, then a particular task is skipped. The task skipping condition is set so that, although a particular task is skipped when that condition is met, the quality of the result is still met. An example of task skipping in use can be seen in (Goiri et al., 2015), where the authors apply (as well as data input sampling) task skipping to MapReduce programs and using these with various statistical theories to reduce run times by approximately 38x with an error of tolerance of 1% with 95% confidence.

2.2.2 Loop Perforation

A basic definition of loop perforation is to skip certain iterations of a loop to output a result which is accurate to a certain (perhaps pre-defined) level of accuracy (Mittal, 2016), e.g. skipping every second term in the addition of a sequence, yet still result in an answer with only a 5% difference to the original solution. Sidiroglou-Douskos *et al.* define a 2-step loop perforation process; firstly to filter out loops which cannot be perforated ('critical loops') and secondly to find loops which can be perforated for certain inputs and a corresponding accuracy bound ('greedy exploration') (Sidiroglou-Douskos et al., 2011). They show performance improvements, whilst preserving accuracy, to applications such as information retrieval, machine-learning and Monte Carlo simulations. In this research, the concept of loop perforation was first studied as an approximate computing technique to be applied to the homomorphic evaluation of the selected functions. However, with no immediately recognisable loops in this process, using the multiplication approach described in (Song, 2017), each depth of the calculation is considered to be a loop. The question then becomes whether it is possible to calculate the evaluation with one depth reduced yet still output

an appropriately accurate answer. This leads to the approach of replacing the higher depth terms with the highest one of the depth below, and analysing the effect this has on the *Evaluation* process. This concept is a form of a known computing technique, depth reduction.

2.2.3 Depth Reduction

By representing a polynomial in the form of an arithmetic circuit, the number of gates (usually AND, XOR) denotes the size of the circuit and the depth is the longest path in the circuit (Tavenas, 2015), i.e. the maximum number of steps required to calculate an operation. For this research, this can be interpreted as the amount of operations required to calculate the term with the highest power of that polynomial. Much research has been carried out into arithmetic sizes and improving the corresponding depths, e.g. (Valiant and Skyum, 1981), (Tavenas, 2015), (Allender et al., 1998). These works do not take the same explicit approach of replacing the terms requiring higher depth calculations with those requiring lower depth calculations as in this research, however the principle of reducing the depth of a circuit to evaluate, for example, a polynomial, for efficiency purposes is the same.

3 PROPOSED APPROXIMATION TECHNIQUES FOR HE

3.1 Preliminaries

The security of this research aligns with the advised parameter sets as defined in the HE Community Standard, (Albrecht et al., 2018); the bits for each prime per level, the vector of inputs (must be a power of 2) and therefore the polynomial modulus degree are all aligned with the advised parameter sets. As with previous work on the CKKS scheme, (Hee Cheon et al., 2017), (Cheon et al., 2019), (Chen et al., 2019a), all methods proposed in this research have at least an 80-bit security level. The ring dimension is set as $N = 2^{14}$, resulting in $\frac{N}{2} = 2^{13}$ plaintext slots, which means that that the functions can be evaluated for each slot in parallel. The scale for this implementation is set at 2^{40} and the ciphertext modulus for the first and last level is set to 60-bit, with all intermediate moduli set to 40-bit. The parameters used here, especially for $2^{13} = 8192$ input slots, can be considered conservative. For further details on parameters see (Hee Cheon et al., 2017) or (Cheon et al., 2019). The accuracy of the defined functions to be calculated (logistic and exponential functions) is kept the same as in (Hee Cheon et al., 2017) to ensure fair comparison. Two sets of inputs have been used: The first set is the vector 0 to 1 split into 2^{13} slots and the second set is the vector 1 to 8192 split into 2^{13} slots, increasing by increments of 1.

