A Technological Storytelling Approach to Nurture Mathematical Argumentation

Giovannina Albano[1][a], Umberto Dello Iacono[2][b] and Giuseppe Fiorentino[3][c]

[1]*Department of Information and Electrical Engineering and Applied Mathematics, University of Salerno, Via Giovanni Paolo II, 132, 84084 Fisciano (SA), Italy*

[2]*Department of Mathematics and Physics, University of Campania "L. Vanvitelli", Viale Abramo Lincoln, 5, 81100 Caserta (CE), Italy*

[3]*Naval Academy of Livorno, Viale Italia 72, Italy*

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Abstract: This research deals with how to foster the attitude of the mathematician facing a problem. In this regard, we identified some fundamental steps, from the initial understanding to its solution, and some key attitudes, such as critical thinking and insight. The steps have been translated into the phases of a digital interactive storytelling in mathematics (DIST-M), while the attitudes have been embodied as characters/roles within the story. The whole didactic design is based on collaborative scripts, and evolves according to the interactions between the characters and the stimuli coming from the expert. In the paper we briefly report the design, it's implementation using ICT tools, a taste of the analysis conducted and the results arising from a DIST-M trial involving 26 first-year high school students.

1 INTRODUCTION

One of the aims of this research concerns how to foster in students the attitude of the mathematician facing a mathematical problem. For this purpose, we identified some of his typical mental activities, framing them as distinct characters of a storytelling framework. We also shaped the story into phases according to the typical process that goes from the initial discovery and inquiry to the conjecture, up to the final proof and rethinking. These phases were translated into learning activities (Leontiev, 1978) within the storytelling.

More in depth, we considered the following fundamental phases involved in the understanding and the formalization of a mathematical problem and its solution:

- ***Inquiry***: the mathematician starts exploring the problem, investigates about the assumptions, makes trials, until a rough hypothesis arises.
- ***Conjecture***: the mathematician starts refining the hypothesis to obtain a clear and complete (though often verbal) conjecture.
- ***Formalization***: in most cases a verbal conjecture is not enough to trigger the mathematical tools needed to prove or disprove it; to this purpose, a more formalized statement is obtained using mathematical language and symbolic representations.
- ***Proof***: the formal language allows to apply all mathematical knowledge to prove (or confute) the conjecture.
- ***Refinement***: once proved the conjecture, mathematicians often retrace their course and refine the proof, producing a clear statement to share with the mathematical community.

Along the above phases, we also foresaw the following key attitudes of a mathematician dealing the solution of a problem:

[a] https://orcid.org/0000-0002-5119-5413
[b] https://orcid.org/0000-0003-0224-1046
[c] https://orcid.org/0000-0003-1021-3969

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- ***Critical thinking***: this is the ability to rethink what is said in order to identify its weaknesses and critical points.
- ***Memory***: as the ability of verbalizing and refining the partial and final formulations of the reached results.
- ***Coordination***: is the ability of planning the actions and keeping track of them, taking into account the purpose of the various tasks.
- ***Insight***: this is the capability of generating new germinal ideas which bring forward towards the goal.
- ***Knowledge***: it stands for previous knowledge, know-how, ability to seek information or to ask for advice when own resources are not enough to overcome difficulties.

These attitudes have been embodied in our storytelling as different characters/roles that all students play in turn, phase after phase.

With such roles and phases in mind, we designed a digital interactive storytelling in mathematics (DIST-M), based on a Vygotskian approach where learning is first socialized and then internalized (Vygotsky, 1980). The instructional design of a DIST-M is based on collaborative scripts within a digital storytelling framework. The narrative framework allows contextualized learning through the integration of logical-scientific and narrative thinking (Bruner, 1986).

In order to increase their cognitive engagement, our storytelling approach does not foresee the construction of the story by the students (which is hard to drive towards the desired learning outcomes). On the contrary, we start from a didactically interesting mathematical problem well integrated in a consolidated story, here students (divided into groups) and experts (teacher or researcher) play as characters along several possible learning paths. The narration evolves according to the interactions between the characters and the stimuli coming from the experts. The analysis of the protocols arising from the classroom experimentations is reported in Section 5.2 and seems to confirm that, in this way, students remain more focused on the plot and its (mathematical) objectives. More details on the implementation of the DIST-M are given in Section 3.

