Integrating Special Rules Rooted in Natural Language Semantics into the System of Natural Deduction

Marie Duží and Michal Fait
VSB - Technical University of Ostrava, Department of Computer Science, Czech Republic
marie.duzi@vsb.cz, michal.fait@vsb.cz

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Abstract: The paper deals with natural language processing and question answering over large corpora of formalised natural language texts. Our background theory is the system of Transparent Intensional Logic (TIL). Having a fine-grained analysis of natural language sentences in the form of TIL constructions, we apply Gentzen's system of natural deduction to answer questions in an 'intelligent' way. It means that our system derives logical consequences entailed by the input sentences rather than merely searching answers by keywords. Natural language semantics is rich, and plenty of its special features must be taken into account in the process of inferring answers. The TIL system makes it possible to formalise all these semantically salient features in a fine-grained way. In particular, since TIL is a logic of *partial* functions, it deals with non-referring terms and sentences with truth-value gaps in an appropriate way. This is important because sentences often come attached with a *presupposition* that must be true in order that a given sentence had any truth-value. Yet, a problem arises how to integrate those special semantic rules into a standard deduction system. Proposal of the solution is one of the goals of this paper. The second novel result is this. There is a problem how to search relevant sentences in the labyrinth of input text data and how to vote for relevant applicable rules to meet the goal, i.e. to answer a given question. To this end, we propose a heuristic method driven by constituents of a given question.

1 INTRODUCTION

Logic and computational linguistics are the disciplines that have much in common; in particular, they should work hand in hand in natural language processing and question answering. In the era of information overload, the systems that can answer questions raised over the large corpora of text data in an 'intelligent' way gain more and more interest in the research community. In this paper, we introduce a system that derives the logical consequences of information recorded in the huge knowledge bases of text data. Thus, the system not only answers the questions by providing explicit knowledge sought by keywords. It answers in an 'intelligent' way and computes *inferable* knowledge (Duží, Menšík, 2017) such that rational human agents would produce if only this were not beyond their time and space capacities. To this end, we apply Gentzen's system of natural deduction adjusted to our background theory Transparent Intensional Logic (TIL) with its *procedural* semantics.[1] In TIL, meanings of natural language sentences are viewed as abstract structured *procedures* that produce Possible World Semantic (PWS-)propositions as their products. Duží and Horák in (2019) introduce the system that applies the goal-driven, backward-chaining strategy of inferring answers by general resolution method adjusted for TIL. It seems to be a natural choice because by applying the goal-driven strategy, we can easily solve the problem of searching for relevant information resources in the huge labyrinth of input data. Yet, a problem arises here, namely the problem of integrating special rules rooted in the rich natural language semantics into the deduction process. These rules include, inter alia, the rules of left and right subsectivity for *property modifiers*, the rules for handling non-referring terms and propositions with truth-value gaps, the rules dealing with *factive*

[1] See, for instance, (Tichý, 1988) or (Duží, Jespersen, Materna, 2010).

verbs like 'knowing' or 'regretting', *presuppositions* of sentences, *de dicto* vs *de re* attitudes, and many other. TIL with its fine-grained procedural semantics is the system in which all these semantically salient features are successfully formalised.

In (Duží, Horák, 2019) and (Duží, Menšík, 2020) it has been assumed that it is possible to pre-process the sentences first so that the special semantic rules are applied prior to the application of a proving method. Yet, it turned out that such a system is under-inferring (Duží, Fait, Menšík, 2019). We have to integrate these special semantic rules into the very process of inferring answers. To this end, we vote for Gentzen's natural deduction here, because enrichment by special rules seems to be easier for the system of natural deduction than for the General Resolution Method where the input sentences must come in Skolem Clausal Form.

The goal of this paper and its novel contribution is to introduce such a system of natural deduction extended by semantic rules for TIL and natural language processing. Yet, another problem crops up here, which is the problem of a proper search strategy in the huge amount of input data. As mentioned above, in GRM it was easy to solve thanks to the goal-driven backward chaining resolution. However, inferring by natural deduction usually applies a forward-chaining strategy. Moreover, even if we apply the backward-chaining strategy, it cannot be strictly goal driven. Sometimes, the process of satisfying one goal after another has to be interrupted by an application of a semantic rule to one or more other constructions, and only then can we continue the inferential process by applying standard rules to answer questions. Thus, another goal of this paper is to introduce a heuristic method of searching for proper input constructions driven by constituents occurring in a given query or goal. Using two case studies, we demonstrate the solutions by an example dealing with property modifiers and an example of dealing with factive verbs and their presuppositions. In both cases, we also deal with anaphora resolution.

The rest of the paper is organized as follows. Section 2 summarises the main principles of TIL. In Section 3 we briefly describe the rules of natural deduction adjusted to TIL. In Section 4 we introduce the semantic rules and their formalization in TIL. Section 5 illustrates our method of intelligent question answering by two case studies. Concluding remarks can be found in Section 6.

[2] A kindred theory of procedural semantics has been introduced by Moschovakis in (2006) and further developed by Loukanova, see, e.g. (Loukanova, 2009).

2 FOUNDATIONS OF TIL

As mentioned above, TIL comes with *procedural* (as opposed to set-theoretical denotational) semantics. Hence, the meaning of a sentence is conceived as an abstract structured procedure encoded by the sentence, the structure of which is isomorphic with the structure of the sentence. These procedures can be viewed as instructions how, in any possible world and time, to evaluate the truth-value of a sentence.[2] They are known as TIL *constructions*. There are six kinds of such constructions defined, namely *variables*, *Trivialization*, *Composition*, (λ-)*Closure*, *Execution* and *Double Execution.* While variables and Trivializations are atomic constructions that supply objects on which molecular constructions operate, Composition and Closure are molecular constructions. Trivialization roughly corresponds to a constant of formal languages; where X is an object whatsoever of TIL ontology, Trivialization ^0X produces X. Variables produce objects of their respective ranges dependently on valuations, they *v-construct.* *Composition* $[F\ A_1 \dots A_m]$ is the procedure of applying the function f produced by F to its arguments produced by $A_1, \dots, A_m$ to obtain the value of f, if any; dually, *Closure* $[\lambda x_1 \dots x_m\ C]$ is the procedure of declaring or constructing a function by abstracting over the values of λ-bound variables in the ordinary manner of lambda calculi. Thus, we define.

Definition (*Constructions*).

(i) *Variables* x, y, … are *constructions* that *construct* objects (elements of their respective ranges) dependently on a valuation v; they *v-construct*.

(ii) Where X is an object whatsoever (even a *construction*), ^0X is the *construction Trivialization* that *constructs* X without any change of X.