The application of depth reduction and task skipping techniques in this research are to the RNS variant of the CKKS scheme, used to calculate the logistic function and the exponential function, using their Taylor Series approximations to orders 9 and 8 respectively as in (Hee Cheon et al., 2017). These functions have been chosen because of their usefulness: the logistic function is commonly used in statistics, neural networks and machine learning and the exponential function for being one of a typical circuit to be evaluated (Hee Cheon et al., 2017). The logistic function can be approximated by the corresponding Taylor Series approximation:

$$\begin{aligned} f(x) &= \tfrac{1}{1+e^{-x}} \\ &\approx \tfrac{1}{2} + \tfrac{1}{4}x - \tfrac{1}{48}x^3 + \tfrac{1}{480}x^5 \\ &\quad - \tfrac{17}{80640}x^7 + \tfrac{31}{1451520}x^9 + O(x^{11}) \end{aligned} \tag{1}$$

The exponential function, can be approximated by the corresponding Taylor Series approximation:

$$\begin{aligned} g(x) &= e^x \\ &\approx 1 + x + \tfrac{x^2}{2} + \tfrac{x^3}{6} + \tfrac{x^4}{24} + \tfrac{x^5}{120} + \tfrac{x^6}{720} \\ &\quad + \tfrac{x^7}{5040} + \tfrac{x^8}{40320} \end{aligned} \tag{2}$$

3.2 Approximate HE via Task Skipping

In this research, it is proposed that instead of explicitly setting a condition for when to skip a task, it is selected on the basis that for a decimal number between 0 and 1, the more it is multiplied by itself the smaller it becomes. This makes the largest powered term in each polynomial negligible, and therefore has less impact on the accuracy of the result if it is skipped. For any other input greater than 1, the removal of the largest powered term has the most impact on the accuracy so cannot be removed. Hence, for inputs greater than 1, the lowest two terms of the polynomials are skipped as they have less impact on the accuracy of the result.

For approximate HE with task skipping, three approaches are proposed and the approximated polynomials defined for the logistic and exponential functions respectively:

1. Approach 1 - input vector 0 to 1, the largest powered term in both polynomials is skipped.

$$f(x) \approx \tfrac{1}{2} + \tfrac{1}{4}x - \tfrac{1}{48}x^3 + \tfrac{1}{480}x^5 - \tfrac{17}{80640}x^7 \tag{3}$$

$$g(x) \approx 1 + x + \tfrac{x^2}{2} + \tfrac{x^3}{6} + \tfrac{x^4}{24} + \tfrac{x^5}{120} + \tfrac{x^6}{720} + \tfrac{x^7}{5040} \tag{4}$$

2. Approach 2 - input vector 0 to 1, the lowest three terms are skipped.

$$f(x) \approx \frac{1}{2} + \frac{1}{480}x^5 - \frac{17}{80640}x^7 + \frac{31}{1451520}x^9 \quad (5)$$

$$g(x) \approx 1 + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^8}{40320} \quad (6)$$

3. Approach 3 - input vector 1 to 'number of slots', the lowest three terms are skipped.

$$f(x) \approx \frac{1}{2} + \frac{1}{480}x^5 - \frac{17}{80640}x^7 + \frac{31}{1451520}x^9 \quad (7)$$

$$g(x) \approx 1 + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^8}{40320} \quad (8)$$

3.3 Approximate HE via Depth Reduction

In this proposed depth reduction approach, instead of certain iterations in a loop being skipped, a certain number of depths are skipped, (usually more than one depth skipped is not plausible due to the large error occurring as more terms are approximated by the largest term in the maximum depth required). Since the functions being evaluated are defined to orders of x, with results in depths of single-figures, the approach in this research is a one-depth reduction. The higher power terms which require the extra depth are replaced with the highest powered term of a lower depth. This replacement procedure aims to provide a more accurate estimation than task skipping alone and since performing an addition is less computationally intensive than multiplication, this results in a more accurate approximation with a lower computational cost. If these terms were not replaced, this would instead be a task skipping technique.

