A Production Model with Continuous Demand for Imperfect Finished Items Resulting from the Quality of Raw Material

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Abstract: The purpose of this paper is to present a production process in which the quality of a single type of raw material used to produce the finished product is considered. The common modeling approach followed by previous research is based on discarding the imperfect quality items of raw material. In this paper, we consider the case where both perfect and imperfect quality items of raw are used in the production process resulting in two types of qualiy of the finished product. It is assumed that both perfect and imperfect quality items of the finished product have continuous demand. This modeling approach has yet to be deployed. Two models that depend on the length inventory cycle of each type of the finished product are developed. Numerical examples are provided to illustrate the determination of the optimal production quantity. Theoretical and practical implications are discussed, and recommendations are presented.

1 INTRODUCTION

Production control and inventory management are two important business functions with the objective of controlling the materials used in manufacturing and trading. The importance of these functions lies in the fact that keeping the right amount of inventory with a good quality level will help organizations avoid excess inventory and shortages, while satisfying customers' demand. This is crucial in an era of globalization, where customer demand for products has been increasing, and where organizations should have the agility required to respond to different demand preferences sufficiently and on time.

In these two functions, the two models, economic production quantity (EPQ) and the economic order quantity (EOQ), are used to identify the optimal quantities to order or produce to meet the demand for a certain product.

The EOQ and EPQ models are simple to apply and built on a number of simplifying assumptions. For instance, the classical EPQ model ignores the cost and quality of raw material used in the production process. Also, the classical EPQ model views the manufacturing process as failure free, implying that items produced have perfect quality. This contrasts with real life production environment, where defective items are generated due to defective raw materials or defective production processes (Pal et al, 2016). This study concurs with this view, and argues that the inventory and production policy guided by the conventional EPQ model is inappropriate as it does not reflect what usually happens in production processes.

Several researchers addressed this unreliable assumption. Recently, a number of research studies have worked on EPQ/EOQ models, taking imperfect quality raw materials or imperfect product items into consideration. This research direction was initiated by Salameh and Jaber (2000), who developed an EPQ model that counted for the imperfect items delivered by a supplier with a known probability density function.

Moreover, several recent studies have considered the effects of the quality of the raw material used in the production process (El-Kassar et al., 2012; Yassine 2016; Yassine & AlSagheer, 2017; Yassine et al., 2018; Yassine & El-Rabih, 2019). Yassine (2018) presented a sustainable EPQ model with quality. In fact, corporations have been engaging in responsible and environmentally friendly activities that enhance performance (El-Kassar & Singh, 2019; Singh et al., 2019, El-Khalil & El-Kassar, 2018; El-Khalil & El-Kassar, 2016). Such activities have been shown to lead to other positive outcomes such as higher level of corporate governance (ElGammal et al., 2018) and favorable employee attitude and behavior (El-Kassar et al. 2017). Firms are also employing information and communication technologies and innovation to improve their competitiveness level (Singh et al., 2019; Balozian et al., 2019; Yunis et al., 2018; Balozian & Leidner, 2017; Yunis et al., 2017). Recently, these factors have been incorporated into the classical EPQ model (Lamba et al., 2019; Yassine, 2018).

Salameh and El-Kassar (2007) extended the EPQ model to account for the raw material used in production. Chan et al. (2003) presented an EPQ model where the imperfect products are reworked or rejected. El-Kassar (2009) presented an EOQ model with quality in which the imperfect items have a continuous demand for both perfect and imperfect quality items. In other directions, several studies considered a supply chain approach was considered (Khan et al., 2011; Khan & Jaber, 2011; Bandaly et al. 2014; Bandaly et al. 2016), while Bandaly & Hassan (2019) considered an integrated production and inventory taking into consideration deterioration and limited storage capacity.

This research paper examines an EPQ model that takes into consideration the situation where a single type of raw material with a percentage of imperfect items is all used in a production process. This EPQ model is material-dependent and hence will yield finished products with a proportion being defective. The model assumes continuous demand for both the good quality and the defective products, making it essential to incorporate both product types in the production model. Two models that depend on the inventory cycle length of each type of finished product are presented. The remaining of this paper is organized as follows. Section 2 presents a review of related work. The mathematical model is developed in section 3. A numerical example is given in section 4 to illustrate the proposed model. Finally, section 5 presents a conclusion and future research recommendations.