(iii) Let $X, Y_1,\dots,Y_n$ be arbitrary *constructions*. Then *Composition* $[X\ Y_1\dots Y_n]$ is the following *construction*. For any v, $[X\ Y_1\dots Y_n]$ is *v-improper* if at least one of the *constructions* $X, Y_1,\dots,Y_n$ is v-improper, or if X does not *v-construct* a function that is defined at the n-tuple of objects *v-constructed* by $Y_1,\dots,Y_n$. If X does *v-construct* such a function, then $[X\ Y_1\dots Y_n]$ *v-constructs* the value of this function at the n-tuple.

(iv) (λ-) *Closure* $[\lambda x_1\dots x_m\ Y]$ is the following *construction*. Let $x_1, x_2, \dots, x_m$ be pair-wise distinct variables and Y a *construction*. Then Closure $[\lambda x_1\dots x_m\ Y]$ *v-constructs* the function f that takes any members $B_1, \ldots, B_m$ of the respective ranges of the variables $x_1, \ldots, x_m$ into the object (if any) that is $v(B_1/x_1,\ldots,B_m/x_m)$-*constructed* by Y, where $v(B_1/x_1,\ldots,B_m/x_m)$ is like v except for assigning B_1 to $x_1, \ldots, B_m$ to x_m.

(v) Where X is an object whatsoever, ${}^1X$ is the *construction Single Execution* that v-constructs what X *v-constructs*. Thus, if X is a v-improper *construction* or not a *construction* as all, ${}^1X$ is v-improper.

(vi) Where X is an object whatsoever, ${}^2X$ is the *construction Double Execution*. If X is not itself a *construction*, or if X does not v-construct a *construction*, or if X *v-constructs* a v-improper *construction*, then ${}^2X$ is v-improper. Otherwise ${}^2X$ *v-constructs* what is *v-constructed* by the *construction v-constructed* by X.

(vii) Nothing is a *construction*, unless it so follows from (i) through (vi).

From the formal point of view, TIL is a typed λ-calculus that operates on functions (intensional level) and their values (extensional level), as ordinary λ-calculi do; in addition to this dichotomy, there is however the highest *hyperintensional* level of procedures producing lower-level objects. And since these procedures themselves can serve as objects on which other higher-order procedures operate, there is a fundamental dichotomy between two modes in which constructions can occur, namely *displayed* (as an object to be operated on) and *executed* to v-construct a lower-level object. In principle, constructions are displayed by Trivialization. A dual operation to Trivialization is the construction called *Double Execution* that executes constructions twice over. Hence, while ${}^0X$ displays X, ${}^{20}X$ voids the effect of Trivialization and is thus equivalent to executed X. Below we refer to this equivalence as to ^{20}-rule.

To avoid vicious circle problem and keep track of particular logical strata in its stratified ontology, TIL ontology is organized into a *ramified hierarchy of types* built over a base. For natural language processing, we use the epistemic base consisting of for atomic types, namely o (the set of truth-values), ι (individuals), τ (times or real numbers) and ω (possible worlds). The type of constructions is $*_n$, where n is the order of construction.

Definition (*Ramified Hierarchy of Types*).
Let B be a *base*, where a base is a collection of pair-wise disjoint, non-empty sets. Then:

$\mathbf{T_1}$ *(types of order 1)*.
i) Every member of B is an elementary *type of order 1 over B*.
ii) Let $\alpha, \beta_1, \ldots, \beta_m$ ($m > 0$) be types of order 1 over B. Then the collection $(\alpha\ \beta_1 \ldots \beta_m)$ of all m-ary partial mappings from $\beta_1 \times \ldots \times \beta_m$ into α is a functional *type of* order 1 *over B*.
iii) Nothing is a *type of order 1 over B* unless it so follows from (i) and (ii).

$\mathbf{C}_n$ *(Constructions of Order n)*
i) Let x be a variable ranging over a type of order n. Then x is a *construction of order n over B*.
ii) Let X be a member of a type of order n. Then ${}^0X$, ${}^1X$, ${}^2X$ are *constructions of order n over B*.
iii) Let $X, X_1, \ldots, X_m$ ($m > 0$) be constructions of order n over B. Then $[X\ X_1 \ldots X_m]$ is a *construction of order n over B*.
iv) Let $x_1, \ldots, x_m, X$ ($m > 0$) be constructions of order n over B. Then $[\lambda x_1 \ldots x_m\ X]$ is a *construction of order n over B*.
v) Nothing is a *construction of order n over B* unless it so follows from $\mathbf{C}_n$ (i)-(iv).

$\mathbf{T}_{n+1}$ (*Types of Order n+1*) Let $*_n$ be the collection of all constructions of order n over B. Then
i) $*_n$ and every type of order n are *types of order n+1*.
ii) If $m > 0$ and $\alpha, \beta_1, \ldots, \beta_m$ are types of order $n+1$ over B, then $(\alpha\ \beta_1 \ldots \beta_m)$ (see T_1 ii)) is a *type of order n+1 over B*.
iii) Nothing is a *type of order n+1 over B* unless it so follows from (i) and (ii).

Empirical sentences and terms denote (PWS-)*intensions*, functions with the domain of possible worlds ω; they are frequently mappings from ω to *chronologies* of α-objects, hence functions of types $((\alpha\tau)\omega)$, or $\alpha_{\tau\omega}$, for short. Where variables w, t range over possible worlds ($w \rightarrow \omega$) and times ($t \rightarrow \tau$), respectively, constructions of intensions are usually Closures of the form $\lambda w \lambda t\ [\ldots w \ldots t \ldots]$.

We model sets and relations by their characteristic functions. Hence, $(o\iota)$, $(o\iota\iota)$ are types of a set of individuals and of a binary relation-in-extension between individuals, respectively. *Quantifiers* $\forall^\alpha$, $\exists^\alpha$ are type-theoretically polymorphic total functions of types $(o(o\alpha))$ defined as follows. Where B is a construction that v-constructs a set of α-objects, $[{}^0\forall^\alpha B]$ v-constructs $\mathbf{T}$ if B v-constructs the set of all α-objects, otherwise $\mathbf{F}$; $[{}^0\exists^\alpha B]$ v-constructs $\mathbf{T}$ if B v-constructs a non-empty set, otherwise $\mathbf{F}$.

Notational Conventions. That an object X belongs to a type α is denoted as 'X/α'; that a construction C v-constructs an α-object (provided not v-improper) is denoted by '$C \rightarrow \alpha$'. Instead of $[{}^0\forall^\alpha\ \lambda x\ A]$, $[{}^0\exists^\alpha\ \lambda x\ A]$ we write '$\forall x\ A$', '$\exists x\ A$' whenever no confusion arises. If $C \rightarrow \alpha_{\tau\omega}$ then the frequently used Composition $[[C\ w]\ t]$, aka extensionalization of the α-intension v-constructed by C, is abbreviated as C_{wt}. We use classical infix notation without Trivialization for truth-value functions $\wedge$ (conjunction), $\vee$ (disjunction), $\supset$ (implication) and $\neg$ (negation). Also, identities $=^{\alpha}$ of α-objects are written in the infix way without Trivialization and the superscript α whenever no confusion arises.