To further understand depth reduction (using the multiplication approach in (Song, 2017), see the following example: To calculate x^{11} homomorphically is a depth 4 calculation, one can reduce this to a depth 3 calculation, cx^7 (where c is a constant). In general, this can be the largest possible term calculated with one lower level of depth, which can be found by defining a new level whenever a polynomial term equals a power of 2. The novelty in this approach is to replace terms of the polynomials resulting in depth 4 calculation with the largest term requiring depth 3 calculations (as opposed to removing the term completely as in Task Skipping Approach 2). Focusing on the two functions in this research, the logistic and exponential functions with depth 4 are approximated by their Taylor Series' requiring depth 3 calculations as follows:

- Logistic function:

$$f(x) \approx \frac{1}{2} + \frac{1}{4}x - \frac{1}{48}x^3 + \frac{1}{480}x^5 - \frac{17}{80640}x^7 - \frac{17}{80640}x^7 \quad (9)$$

- Exponential Function:

$$g(x) \approx 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^7}{5040} \quad (10)$$

The double negative, highest powered term in equation (9) greatly impacts the accuracy of the approximation, especially for larger input values. To combat this, yet still to retain the benefits of approximating the logistic function using depth reduction, this equation will be slightly modified; the highest powered term(s) will be multiplied by -1 to be turned into positive terms, therefore to compensate this change without altering the approximation in (9) by a large amount, the x^5 term is also multiplied by -1. This results in the depth reduced approximation to be implemented as follows:

$$f(x) \approx \frac{1}{2} + \frac{1}{4}x - \frac{1}{48}x^3 - \frac{1}{480}x^5 + \frac{17}{80640}x^7 + \frac{17}{80640}x^7 \quad (11)$$

3.4 Approximate HE via Task Skipping and Depth Reduction

The functions to be evaluated can be approximated further by combining and applying together the two previously proposed techniques. Applying depth reduction to Task Skipping Approach 1 has been computed in the form of the basic depth reduction technique. However, depth reduction can be applied to Task Skipping Approaches 2 and 3, resulting in the following polynomials being evaluated for the inputs defined in Section 3.1:

- Logistic function:

$$f(x) \approx \frac{1}{2} + \frac{1}{480}x^5 - \frac{17}{80640}x^7 - \frac{17}{80640}x^7 \quad (12)$$

- Exponential Function:

$$g(x) \approx 1 + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + \frac{x^7}{5040} + \frac{x^7}{5040} \quad (13)$$

Due to the alternating signs in the approximation of the logistic function, in addition to applying task skipping and depth reduction techniques, the resulting approximation gives a large error and thus needs modification to give a more accurate evaluation. Hence, the non-constant terms in the approximation defined in equation (12) are multiplied by -1, giving the following approximation to be implemented:

$$f(x) \approx \frac{1}{2} - \frac{1}{480}x^5 + \frac{17}{80640}x^7 + \frac{17}{80640}x^7 \quad (14)$$

By using equation 13 as the approximation of the exponential function for this approach, the calculation still satisfies task skipping and depth reduction techniques, but gives a more accurate result with an additional reduction in running time. All of the approximate functions, as given in Equations (1-8), (11), (13) and (14), are homomorphically evaluated.

3.5 State of the Art in Approximate HE

Table 1 shows the current state-of-the-art running times of the homomorphic evaluation of the logistic and exponential functions, as given in (Hee Cheon et al., 2017)[1], (Cheon et al., 2019)[2]. It also shows that the RNS variant of the CKKS scheme provides a much faster evaluation time for the logistic and exponential functions. In this research the depths and polynomial degree have been kept the same as is in Table 1 to ensure fair comparisons. It is important to note that the results presented in this research cannot directly be compared to those in Table 1 due to differences in implementation arising from different libraries used.

Table 1: State-of-the-art implementation results for the evaluation of the logistic and exponential functions using the HEAAN library.