Another aim of this research was to understand whether technology is an essential component and to which extent it can play an active role in teaching and learning processes of this kind. We try a tentative answer to this question in Section 6.

2 THEORETICAL BACKGROUND

The design of the learning activities has been informed by a network of theories (Bikner-Ahsbahs & Prediger, 2014), where different approaches are combined and coordinated to investigate the same phenomenon from different points of view. In the following, we briefly summarize the main theoretical references taken into account in the instructional design.

Initially developed in the field of psychology and social sciences, the Activity Theory studies individuals from the analysis of their activities. The key concept is the activity, understood as something characterized by interactions (it is an action in the world) and intentionality (the action responds to a purpose). The activity is therefore a high-level, usually collaborative, construct at the top of a hierarchy with two other components: actions and operations, characterized respectively by awareness (having a goal) and unconsciousness (task).

The intrinsic collaborative nature of these activities led us to explore some aspects of collaborative learning, particularly computer-based learning. It is well known that the effectiveness of collaborative learning is not to be taken for granted, especially in computer-supported environments (Weinberger et al., 2009), where the need to pre-structure and regulate social and cognitive processes is even more evident. This is why researchers in the field have used the concept of scripts as introduced in cognitive psychology. It refers to an internal memory scheme corresponding to a sequence of actions that define a well-known situation (Schank and Abelson, 1977). Here each actor has a specific role and specific actions to perform and the script is activated every time a similar situation happens. In education, scripts are externally imposed to support the students in collaborative learning contexts through the regulation of roles and actions that students must take and perform to achieve a successful learning (King, 2007). Over time, through social practice, the expectation is that the student will internalize such scripts (Vygotsky, 1980) and shift from hetero regulation to self-regulation.

In mathematics education, the importance of assigning roles to the members of cooperative groups has been recognized in order to achieve a successful learning. Pesci (2009) devises five roles: the first is *oriented to the task*, in charge of getting the group to solve the mathematical task as well as they can; the second is *oriented to the group*, responsible of the active participation of members of the group; the third is *the memory*, who takes care of verbalizing the results of the group; the fourth is the *speaker*, in charge of reporting the results outside of the group; the last one is the *observer*, who takes notes of all the processes and of the engagement of each member of the group. A key remark highlighted by Pesci concerns the importance of rotating such roles among the students, so that each of them has the chance to experience each role and thus develops the corresponding skill. A further role is foreseen for the teacher, who acts as a supervisor, refraining from giving hints on the mathematical side.

3 DESIGN OF DIST-M EXPERIENCES

The theoretical background has informed our project and suggested how to design the DIST-M experience in order to engage students in developing the mathematician attitudes outlined in the introduction. The general framework of the DIST-M consists in a digital storytelling (Albano & Dello Iacono, 2018; Albano, Dello Iacono, Fiorentino, Polo, 2018). The story evolves along various episodes where students and teachers are involved impersonating Characters corresponding to the roles introduced in Section 1. Following Pesci, the student plays different roles in each episode, allowing him to experience the story from different perspectives and to practice different skills. Teachers and researchers play the Guru role. and, differently from Pesci, in our design they can also intervene mathematically, since their role embodies the professional knowledge.

To cope with classes with more than 20 pupils, one may be tempted to adopt two opposite solutions. The first one foresees many students playing each role/Character; however, the resulting "shared responsibility" can easily lead to situations where only a few members of each group are really engaged. The opposite approach allows many smaller teams to act along the story as (nearly) isolated "threads". However, it is hard to monitor all the teams by means of their simultaneous real-time online conversations. We tried a different approach, broadening the roles and their rotation, with the Observers (Pesci, 2009). Indeed, when a DIST-M is experienced in a class, various groups of four students (one for each role) are created leading to five or six groups. For each episode, only one group is directly involved as the Characters of the story, whilst the others are the Observers (each member of the group observes a different Character). In the following episode, the active group and all the roles are rotated. In this way, each students is active once and will experience all the roles, either as a Character or as an Observer. In this way, also the Observers are engaged in the story, both on the cognitive level ("What would I have done in his shoes?") and on the metacognitive one ("Is the observed Character playing well his role? Is he effective?").