For a simple example, where $Student/(o\iota)_{\tau\omega}$ is a property of individuals and John/ι an individual, the sentence "John is a student" encodes as its meaning the hyper-proposition

$$\lambda w \lambda t\ [{}^0Student_{wt}\ {}^0John]$$

The property *Student* must be extensionalized first, $Student_{wt} \to (o\iota)$ and only then can it be applied to John, $[{}^0Student_{wt}\ {}^0John] \to o$. Abstracting over the values of variables w, t the proposition of type $o_{\tau\omega}$ that John is a student is produced.

3 NATURAL DEDUCTION IN TIL

The rules of natural deduction adjusted to TIL have been described in (Duží, Menšík, 2020). Here we just briefly recapitulate. For a correct application of the rules of a proof calculus in TIL it is important to realize that the rules are applicable to *constituents* of a given construction producing propositions and or truth-values. As described above, constituents of a procedure are not the input/output objects on which the procedure operates; they are beyond the procedure. Rather, constituents of a procedure are its sub-procedures occurring in executed mode.

When a construction C occurs in the displayed mode in D, then the construction C itself becomes the object on which other sub-constructions of D can operate; we also say that the context of its occurrence is *hyperintensional*, because all the sub-constructions of a displayed construction occur neither intensionally nor extensionally; they are displayed as well. When a construction C occurs in the executed mode in D, then the *product* (if any) of C is the object to be operated on. In this case the executed construction C is a *constituent* of its super-construction D.

The rules follow the general pattern of natural deduction and are thus introduced in I/E pairs. The rules dealing with truth-functions, namely conjunction introduction ($\wedge$-I) and elimination ($\wedge$-E), disjunction introduction ($\vee$-I) and elimination ($\vee$-E), implication introduction ($\supset$-I) and elimination ($\supset$-E, known also as modus ponendo ponens MPP) are standard, as in propositional logic. Additionally, there are rules for quantifiers (general $\forall$ and existential $\exists$). Again, these additional rules are of two kinds, namely introduction and elimination rules. Yet, quantifiers in TIL (see above) are not special symbols; rather, they are *functions* applicable to classes of objects. Hence, the rules must be adjusted for the TIL system. Here is how.

Let $x,y \to \alpha$, $B(x) \to o$: the variable x is free in B; $[\lambda x\ B] \to (o\alpha)$, $\forall/(o(o\alpha))$, $C \to \alpha$. Then general quantifier elimination in full detail consists of these steps:

$[{}^0\forall\lambda x\ B]$	$\varnothing$
$[[\lambda x\ B]\ y]$	$\forall$-E
$B(y)$	β–reduction
$B(C/y)$	*substitution*

where $B(C/y)$ arises from B by a collision-less, valid substitution of the construction C for all occurrences of the variable y in B.

For the sake of simplicity, we write this rule in the shortened form:

$$\frac{X \vdash [{}^0\forall\lambda x\ B]}{X \vdash B(C/x)} \qquad (\forall\text{-E})$$

The dual rule $\forall$-I then comes down in this form:

$$\frac{X \vdash B(y/x)}{X \vdash [{}^0\forall\lambda x\ B]} \qquad (\forall\text{-I})$$

Furthermore, there are rules for λ-introduction (λ-I) and elimination (λ-E). They are used in particular when dealing with empirical propositions. Since in any world w and time t the proof sequence must be truth-preserving from premises to a conclusion, the first steps of each such proof are λ-elimination (λ-E) of the left-most $\lambda w \lambda t$ to obtain constructions of truth-values, and the last steps introduce these $\lambda w \lambda t$ again (λ-introduction (λ-I).

4 SEMANTIC RULES

There are many features of the rich semantics of natural language that must be formalized by special rules that are not found in the formal logical languages. TIL is a logical system that has been primarily applied to the analysis of natural language because it is a powerful system in which almost all the semantically salient features of a language can be captured by rigorous, fine-grained analysis. Since it is out of the scope of this paper to deal with all the natural language semantic peculiarities, we refer for details to (Duží, Jespersen, Materna, 2010). To illustrate the problems we have to deal with when building up a question answering system over natural language corpora, we are now going to deal with factive verbs and presuppositions triggered by them, property modifiers and anaphoric references.

4.1 Factive Attitudes and Presuppositions

Factive verbs like to 'know that', 'regret that', 'be sorry', 'be proud', 'be indifferent', 'be glad that', 'be sad that', etc., presuppose that the embedded clause denotes a true proposition. For, if one asks, "Does John regret that he came late?" and John did not come late, there is no direct answer Yes or No. For, both answers entail that John did come late. In such a case an appropriate answer conveys information that the presupposition is not true, like "It is not true that John regrets his coming late because he did not come late". Note that while the direct answer applies *narrow scope* negation, the complete answer denies by *wide scope* negation.[3] Hence, both John regrets and John does not regret his coming late entail that John did come late. If John did not come late, he could neither regret nor not regret it, the proposition that he regrets it has a truth-value gap. Schematically, if K is a factive verb and X its complement clause, the following rules are valid: $K(X) \vdash X, \neg K(X) \vdash X$.

Factive verbs should be distinguished from implicative verbs like 'to manage' or 'to dare'. While sentences applying factive verbs presuppose the truth of the embedded clause, those with implicative verbs only entail it.[4] Schematically, where I is an implicative verb and X the complement clause, we have the following rules. $I(X) \vdash X, \neg I(X) \vdash \neg X$.

TIL is a logic of partial function, and as such is apt for dealing with presuppositions and truth-value gaps. Yet, partiality, as we all know very well, brings about technical complications. To manage them properly, we define properties of propositions *True*, *False* and *Undefined*, all of type $(\text{o}\text{o}_{\tau\omega})_{\tau\omega}$, as follows ($P \rightarrow \text{o}_{\tau\omega}$):

$$[{}^0True_{wt}\,P] \ v\text{-constructs } \mathbf{T} \text{ if } P_{wt}, \text{ otherwise } \mathbf{F};$$
$$[{}^0False_{wt}\,P] \ v\text{-constructs } \mathbf{T} \text{ if } \neg P_{wt}, \text{ otherwise } \mathbf{F};$$
$$[{}^0Undefined_{wt}\,P] = \neg[{}^0True_{wt}\,P] \wedge \neg[{}^0False_{wt}\,P].$$

Now we can rigorously define the difference between presupposition and a mere entailment. Let P, Q be constructions of propositions. Then

$$Q \textit{ is entailed by } P \text{ iff}$$
$$\forall w \forall t\, [[{}^0True_{wt}\,P] \supset [{}^0True_{wt}\,Q]];$$
$$Q \textit{ is a presupposition of } P \text{ iff}$$
$$\forall w \forall t\, [[[{}^0True_{wt}\,P] \vee [{}^0False_{wt}\,P]] \supset [{}^0True_{wt}\,Q]].$$

Hence, we have: *Q is a presupposition of P* iff $\forall w \forall t$ $[\neg[{}^0True_{wt}\,Q] \supset [{}^0Undefined_{wt}\,P]]$. If a presupposition of a proposition P is not true, then P has no truth value.