Method	Depth	N	log q	log Q_l	Polynomial Degree	Total Time	Amortised Time
Logistic Function							
HEAAN [1]	3	2^{13}	30	155	7	0.54 s	130 μs
HEAAN [1]	4	2^{14}	30	185	9	0.78 s	90 μs
HEAAN-RNS [2]	3	2^{14}	55	281	7	0.161 s	19 μs
Exponential Function							
HEAAN [1]	3	2^{13}	30	155	7	0.65 s	160 μs
HEAAN-RNS [2]	3	2^{14}	55	281	7	0.164 ms	20 μs

4 RESULTS AND ANALYSIS

In this section, the proposed approximate techniques (from Section 3) are implemented and compared against implementations of the functions without approximate techniques, both in the SEAL library (SEAL, 2019). Table 2 displays the results. The amortised running times (i.e. the running time per individual ciphertext slot) are defined in microseconds, as the time taken for the algorithm to run from the encoding step to the decoding stage. The approximate computing techniques is applied to the *Evaluation* stage of the HE scheme, hence the *KeyGen* running times are not included. The error terms are calculated as absolute errors of the approximate evaluation against the actual evaluation of the functions. The calculation of the average error uses the absolute values to work out an average error from all the input values. The computations have been run on an Intel Core i7-8700 CPU @3.20 GHz with 16GB RAM using Microsoft's SEAL library (Version 3.4) (SEAL, 2019). Table 2 shows running times, with and without approximate optimisations, for homomorphically evaluated logistic and exponential functions using the inputs as described in Section 3.2. For each optimisation the percentage speed-up is calculated against the running time without optimisation and the average error is calculated against the 'actual' input evaluated in each function. In all cases the input vector is of size 2^{13}, i.e. 8192 slots.

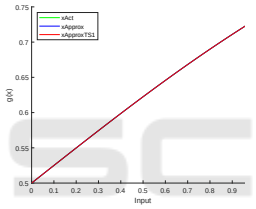
Task Skipping Approach 3 and Depth Reduction with Task Skipping Approach 3 are compared with second row for each function in Table 2, for input values 1-8192. All other approximate technique approaches are compared with the first row, where input values range from 0 to 1. The results for 'without optimisations' in Table 2 cannot be directly compared with the results outlined in (Cheon et al., 2019), due to differing underlying libraries used. It is also unfair to compare the HEAAN and SEAL libraries directly. Here SEAL is used to measure the effect the proposed approximations have on homomorphic evaluations. Further generic software optimisations could improve performance. All approaches show a speedup of 12.1-45.5% for task skipping, 35-45.5% for depth reduction and 53.3-57.6% for both techniques applied together across all the functions evaluated.

For input values 0-1, as shown in Table 2, both logistic and exponential functions benefit from Task Skipping Approach 1. It runs on average 40% faster with a larger error for the logistic function and around 10% better approximation for the exponential function. For depth reduction, the logistic function shows a similar speed up compared to Task Skipping Approach 1 but with a larger error. The exponential function, however, benefits from an approximately 4x smaller error than Task Skipping Approach 1, with a similar speed-up. When applying both techniques together, both functions run in around half the time compared to when no optimisation is used, 53.3% (exponential) and 57.6% (logistic), with an expected larger error value than when either technique applied on its own.

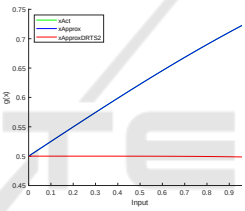
For the evaluations of the logistic and exponential functions shown in Figure 1, Figures 1a and 1c (equations (3) and (11) respectively) show the techniques which fare best for high accuracy applications and Figures 1b and 1d (equations (14) and (13) respectively) show the techniques for when running time is most important. For Figures 1b, 1d and Table 2, the approximations do not follow the general trend of the outputs for the inputs 0-1. As shown in Figures 1b and 1d, as the input value increases the error value increases as well, i.e. unlike in Figures 1a and 1c the trend of function approximation is not continuous.

Table 2: Table showing results for homomorphic evaluation of approximated logistic and exponential functions respectively. Three task skipping techniques (outlined in Section 3.2) and the depth reduction technique (outlined in Section 3.3) are presented. Results are calculated using the RNS-CKKS scheme implemented in the SEAL library. The corresponding equation numbers are shown in brackets. The rows with N/A in the speed-up column are the baseline results used for comparison.