3.1 Macro Design

According to the Activity Theory, each phase corresponds to an activity, characterized by interactions (inside the digital environment, along the story, using the available tools, with the other Characters) and intentionality (each activity has a purpose). The macro script consists of 5 activities corresponding to the phases of a mathematician work introduced in Section 1: *Inquiry*, *Conjecture*, *Formalization*, *Proof*, *Refinement*. Each phase corresponds to one episode of the story. In the *Inquiry* phase (Episode 1) students explore a given mathematical situation. The aim of this phase is to produce a short description of what has been found. Subsequently, in the *Conjecture* phase (Episode 2), the description is refined. The students, starting from what was found, are required to formulate an agreed mathematical conjecture. The latter is usually expressed in verbal form and, as such, it may not be suitable for the formal manipulation required to obtain a proof. So, in Episode 3 (the *Formalization* phase), the students are asked to formalize the conjecture in a proper mathematical language. This new form is exploited in Episode 4 (the *Proof* phase) making them organize the arguments in a deductive chain, to build a (mathematical) proof. Finally, in Episode 5 (that is the *Refinement* phase) students are required to write a report about the process carried out and the results, explaining the purpose of each episode.

In order to fulfill the objective of each activity, every student has some actions to perform, sometimes individually, sometimes collaboratively. The actions depend on the mathematical problem and are designed to accomplish the purpose of the episode.

3.2 Micro Design

As already stated, the roles within the DIST-M arise from the need to regulate group work in computer-based environments, but they also represent functions describing the attitude of a mathematician facing a problem. Taking into account both the devised functions (section 1) and the information from the theoretical background (section 2), we have foreseen the following Characters, all played by students:

- **The *Boss*** (coordination)**:** is both *task* and *group oriented* (Pesci, 2009), takes care of the participation climate in the group, keeps the focus of the group on the actions to be carried out according to the purpose of the episode and to the mathematical task; the responsibility of the mathematical success, however, is in charge of the whole group.
- **The *Blogger*** (memory)**:** is both *memory* and *speaker* (Pesci, 2009), summarizing and communicating to the Guru the results obtained by the whole group.
- **The *Pest*** (critical thinking)**:** acts as the devil's advocate (Soldano & Arzarello, 2016), asking questions to check the robustness of all results obtained by the group.
- **The *Promoter*** (insight)**:** is the creative of the group, providing new insights to proceed towards the objective of the episode.

We have also a Character played by the teacher or the researcher:

- **The *Guru*** (knowledge): is the one to ask for information and advice in case of lack of own resources, the expert in the Vygotskian perspective.

To avoid deadlock situations, we have foreseen a private communication channel between the Guru and the Promoter. So if the Guru believes that the group is in trouble, unable to ask for advice, or if someone is not playing his role correctly without someone else pointing it out, then the Guru can give advice to the Promoter to overcome the difficulties.

In each episode, each member of the groups assumes the perspective of one of the above roles, and their role changes in the following episode. As already stated, in each episode only one group is active playing the Characters of story while the members of all other groups are Observers (of a different Character). The rotation of the roles also involves the episodes where the group members are Observers. So, as stated, all students will play all roles.

During the observation phases, the student is asked to take the perspective of the assigned role, watch the corresponding active Character and comment in a personal spaces (the Logbook) how the player behaves with respect to what is needed on the mathematical plane to accomplish the episode objective. In these observation phases the student is urged to switch from the cognitive to the metacognitive level.

This continuous change of perspective, from Character to Observer, from one role to another and from the cognitive to the metacognitive level are intended to help internalise all the roles and phases introduced in Section 1.

3.3 The (self-)assessment

As each DIST-M experience is intended within the curriculum, the assessment of such experience should be part of the didactical contract. Thus we have foreseen two different kind of evaluation:

- A ***Collective Assessment*****:** it refers to the Episode 5 (section 3.1); each group of students is asked to retrace all the previous episodes and fill-in a report describing what happened in each episode, both at the story level and at the mathematical level; in particular, students are expected to recognize and describe the purpose of each episode.
- An ***Individual Assessment*****:** all students are required to review the whole activity, commenting on how they played the assigned roles, both as a Character and as an Observer; and say what they would change in hindsight.

The collective assessment is essentially cognitive and refers mainly to the mathematical content. However, it makes the students aware of the attitudes of a mathematician and the steps he makes, as experienced along the episodes Eventually, the whole path (the script underpinning the story) will become clear to the students and they will realize that it can be followed every time they find themselves involved in similar situations.