Factive verbs being a special case of attitudinal verbs, they thus denote relations-in-intension of an individual to the meaning of the embedded clause, which is a construction of a proposition. Hence, if K is the meaning of a factivum, then $K \rightarrow (\text{o}\iota *_n)_{\tau\omega}$. Furthermore, let $c/*_{n+1} \rightarrow *_n$, ${}^2c \rightarrow \text{o}_{\tau\omega}$ be a variable ranging over constructions of propositions, $a \rightarrow \iota$. Then the rules for factive propositional attitudes are:

$$\frac{[{}^0K_{wt}\,a\ c]}{[{}^0True_{wt}\ {}^2c]} \qquad \frac{\neg[{}^0K_{wt}\,a\ c]}{[{}^0True_{wt}\ {}^2c]}$$

4.2 Property Modifiers

Property modifiers are denoted by adjectives and they are functions in extension that applied to a root property return as a value the modified property. Here we deal with properties of individuals and modifiers of such properties of type $((\text{o}\iota)_{\tau\omega}(\text{o}\iota)_{\tau\omega})$. There are three basic kinds of modifiers, namely *intersective*, *subsective* and *privative.* Here are the examples.

a) Intersective. "A yellow elephant is yellow and is an elephant."
b) Subsective. "A skilful surgeon is a surgeon."
c) Privative. "Forged passport is non-passport."

We are not going to analyse these modifiers in detail here. TIL analysis has been introduced in numerous papers, see, e.g. (Jespersen, Carrara, Duží, 2017), (Duží, 2017) or (Jespersen, 2015), (Jespersen, 2016). The issue we deal with bellow is the rule of left subsectivity.[5]

The principle of left subsectivity is trivially (by definition) valid for intersective modifiers. If Jumbo

[3] For details on narrow and wide scope negation see (Duží, 2018b) and for answering questions with presuppositions, see (Duží, Číhalová, 2015).

[4] We are not going to deal with implicative verbs here; yet, see (Nadathur, 2016), and also (Baglini, Francez, 2016) for detail. Note however, that the notion of presupposition that these authors deal with is pragmatic in nature, while we deal with logical presuppositions the definition of which comes below. It appears the implicative verbs listed above presuppose a weaker version of a presupposition; 'to manage something' presupposes 'to try that something' (and a certain difficulty of the task) and 'to dare' presupposes a sort of 'want'. We are grateful to an anonymous referee for this note.

[5] Here we partly draw on material from (Duží et al., 2010, §4.4).

is a yellow elephant, then Jumbo is yellow. Yet how about the other modifiers? If Jumbo is a small elephant, is Jumbo small? If you factor out *small* from *small elephant*, the conclusion says that Jumbo is small. Yet this would seem a strange thing to say, for something appears to be missing: Jumbo is a small *what*? Nothing or nobody can be said to be small — or forged, skilful, good, notorious, or whatnot, without any sort of qualification. A complement to provide an answer to the question, 'a … *what*?' is required. We are going to introduce the rule of left subsectivity that is valid for all kinds of modifiers including subsective and privative ones. The idea is simple. From a is an $[MP]$ we infer that a is an *M-with respect to something.*

Here is the scheme of defining left subsectivity rule, SI being substitution of identical properties (Leibniz's Law).

(1)	a is an MP	assumption
(2)	a is an (M something)	1, EG
(3)	M^* is the property (M something)	definition
(4)	a is an M^*	2, 3, SI

To put the rule on more solid grounds of TIL, let $\pi = (o\iota)_{\tau\omega}$ for short, $M \to (\pi\pi)$ be a modifier, $P \to \pi$ an individual property, $[MP] \to \pi$ the property resulting from applying M to P, Further, let $=/(o\pi\pi)$ be the identity relation between properties, and let $p \to_v \pi$ range over properties, $x \to_v \iota$ over individuals. Then the *proof* of the rule is this:

1.	$[[MP]_{wt}\ a]$	assumption
2.	$\exists p\ [[Mp]_{wt}\ a]$	1, EG
3.	$[\lambda x\ \exists p\ [[Mp]_{wt} x]\ a]$	2, β-expansion
4.	$[\lambda w'\lambda t'\ [\lambda x\ \exists p\ [[Mp]_{w't'} x]]_{wt}\ a]$	3, β-expansion
5.	$M^* = \lambda w'\lambda t'\ [\lambda x\ \exists p\ [[Mp]_{w't'}\ x]]$	definition
6.	$[M^*_{wt}\ a]$	4, 5, SI

Any valuation of the free occurrences of the variables w, t that makes the first premise true will, together with step five, make the conclusion true. Left subsectivity (LS), dressed up in full TIL notation, is this:

$$\text{(LS)}\qquad \frac{[[MP]_{wt}\ a] \quad [M^* = \lambda w\lambda t\ \lambda x\ \exists p\ [[Mp]_{wt} x]]}{[M^*_{wt}\ a].}$$

Additional type: $\exists/(o(o\pi))$.

This specification of the rule easily dismantles objections raised against the (LS) principle by Gamut (1991, §6.3.11) and Geach (1956). Summarising briefly, there are three such arguments against (LS).

First Objection. If Jumbo is a small elephant and a large mammal, then Jumbo is small and large — contradiction! Yet, there is no contradiction, because Jumbo is small as an elephant and large as a mammal. Hence the properties p, q with respect to which Jumbo is a $[^0Small\ p]$ and $[^0Large\ q]$ are distinct.

The conclusion ought to strike us as being trivial. If we grant, as we should, that nobody and nothing is absolutely small or absolutely large, then everybody is made small by something and made large by something else. And if we grant, as we should, that nobody is absolutely good or absolutely bad, then everybody has something they do well and something they do poorly. That is, everybody is both good and bad, which here just means being good at something and being bad at something else, without generating paradox ($Good$, $Bad/(\pi\pi)$):

$$\lambda w\lambda t\ \forall x\ [\exists p\ [[^0Good\ p]_{wt}\ x] \wedge \exists q\ [[^0Bad\ q]_{wt}\ x]].$$

But nobody can be good at something and bad *at the same thing* simultaneously:

$$\forall w \forall t\ \forall x\ \neg\exists p\ [[[^0Good\ p]_{wt}\ x] \wedge [[^0Bad\ p]_{wt}\ x]].$$

The Second Objection is rejected in a similar way. The argument goes as follows. If Jumbo is a small elephant and Mickey is a large mouse, then Jumbo is small, and Mickey is large; hence Jumbo is smaller than Mickey. Again, to derive the conclusion, it would have to be granted that Jumbo is small with respect to the same property as Mickey, which is not so.