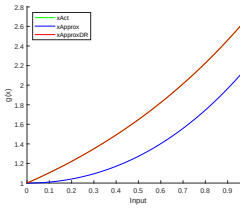
Homomorphic Evaluation of Function	**Degree**	**log Q_l**	**Total Time**	**Amortized Time**	**% speed-up**	**Average error from actual**
Logistic Function						
Without Optimisation (1), inputs 0-1	9	280	0.274354 s	33 μs	N/A	1.52 x 10^{-7}
Without Optimisation (1), inputs ≥ 1	9	280	0.275538 s	34 μs	N/A	3.55 x 10^{29}
Task Skipping Approach 1 (3)	7	240	0.144749 s	18 μs	45.5 %	2.08 x 10^{-6}
Task Skipping Approach 2 (5)	9	280	0.234501 s	29 μs	12.1 %	0.11979
Task Skipping Approach 3 (7)	9	280	0.235498 s	29 μs	12.1 %	3.55 x 10^{29}
Depth Reduction (11)	7	240	0.145621 s	18 μs	45.5 %	6.18 x 10^{-4}
Task Skipping Approach 2 with Depth Reduction (14)	7	240	0.114206 s	14 μs	57.6 %	0.12041
Task Skipping Approach 3 with Depth Reduction (14)	7	240	0.114705 s	14 μs	57.6%	3.55 x 10^{29}
Exponential Function						
Without Optimisation (2), inputs 0-1	8	280	0.495471 s	60 μs	N/A	0.333321
Without Optimisation (2), inputs ≥ 1	9	280	0.495397 s	60 μs	N/A	5.59 x 10^{25}
Task Skipping Approach 1 (4)	7	240	0.316672 s	39 μs	35 %	0.08334
Task Skipping Approach 2 (6)	8	280	0.38291 s	47 μs	21.6 %	0.70835
Task Skipping Approach 3 (8)	8	280	0.384384 s	47 μs	21.6 %	5.59 x 10^{25}
Depth Reduction (10)	7	240	0.318610 s	39 μs	35 %	2.42 x 10^{-5}
Task Skipping Approach 2 with Depth Reduction (13)	7	240	0.228299 s	28 μs	53.3 %	0.70833
Task Skipping Approach 3 with Depth Reduction (13)	7	240	0.227856 s	28 μs	53.3 %	5.59 x 10^{25}



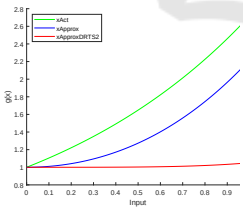
(a) Logistic Function Task Skipping Approach 1, equation (3)



(b) Logistic Function Depth Reduction with Task Skipping Approach 2, equation (14)



(c) Exponential Function with Depth Reduction, equation (11)



(d) Exponential Function Depth Reduction with Task Skipping Approach 2, equation (13)

Figure 1: Graphical representation of the approaches for both functions where accuracy (1a, 1c) and improved running time (1b, 1d) are the focus.

As shown by the large average error values in Table 2, for large input vectors (inputs ≥ 1) the average error is much worse and thus for such inputs, the reduction in accuracy caused by the proposed approximation techniques in their current form is too costly to justify the gain in terms of run time. This is considered upon in the recommendations in Section 5.

5 CONCLUSIONS

This research shows the proposed approximate computing techniques provide a decent run-time speedup of homomorphic evaluation, from 12.1-45.5% (task skipping), 35-45.5% (depth reduction) and 53.3-57.6% (the combination of techniques). The SEAL library is used to indicatively show how approximate computing for homomorphic evaluation of the logistic and exponential functions affects performance, targeting the CKKS scheme.

As shown in Table 2, the error values for when these techniques are applied to inputs ≥ 1 are very large, therefore, giving unreliable estimations, even though there is a speed-up. Thus, these techniques in their current form, are recommended for input values in the range 0-1.

Based on this research's results, for a balanced trade-off between performance and accuracy, the recommended approximate computing techniques for the logistic and exponential functions respectively are (for input values 0 to 1): Task Skipping Approach 1 and Depth Reduction.

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