The individual assessment is essentially metacognitive and affective. It concerns the functions involved in a mathematician's work and how they showed up in the corresponding roles.

4 A CASE STUDY

The educational activities of the DIST-M take place in a sci-fi setting. The Characters of the story are four friends: Marco, Federico, Sofia and Clara. Marco is the leader of the group and is trusted by his friends: he is the *Boss*. Federico is a computer expert, convinced of the existence of the aliens and always eager to investigate and look for creative ideas: he is the *Promoter*. Sofia loves reading and writing and wants to become a journalist: she is the *Blogger*. Clara is wary and often annoys her friends with countless doubts and questions: she is the *Pest*. A further Character goes along with the four friends: Gianmaria, Federico's uncle. He is a smart adult, with a great knowledge, experience and understanding: he is the *Guru.*

4.1 The Mathematical Problem

As already stated, the DIST-M starts from a didactically interesting mathematical problem, and aims to develop argumentative and proving skills in the students. The problem is the following: given four consecutive numbers, the difference between the product of the two middle ones and the product of the two extremes is always 2 (Mellone & Tortora, 2015). This problem is suitable to introduce students to algebraic proofs. In fact, to prove that the result is always 2 needs some algebraic modelling; for example, from n, n+1, n+2, n+3, we may write the given statement as (n+1) (n+2) - n(n+2) and manipulate it to obtain 2. This problem is also suitable to promote discussions on terms as "all" and "always" (i.e. universal quantifier), encourages reflection on key mathematical concepts such as "consecutive" in the set of natural numbers, integers, or within a generic numerical sequence.

Within the story, the mathematical problem has been intentionally reformulated and partly hidden: one day Federico (the Promoter), with a self-built supercomputer, finally picks up a strange message from the aliens. It is made of numbers and mathematical signs and demands a decipherment (Figure 1). Unfortunately, due to interference, he cannot write down all the numbers.

2	3	4	5	3	×	4	−	2	×	5
11	12	13	14	12	×	13	−	11	×	14
5	6	7	8	6	×	7	−	5	×	8
15	16		18	16	×	17	−	15	×	18
21	22	23	24	22	×	23	−		×	24
	13	14	15	13	×	14	−		×	15
10	11	12		11	×	12	−	10	×	

Figure 1: the mathematical problem in Episode 1.

Federico immediately involves his friends and calls his uncle Gianmaria (the Guru) for help since he is also fond of mathematical puzzles.

Starting from the recognition that each quadruple on left side of Figure 1 is always formed by consecutive integer numbers, pupils are guided by the story towards the production of a conjecture about the message such as that 2 is always the result of all operations on the right side of Figure 1.

To achieve this goal they will need and will be guided by the story, to formulate their conjecture, formalise it in mathematical language, prove it and finally send a message back to the aliens, who are supposed to understand mathematical language.

4.2 Implementation of the DIST-M

The DIST-M is mainly implemented using Moodle, which provides useful tools for both educational activities and social interactions (Albano, Dello Iacono, Fiorentino, 2016).

Moodle's tools and resources have been carefully chosen and configured to meet the educational needs of each design step. For instance, we used *Chats* for all informal communication between students in the same group and for the privileged channel between Gianmaria the *Guru* and Federico the *Promoter*. While *Forums* and *Questionnaires* (suitably adapted to simulate the sending of an email) were used for all formal communications between the Characters and the *Guru*. This shift, which mimics the transition from spoken to written language, is also intended to facilitate the transition to higher literate registers (Ferrari, 2004).

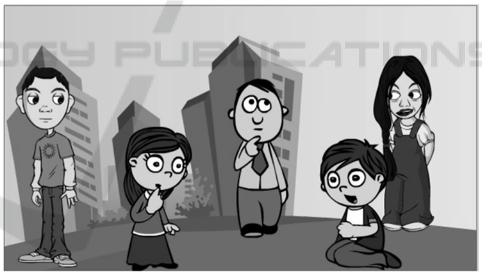

Figure 2: The comics setting and the 5 characters.

The whole learning environment has been customised to look like a strip cartoon. The comics were made with the online environment Toondoo and PowerPoint (as in Figure 2) and inserted into Moodle Books to implement all episodes of the story. So, students can navigate the story flipping through pages and chapters. Moreover, a few lines of custom CSS, the use links within of Moodle's Labels allowed access to "stealth" activities (available, but not shown on course home page) so hiding some other elements of Moodle's user interface.