Third Objection. If we do not hesitate to use 'small' not only as a modifier but also as a predicate, then it would seem we could not possibly block the following fallacy:

Jumbo is small
Jumbo is an *elephant*

Jumbo is a small elephant.

But we can and must block it, for this argument is obviously not valid. The premises do not guarantee that the property p with respect to which Jumbo is small is identical to the property *Elephant*.

4.3 Anaphoric References and Substitution Method

Resolving anaphoric references is a hard nut for every linguist dealing with the semantics of natural languages because there are frequently many ambiguities as for to which part of the foregoing discourse the anaphoric pronoun refers. Logic cannot disambiguate any sentence, of course. Instead, logic can contribute to disambiguation and better communication by making these hidden features explicit and logically tractable. If a sentence or term is ambiguous, we furnish it with multiple constructions as its proposed meanings and leave it to the agent to decide which of these meanings is the intended one.

To deal with anaphoric references, we apply generalized Hans Kamp's Discourse Representation Theory (DRT), see (Kamp, 1981), (Kamp, Reyle, 1993). 'DRT' is an umbrella term for a collection of logical and computational linguistic methods developed for a *dynamic interpretation* of natural language, where each sentence is interpreted within a certain discourse. DRT as presented in (Kamp, 1981) is a first-order theory. Thus, only terms denoting individuals (indefinite or definite noun phrases) can introduce so-called discourse referents, which are free variables that are updated when interpreting the discourse.

Since TIL semantics is procedural, hence hyperintensional and higher-order, not only individuals, but entities of any type, like properties of individuals, propositions, relations-in-intension, and even constructions (i.e., meanings of antecedent expressions), can be linked to anaphoric variables. Moreover, the thoroughgoing typing of the universe of TIL makes it possible to determine the respective *type-theoretically appropriate antecedent*, which also contributes to disambiguation.[6]

For instance, the ambiguous anaphoric reference to *properties* as in Neale's example "John loves his wife and so does Peter" has been analysed in (Duží, Jespersen, 2013). The authors prove that the sentence entails that John and Peter share a property. Only that it is ambiguous which one; there are two options, (i) loving John's wife and (ii) loving one's own wife. The property predicated of Peter in 'so does Peter' is a function of the property predicated of John in 'John loves his wife'. Since the source clause is ambiguous between attributing (i) or (ii) to John, the target clause is likewise ambiguous between attributing (i) or (ii) to Peter. The ambiguity of the anaphoric expression 'his wife' as applied to John is visited upon the likewise anaphoric expression 'so does'. The authors propose the analyses of both readings and show that unrestricted β-reduction 'by name' reduces both readings to the strict one on which John and Peter love John's wife, which is undesirable.[7]

The solution consists in the application of β-reduction 'by value' that makes use of the functions *Sub* and *Tr* defined as follows. The function $Sub/(*_n *_n *_n *_n)$ operates on constructions so that the Composition $[{}^0 Sub\ C_1\ C_2\ C_3]$ produces a construction D that is the result of the collision-less substitution of the product of C_1 for the product of C_2 into C_3. The function $Tr/(*_n\ \alpha)$ produces the Trivialization of the α-object.

What is also special about "John loves his wife, and so does Peter" is that it involves *two anaphoric* terms, namely 'his' and 'so does'. It might seem tempting, though, to analyse "John loves his wife" as though it were synonymous with "John loves John's wife". Then "So does Peter" would unambiguously attribute to Peter the property of loving John's wife. But this analysis would not be plausible as it would entirely annihilate the *anaphoric* character of 'his'. Instead, the form of the solution must be in terms of resolution of verb-phrase ellipsis. It needs to be spelt out which of two properties applies to John in "John loves his wife" and so applies to Peter in "So does Peter".

The property (i) of loving John's wife is produced by

$$\lambda w \lambda t\ \lambda x\ [{}^0 Love_{wt}\ x\ [{}^0 Wife\_of_{wt}\ {}^0 John]]$$

while the property (ii) of loving one's own wife is produced by

$$\lambda w \lambda t\ \lambda x\ [{}^0 Love_{wt}\ x\ {}^2[{}^0 Sub\ [{}^0 Tr\ x]\ {}^0 y\ {}^0[{}^0 Wife\_of_{wt}\ y]]]$$

From the logical point of view, anaphoric pronouns denote variables, valuation of which is supplied by referring to an appropriate antecedent. To this end, we developed a *substitution method* that exploits the functions *Sub* and *Tr* defined above

To adduce an example of referring to the *meaning* of a term, i.e. to the encoded *construction*, the sentence "Sin of π equals zero and John knows *it*" encodes the following construction as its meaning[8].

$$\lambda w \lambda t \left[[[{}^0 Sin\ {}^0\pi] = {}^0 0] \wedge {}^2[{}^0 Sub\ [{}^0 Tr\ {}^0[[{}^0 Sin\ {}^0\pi] = {}^0 0]]\ {}^0 it\ {}^0[{}^0 Know_{wt}\ {}^0 John\ it]] \right]$$

[6] The algorithm for dynamic discourse representation within TIL has been specified in (Duží, 2018a) and implemented by Kotová, (2018). It is applied in a multi-agent system to govern the communication of individual agents by messaging.

[7] (Loukanova, 2009) also warns against unrestricted β-reduction and its undesirable results.

[8] We analyse $Know(ing)/(o\iota *_n)_{\tau\omega}$ as a hyperintensional attitude, i.e. the relation-in-intension of an individual to a *hyperproposition* (construction of a truth value or a PWS proposition). In case of mathematics it is obvious that such attitudes must relate an individual to the very procedure rather than its product; it makes no sense to know a truth value without any mathematical operation producing it. In an empirical case intensional attitudes are also thinkable. Yet, since intensional attitudes inevitably yield a variant of the well-known paradox of logical/mathematical omniscience, we vote for the hyperintensional analysis here.

Types. $Sin/(\tau\tau)$; $0,\pi/\tau$; $[[{}^0Sin\ {}^0\pi] = {}^00]/*_1 \rightarrow o$; $Know/(o\iota*_n)_{\tau\omega}$; John/$\iota$; it/$*_2 \rightarrow *_1$.

Note that the result of the substitution (application of the *Sub* function) is an adjusted *construction* $[{}^0Know_{wt}\ {}^0John\ {}^0[[{}^0Sin\ {}^0\pi] = {}^00]]$. But the second argument of conjunction must be a truth-value; hence, the adjusted construction must be executed—therefore Double Execution.

This analysis is fully compositional. The meaning of "John knows it"—$\lambda w\lambda t\ [{}^0Know_{wt}\ {}^0John\ it]$—contains a *free* variable *it* as its constituent. If the sentence is uttered in isolation, the valuation assignment is a pragmatic matter of a speaker/interpreter. However, if the sentence is embedded in the discourse context, the variable *it* becomes *bound,* and the value assignment is provided by the substitution method.[9]

5 TWO CASE STUDIES

5.1 Reasoning with Property Modifiers

Scenario. John is a married man. John's partner is Eve. John is a member of a sports club and a student. All students like holidays. Everybody who is married believes that his/her partner is fantastic. Frank is a student. Frank thinks that Peter is an actor.