Making an extensive use of access condition, some activities and resources are visible and available only to specific roles; for instance, the Chat between

Federico (the Promoter) and Gianmaria (the Guru) is only visible to the Promoter role, while the Questionnaire that simulates the sending of an email can be seen only by the Blogger role. The Observers were implemented with a new Moodle role which allows to see all chats and forums, without being able to edit them.

Some digital application, implemented with GeoGebra (Albano & Dello Iacono, 2019a), have been embedded to support students in the production of conjecture, argument and formalization process (Albano & Dello Iacono, 2019b). In particular, in the first episode, a GeoGebra spreadsheet is embedded in a Moodle Page to support students in their exploration in search of regularities. Students can exploit the potential of spreadsheets quickly performing many tests on all the quadruples. Another Moodle Page with an embedded GeoGebra spreadsheet is also available with a short tutorial on how to use it (assuming that students were able to use technologies in a didactically meaningful way is a common mistake that we tried to avoid). The student can choose if and when follow the tutorial. In the second and third episodes of the story, two Interactive Semi-open Questions (ISQ) (Albano & Dello Iacono, 2019b) were implemented to support students in the production of a conjecture and for its formalization respectively.

An ISQ is a digital GeoGebra application embedded in Moodle that allows the student to build a statement or a sentence by dragging digital tiles. Figure 3 and 4 show the digital tiles in the two ISQs needed to construct a verbal statement and its symbolic expression, in Episodes 2 and 3 respectively.

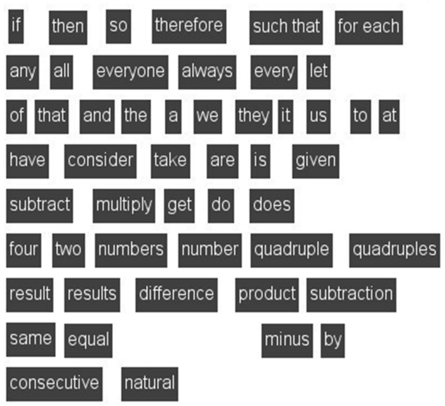

Figure 3: Interactive Semi-open Question for Episode 2.

Figure 4: Interactive Semi-open Question for Episode 3.

These GeoGebra applications are initially hidden, the expert (i.e. the Guru) can activate these resources and make them available to students, according to the technological literacy of the class.

All students are asked to keep a personal Logbook for all episodes. The Observers will take notes while the (observed) Characters are playing. The Characters will describe the episode in which they were active at the end of the experience. The Logbook is implemented as a Google Doc with preloaded questions to guide the (cognitive and metacognitive) analysis of each episode. The same document will be useful in the final (self-assessment) phase when all students are asked (with an open text Moodle assignment) to rethink about how they played during the whole story.

5 EXPERIMENTATION

In this section we briefly report the methodology and a short analysis of some excerpts arising from an experimentation of the outlined DIST-M.

5.1 Methodology

This DIST-M was experimented with 26 first-year high school students. The trial was carried out in a computer lab at the University of Salerno. All students worked individually on a computer and communicated with their classmates by means of one of the platform's tools. Groups of 4 or 5 students have been set up. In groups with 5 members the Pest and Pest Observer roles were duplicated. The role of the expert, Gianmaria, was played by a researcher together with the teacher. In the following section we give a taste of the engagement of the students in their roles (as Characters or Observers), focusing on the first episode of the story (i.e. the Inquiry phase).

5.2 Data Analysis and Results

A qualitative analysis is conducted with the aim of understanding how much the environment has influenced the students' appropriation of roles with respect to the mathematical problem to be solved. In this regard, we will look at the transcriptions produced by the students in the Group Chat and in the private Chat between Federico and Gianmaria, and the Observers' Google Doc files.