Question. Does John believe that Eve is fantastic?
To formalise our mini knowledge base, we start with assigning types to the objects that receive mention in the text:

Types: *John*, *Eve, Peter, Frank, S(port)C(lub)*/ι; *Partner-of*/$(\iota\iota)_{\tau\omega}$; $Married^m/((o\iota)_{\tau\omega}(o\iota)_{\tau\omega})$; *Married, Actor*, *Student*, *Fantastic*/$(o\iota)_{\tau\omega}$; *Member*, *Like* /$(o\iota\iota)_{\tau\omega}$; *Holidays*/$\iota$; *Believe*, *Think*/$(o\iota*_n)_{\tau\omega}$; $w \rightarrow \omega$; $t \rightarrow \tau$; $x, y \rightarrow \iota$.

Analysis of the sentences of our scenario comes down to these constructions:

A. $\lambda w\lambda t\ [[{}^0Married^m\ {}^0Man]_{wt}\ {}^0John]$
B. $\lambda w\lambda t\ [[{}^0Partner\text{-}of_{wt}\ {}^0John] = {}^0Eve]$
C. $\lambda w\lambda t\ [[{}^0Member_{wt}\ {}^0John\ {}^0SC] \wedge [{}^0Student_{wt}\ {}^0John]]$
D. $\lambda w\lambda t\ \forall x\ [[{}^0Student_{wt}\ x] \supset [{}^0Like_{wt}\ x\ {}^0Holidays]]$
E. $\lambda w\lambda t\ \forall x\ [[{}^0Married_{wt}\ x] \supset$
$[{}^0Believe_{wt}\ x\ [{}^0Sub\ [{}^0Tr\ [{}^0Partner\text{-}of_{wt}\ x]]\ {}^0y$
${}^0[\lambda w\lambda t\ [{}^0Fantastic_{wt}\ y]]]]]$
F. $\lambda w\lambda t\ [{}^0Student_{wt}\ {}^0Frank]$
G. $\lambda w\lambda t\ [{}^0Think_{wt}\ {}^0Frank\ {}^0[\lambda w\lambda t\ {}^0Actor_{wt}\ {}^0Peter\ ]]$

Conclusion/question:
Q $\lambda w\lambda t\ [{}^0Believe_{wt}\ {}^0John\ {}^0[\lambda w\lambda t\ [{}^0Fantastic_{wt}\ {}^0Eve]]]$

To derive the answer, we are going to apply the system of Gentzen's natural deduction (ND) adjusted for TIL. In addition to the standard rules of the ND system, we need the rule of left subsectivity (LS) for dealing with the property modifier $Married^m$.

The rule results in

$$[[{}^0Married^m\ {}^0Man]_{wt}\ x\ ] \vdash [{}^0Married_{wt}\ x]$$

Informally, this rule represents the fact that "Married man is married".

We must also deal with technical rules and functions specific for TIL. For instance, application of the functions *Sub* and *Tr* must be properly evaluated, or Leibniz's law of substitution of identicals specified for TIL in (Duží, Materna, 2017) and (Fait, Duží, 2020) must be properly applied.

Table 1 presents the proof. The answer to the question Q is Yes, of course; it follows from our mini knowledge base that John indeed believes that Eve is fantastic.

However, in this proof, we simplified the situation. We took into account only the premises relevant for deriving the conclusion, ignoring the others. For instance, from premises D and F one can infer (by applying $\forall$-E and MPP) that "Frank likes holidays". Similarly, by applying $\wedge$-E, $\forall$-E and MPP to the premises C and D we can infer that John likes holidays. Yet, these conclusions are pointless for answering the question Q.

In practice, there are a huge number of sentences formalised in the form of TIL constructions so that extracting the relevant ones is not so easy. Moreover, implementation of the method within the interactive question answering system calls for an algorithm of selecting relevant input sentences so that to reduce inferring consequences that are not needed. To this end, we propose a simple solution that nevertheless restricts the number of input premises and thus also the length of the proofs significantly. We select only those sentences that talk about the objects that receive mention in a given question.

In our example, the following constructions would be selected because they contain the constituents ${}^0Believe$, ${}^0John$, ${}^0Fantastic$ and ${}^0Eve$, which they have in common with the question Q.

[9] A similar stance and solution can be found in (Loukanova, 2012).

Table 1: Derivation of the answer.

1.	$\lambda w \lambda t\ [[{}^0\mathit{Married}^m\ {}^0\mathit{Man}]_{wt}\ {}^0\mathit{John}]$	$\varnothing$
2.	$\lambda w \lambda t\ [[{}^0\mathit{Partner\text{-}of}_{wt}\ {}^0\mathit{John}] = {}^0\mathit{Eve}]$	$\varnothing$
3.	$\lambda w \lambda t\ \forall x\ [[{}^0\mathit{Married}_{wt}\ x] \supset [{}^0\mathit{Believe}_{wt}\ x\ [{}^0\mathit{Sub}\ [{}^0\mathit{Tr}\ [{}^0\mathit{Partner\text{-}of}_{wt}\ x]]\ {}^0y\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Fantastic}_{wt}\ y]]]]]$	$\varnothing$
4.	$[[{}^0\mathit{Married}^m\ {}^0\mathit{Man}]_{wt}\ {}^0\mathit{John}]$	1, λ-E
5.	$[[{}^0\mathit{Partner\text{-}of}_{wt}\ {}^0\mathit{John}] = {}^0\mathit{Eve}]$	2, λ-E
6.	$\forall x\ [[{}^0\mathit{Married}_{wt}\ x] \supset [{}^0\mathit{Believe}_{wt}\ x\ [{}^0\mathit{Sub}\ [{}^0\mathit{Tr}\ [{}^0\mathit{Partner\text{-}of}_{wt}\ x]]\ {}^0y\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Fantastic}_{wt}\ y]]]]]$	3, λ-E
7.	$[[{}^0\mathit{Married}_{wt}\ {}^0\mathit{John}] \supset [{}^0\mathit{Believe}_{wt}\ {}^0\mathit{John}\ [{}^0\mathit{Sub}\ [{}^0\mathit{Tr}\ [{}^0\mathit{Partner\text{-}of}_{wt}\ {}^0\mathit{John}]]\ {}^0y\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Fantastic}_{wt}\ y]]]]]$	6, ∀-E, ${}^0\mathit{John}/x$
8.	$[[{}^0\mathit{Married}_{wt}\ {}^0\mathit{John}] \supset [{}^0\mathit{Believe}_{wt}\ {}^0\mathit{John}\ [{}^0\mathit{Sub}\ [{}^0\mathit{Tr}\ {}^0\mathit{Eve}]\ {}^0y\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Fantastic}_{wt}\ y]]]]]$	5,7, SI (Leibnitz)
9.	$[{}^0\mathit{Married}_{wt}\ {}^0\mathit{John}]$	4. LS
10.	$[{}^0\mathit{Believe}_{wt}\ {}^0\mathit{John}\ [{}^0\mathit{Sub}\ [{}^0\mathit{Tr}\ {}^0\mathit{Eve}]\ {}^0y\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Fantastic}_{wt}\ y]]]]$	8,9 MPP
11.	$[{}^0\mathit{Believe}_{wt}\ {}^0\mathit{John}\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Fantastic}_{wt}\ {}^0\mathit{Eve}]]]$	10, *Sub*, *Tr*
12.	$\boldsymbol{\lambda w \lambda t\ [{}^0\mathit{Believe}_{wt}\ {}^0\mathit{John}\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Fantastic}_{wt}\ {}^0\mathit{Eve}]]]}$	**11, λ-I**

A. $\lambda w \lambda t\ [[{}^0\mathit{Married}^m\ {}^0\mathit{Man}]_{wt}\ {}^0\mathit{John}]$
B. $\lambda w \lambda t\ [[{}^0\mathit{Partner\text{-}of}_{wt}\ {}^0\mathit{John}] = {}^0\mathit{Eve}]$
C. $\lambda w \lambda t\ [[{}^0\mathit{Member}_{wt}\ {}^0\mathit{John}\ {}^0\mathit{SC}] \wedge [{}^0\mathit{Student}_{wt}\ {}^0\mathit{John}]]$
E. $\lambda w \lambda t\ \forall x\ [[{}^0\mathit{Married}_{wt}\ x] \supset [{}^0\mathit{Believe}_{wt}\ x\ [{}^0\mathit{Sub}\ [{}^0\mathit{Tr}\ [{}^0\mathit{Partner\text{-}of}_{wt}\ x]]\ {}^0y\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Fantastic}_{wt}\ y]]]]]$

The premises D, F and G are irrelevant because they do not have any constituent in common with the question Q. This heuristic method does not guarantee that all the selected constructions are necessary for deriving the answer (in our case the premise C is spare), nor that the selected set is sufficient for deriving the answer. It may happen that in the proof process the heuristic method must be iterated to select additional input sentences. Anyway, it turns out that in most cases one-step heuristic is sufficient, and the process of proving is effectively optimized.

5.2 Factive Propositional Attitudes

Scenario. The Mayor of Ostrava is Tomáš Macura. Prof. Vondrák likes teaching. The Mayor of Ostrava knows that the President of Technical University of Ostrava (TUO) does not know (yet) that he (the President of TUO) will go to Brussels. The President of TUO is prof. Snášel. Prof. Snášel likes swimming. Prof. Vondrák is a politician.

Question. Will prof. Snášel go to Brussels?

Types: Snasel, *Macura*, *Vondrak, Brussels*/ι; *President*(-of TUO), *Mayor* (-of Ostrava)/$\iota_{\tau\omega}$; *Go*/$(o\iota\iota)_{\tau\omega}$; *Like*/$(o\iota\alpha)_{\tau\omega}$; *Know*/$(o\iota *_n)_{\tau\omega}$. *Swimming, Teaching*/α;[10] *Politician*/$(o\iota)_{\tau\omega}$.

Knowledge base:
A. $\lambda w \lambda t\ [{}^0\mathit{Mayor}_{wt} = {}^0\mathit{Macura}]$
B. $\lambda w \lambda t\ [{}^0\mathit{Like}_{wt}\ {}^0\mathit{Vondrak}\ {}^0\mathit{Teaching}]$
C. $\lambda w \lambda t\ [{}^0\mathit{Know}_{wt}\ {}^0\mathit{Mayor}_{wt}\ {}^0[\lambda w \lambda t\ \neg[{}^0\mathit{Know}_{wt}\ {}^0\mathit{President}_{wt}\ [{}^0\mathit{Sub}\ [{}^0\mathit{Tr}\ {}^0\mathit{President}_{wt}]\ {}^0\mathit{he}\ {}^0[\lambda w \lambda t\ [{}^0\mathit{Go}_{wt}\ \mathit{he}\ {}^0\mathit{Brussels}]]]]]]$
D. $\lambda w \lambda t\ [{}^0\mathit{President}_{wt} = {}^0\mathit{Snasel}]$
E. $\lambda w \lambda t\ [{}^0\mathit{Like}_{wt}\ {}^0\mathit{Snasel}\ {}^0\mathit{Swimming}]$
F. $\lambda w \lambda t\ [{}^0\mathit{Politician}_{wt}\ {}^0\mathit{Vondrak}]$

Question:
Q. $\lambda w \lambda t\ [{}^0\mathit{Go}_{wt}\ {}^0\mathit{Snasel}\ {}^0\mathit{Brussels}]$

What is interesting about this example is that it makes it possible to demonstrate a *top-down derivation* from hyperintensional level of the complement of knowing/not knowing that "he will go to Brussels" to the extensional level of Snasel's going to Brussels. It is made possible by application of the rules for factive attitudes defined above, plus the rule for *True-Elimination* and resolution of anaphoric references by the substitution method. To recapitulate, here are the rules ($c \rightarrow *_n$, ${}^2c \rightarrow o_{\tau\omega}$; $p \rightarrow o_{\tau\omega}$; *True*/$(oo_{\tau\omega})_{\tau\omega}$).

(F1) $[{}^0\mathit{Know}_{wt}\ a\ c] \vdash [{}^0\mathit{True}_{wt}\ {}^2c]$

(F2) $\neg[{}^0\mathit{Know}_{wt}\ a\ c] \vdash [{}^0\mathit{True}_{wt}\ {}^2c]$

(*True*-E) $[{}^0\mathit{True}_{wt}\ p] \vdash p_{wt}$

[10] For the sake of simplicity, we assign type α to these activities, because this simplification is harmless to the derivation we are going to demonstrate.

Table 2: Top-down derivation of the answer.