In the following we report an excerpt from the discussion between the Characters in the Group Chat:

1 BOSS: since in the previous rows the difference between the numbers was one, then in the seventh row (the last one), as it ends with 15 and the difference is one, then the four numbers should be: 12, 13, 14, 15.
2 BOSS:So, it will be 12*14-13*15. Do you agree?
3 PEST: Ehm, excuse me, I don't understand this
4 BOSS: sorry, I was wrong, 13*14-12*15
5 PROMOTER: guys, since I see the row with the numbers 15,16 and 18, since in the row on the right there is *17, in my opinion the missing number should be 17, yielding 16*17-15*18
6 PEST: product of the middle ones minus the product of the extremes?
7 BLOGGER: row 5 I suppose should be 7-8-9-10 8*10-7*10. Do you agree?
8 BLOGGER: Boss, what do you think about?
9 PEST: Are we supposed to compute the result?
10 BLOGGER: we are expected to find the missing numbers

In the above discussion, we can note how the Boss, in explaining his idea to the group (line 1) makes a mistake. The timely intervention of the Pest, in search of clarification (line 3) allows the Boss to correct his statement. The intervention of the Boss leads both the Promoter and the Blogger to explain their idea about the quadruples. The frequent questions of the Pest (e.g. line 9) lead the Blogger to come back to the story request and to keep the teammates focused on what they are expected to carry out. It's worthwhile to note how the members of the group consider the Boss as a reference for the group, asking his opinion (e.g. line 8).

The above dialogue can be considered fruitful, as shown by the few interactions between Gianmaria (the Guru) and Federico (the Promoter) in their private chat:

11 GIANMARIA: ehm... how the numbers are arranged in each row? Have you and your friends "solved" the enigma? What comes out?
12 PROMOTER: it appears that the second number is multiplied by the third one minus the first number by the fourth one. For instance, looking at the first row (15,16,17,18) we have (16*17-15*18)
13 PROMOTER: the result could change; we should check computing the result for each row

The suggestions of Gianmaria are reported in the Group Chat by the Promoter, allowing the whole group to come out of a stalemate, as shown by the following excerpt:

14 PROMOTER: my uncle said that we should check whether the result changes from row to row
15 BLOGGER: I don't understand
16 BOSS: maybe we should perform the computations shown in each row and see if the result changes
17 PEST: we should check whether the result it is always the same one
18 BLOGGER: are we expected to compute the mathematical expression?
19 PROMOTER: I think so
20 BOSS: the first one is 2
21 PEST: they are all 2!
22 PROMOTER: so, guys, we may state that in all the rows the result is 2

The excerpts from the Characters show that they were able to disclose the consecutivity relation and regularity of the quadruples at the end of the first episode. The success of the activity seems to have been influenced by the conscious appropriation of the roles by the students and, therefore, by the responsibility that each specific role implied. This is also highlighted by the Observers, as shown below.

BOSS OBSERVER: The fact that the boss recognises his mistake and cares of the group work is a positive thing. At the end of the discussion he greatly helped.
PEST OBSERVER: She [the Pest] is involved by frequently asking questions. So she poses doubts, stimulating the group to validate what has been conjectured.
PROMOTER OBSERVER: He recognises the mistakes made by the group and look for a correct solution.
BLOGGER OBSERVER: He helps the others and in my opinion he is working well. He is carrying out all the steps correctly and is arranging the email to the promoter's uncle despite the difficulties.

6 CONCLUSIONS

In this paper we presented an overview of a research project aimed at investigating how to promote some of the attitudes of a mathematician. To this aim, we used an immersive storytelling approach where students can experience the whole process and skills involved in a mathematical discovery. The mental activities characterizing such process are the Characters of the story with their specific roles, and the story itself has been carefully designed to address all the problem-solving steps. Both cognitive and metacognitive aspects were taken into account, engaging the students with active playing and observation phases from one episode to the other of the story. We think that, in the long run, this will help them to interiorise such roles. It is worth noting that the designed learning activity is part of their curriculum, so it also includes individual and collective assessment steps. The first one allows to make evident to the students the process and the mental activities constituting the mathematician's attitude, and to institutionalize the mathematical knowledge. The second moves along the affective level of learning, focusing on the engagement of the student as Characters or Observers.

The implemented technological environment provides added-value to both students and teachers. In fact, it allows immediate and persistent access to (far) more information than in real situation (e.g. chat and forum transcriptions with the details of all discussions). So, students can really act as Observers without losing any detail. Analogously, teachers have the real chance to observe their students working in a more authentic context.

The first trials show the engagement of the students and some signs towards the appropriation of the roles can be recognised.

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