1	$\lambda w\lambda t\ [{}^0Know_{wt}\ {}^0Mayor_{wt}\ {}^0[\lambda w\lambda t\ \neg[{}^0Know_{wt}\ {}^0President_{wt}\ [{}^0Sub\ [{}^0Tr\ {}^0President_{wt}]$ ${}^0he\ {}^0[\lambda w\lambda t\ [{}^0Go_{wt}\ he\ {}^0Brussels]]]]]]$	$\varnothing$
2.	$\lambda w\lambda t\ [{}^0President_{wt} = {}^0Snasel]$	$\varnothing$
3.	$\lambda w\lambda t\ [{}^0Like_{wt}\ {}^0Snasel\ {}^0Swimming]$	$\varnothing$
4.	$\lambda w\lambda t\ [{}^0Mayor_{wt} = {}^0Macura]$.	$\varnothing$
5.	$[{}^0Know_{wt}\ {}^0Mayor_{wt}\ {}^0[\lambda w\lambda t\ \neg[{}^0Know_{wt}\ {}^0President_{wt}\ [{}^0Sub\ [{}^0Tr\ {}^0President_{wt}]\ {}^0he$ ${}^0[\lambda w\lambda t\ [{}^0Go_{wt}\ he\ {}^0Brussels]]]]]]$	1, λ-E
6.	$[{}^0President_{wt} = {}^0Snasel]$	2, λ-E
7.	$[{}^0Like_{wt}\ {}^0Snasel\ {}^0Swimming]$	3, λ-E
8.	$[{}^0Mayor_{wt} = {}^0Macura]$	4, λ-E
9.	$[{}^0True_{wt}\ {}^{20}[\lambda w\lambda t\ \neg[{}^0Know_{wt}\ {}^0President_{wt}\ [{}^0Sub\ [{}^0Tr\ {}^0President_{wt}]\ {}^0he$ ${}^0[\lambda w\lambda t\ [{}^0Go_{wt}\ he\ {}^0Brussels]]]]]]$	5, F1
10.	${}^{20}[\lambda w\lambda t\ \neg[{}^0Know_{wt}\ {}^0President_{wt}\ [{}^0Sub\ [{}^0Tr\ {}^0President_{wt}]\ {}^0he$ ${}^0[\lambda w\lambda t\ [{}^0Go_{wt}\ he\ {}^0Brussels]]]]]_{wt}$	9, *True* - E
11.	$[\lambda w\lambda t\ \neg[{}^0Know_{wt}\ {}^0President_{wt}\ [{}^0Sub\ [{}^0Tr\ {}^0President_{wt}]\ {}^0he$ ${}^0[\lambda w\lambda t\ [{}^0Go_{wt}\ he\ {}^0Brussels]]]]]_{wt}$	10, ^{20}-E
12.	$\neg[{}^0Know_{wt}\ {}^0President_{wt}\ [{}^0Sub\ [{}^0Tr\ {}^0President_{wt}]\ {}^0he\ {}^0[\lambda w\lambda t\ [{}^0Go_{wt}\ he\ {}^0Brussels]]]]$	11, β-r
13.	$[{}^0True_{wt}\ {}^2[{}^0Sub\ [{}^0Tr\ {}^0President_{wt}]\ {}^0he\ {}^0[\lambda w\lambda t\ [{}^0Go_{wt}\ he\ {}^0Brussels]]]]$	12, F2
14.	${}^2[{}^0Sub\ [{}^0Tr\ {}^0President_{wt}]\ {}^0he\ {}^0[\lambda w\lambda t\ [{}^0Go_{wt}\ he\ {}^0Brussels]]]_{wt}$	13, *True* – E
15.	${}^2[{}^0Sub\ [{}^0Tr\ {}^0Snasel]\ {}^0he\ {}^0[\lambda w\lambda t\ [{}^0Go_{wt}\ he\ {}^0Brussels]]]_{wt}$	14,6, SI
16.	${}^2[{}^0Sub\ {}^{00}Snasel\ {}^0he\ {}^0[\lambda w\lambda t\ [{}^0Go_{wt}\ he\ {}^0Brussels]]]_{wt}$	15, *Tr*
17.	${}^{20}[\lambda w\lambda t\ [{}^0Go_{wt}\ {}^0Snasel\ {}^0Brussels]]_{wt}$	16, *Sub*
18.	$[\lambda w\lambda t\ [{}^0Go_{wt}\ {}^0Snasel\ {}^0Brussels]]_{wt}$	17, ^{20}-E
19.	$[{}^0Go_{wt}\ {}^0Snasel\ {}^0Brussels]$	18, β-r
20.	$\lambda w\lambda t\ [{}^0Go_{wt}\ {}^0Snasel\ {}^0Brussels]$	19, λ-I

For technical reasons, we also need the rule of ^{20}-Elimination, a simple technical adjustment, which holds for any construction C that is typed to v-construct a non-procedural object of a type of order 1.

(^{20}-E) $\quad {}^{20}C = C$

For the selection of constructions that are relevant for deriving the answer we now apply the heuristics described above. Constituents of the question Q are ${}^0Go$, ${}^0Snasel$ and ${}^0Brussels$. These constituents occur as sub-constructions of the sentences C, D and E.

C. $\lambda w\lambda t\ [{}^0Know_{wt}\ {}^0Mayor_{wt}\ {}^0[\lambda w\lambda t\ \neg[{}^0Know_{wt}$ ${}^0President_{wt}\ [{}^0Sub\ [{}^0Tr\ {}^0President_{wt}]\ {}^0he$ ${}^0[\lambda w\lambda t\ [{}^0Go_{wt}\ he\ {}^0Brussels]]]]]]$
D. $\lambda w\lambda t\ [{}^0President_{wt} = {}^0Snasel]$
E. $\lambda w\lambda t\ [{}^0Like_{wt}\ {}^0Snasel\ {}^0Swimming]$

In the sentence C there is another constituent, namely ${}^0Mayor$, and this same constituent also occurs in the premise A. By iterating the heuristics, we include A among the premises as well:

A. $\lambda w\lambda t\ [{}^0Mayor_{wt} = {}^0Macura]$.

The proof of the argument, i.e. the derivation of the answer to the question Q from premises A, C, D and E can be found in Table 2. Since we proved that the premises A, C, D and E entail that Snášel is going to Brussels, the answer to the question Q is YES.

6 CONCLUSION

In this paper, we introduced the system for 'intelligent' question answering over natural language texts. The system derives answers to the questions as logical consequences of assumptions extracted from given text corpora. When designing such a system, one has to solve several problems. First, natural language sentences must be analysed in a fine-grained way so that all the semantically salient features of a language are captured by an adequate formalization. To this end, we exploited the system of Transparent Intensional Logic (TIL). Second, there are special rules rooted in the rich semantics of natural language which are not found in standard proof calculi. The problem is how to integrate these rules with a given proof system. And the third problem is how to extract just those sentences that are needed for deriving the answer from the large corpora of input text data. There are two novel contributions of the paper. While in the previous proposals based on TIL it has been tacitly presupposed that it is possible to pre-process the natural language sentences first, and then to apply a standard proof calculus, we gave up this assumption, because it turned up to be unrealistic. Instead, we voted for Gentzen's natural deduction system so that those special semantic rules could be smoothly inserted into the derivation process together with the standard I/E rules of the proof system. Yet, by applying the forward-chaining strategy of the natural deduction system, we faced up the problem of extracting those sentences that are relevant for the derivation of the answer. As a solution, we proposed a heuristic method that extracts those sentences that have some constituents in common with the posed question.

Future research will concentrate on the comparison of this approach with the system of deriving answers by means of the backwards-chaining strategy of general resolution method and/or sequent calculus, and an effective implementation thereof. Moreover, we will also deal with Wh-questions like "Who is going to Brussels?", "When did an American president visit Prague?", analyse them and propose a method of their intelligent answering.

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