2 MATHEMATICAL MODEL

Consider the case where items of raw material received from a supplier are of perfect and imperfect quality. Both types are used in the production process resulting in perfect and imperfect finished products. It is assumed that both types of finished product have continuous demand.

2.1 Notation

The following notation is used for this model are:

- Q: Number of units per order (units)
- Q*: Optimal number of units per order (units)
- D_p: Demand rate for perfect finished products (units/unit time)
- D_i: Demand rate for imperfect finished products (units/unit time)
- D: Demand rate (units/unit time): D_p+D_i
- C: Purchasing cost per unit ($/unit)
- C_p: Unit production cost ($/unit)
- K_o: Ordering cost of raw material ($)
- K_s: Set-up cost of production ($)
- C_s: Unit screening cost ($/unit)
- C_{hr}: Holding cost of raw material ($/unit/unit time)
- C_{hf}: Holding cost of finished products ($/unit/unit time)
- q: Percentage of perfect quality of raw material
- S_p: Selling price of perfect quality products ($)
- S_i: Selling price of imperfect quality products ($)
- S_d: Discounted selling price of imperfect quality products ($)
- T: Inventory cycle length (unit time) = Q/D
- T_p: Perfect items inventory cycle (unit time)
- T_i: Imperfect items inventory cycle (unit time)
- X: Screening rate (unit/unit time)
- P_p: Production rate of perfect products (units/ unit time)
- P_i: Production rate of imperfect products (units/ unit time)
- P: Production rate (units/ unit time) = P_p+P_i
- T_s: Screening time (unit time) = Q/X
- T_{pr}: Production period (unit time) = Q/P

The decision variable is the order quantity Q and the aim is to determine the optimal order quantity Q* that maximizes the total profit per unit time function.

2.2 The Case Tp ≤ Ti

The objective of this model is to find the optimal number of units per order Q* that maximizes the expected value of the total profit per unit time function.

Figures 1a and 1b depict the inventory levels of raw material and the finished goods. The raw material are screened for imperfect quality items. The percentage q of perfect quality raw material is a random variable having a known probability distribution with an expected value of E[q]. After screening, the perfect quality items of raw material are used to produce qQ perfect quality finished items. The remaining imperfect quality items of raw material are also used in the production process resulting in (1-q)Q imperfect quality finished items. A linear relationship in the production of the two types of items is assumed.

The combined production period as well as the inventory cycle for the perfect and imperfect finished items are:

$$T_{pr} = \frac{Q}{P} \tag{1}$$

$$T_p = \frac{qQ}{D_p} \tag{2}$$

$$T_i = \frac{(1-q)Q}{D_i} \tag{3}$$

Let $T_1 = \min\{T_p, T_i\}$ and let $T_2 = \max\{T_p, T_i\}$. Assuming that $T_p > T_i$, we have $T_1 = T_i$ and $T_2 = T_p$.

The total cost per cycle function, TC(Q), comprises the following costs: Ordering cost, Set-up cost, Purchasing cost, Production cost, Cost of holding raw material, Cost of holding finished products, and Screening cost. Hence,

$TC(Q) = K_o + K_s + CQ + C_pQ + C_sQ + C_{hr} \times$ (Area under curve of figure 1a) $+ C_{hf} \times$ (Area under curve of figure 1b).

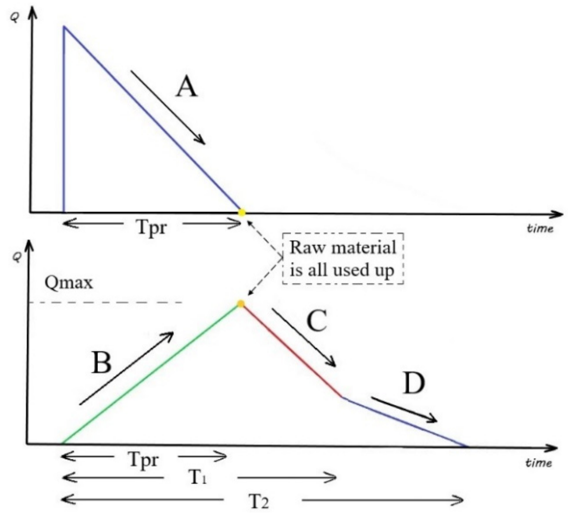

Figure 1a: Raw material inventory (top) Figure 2b: Finished product inventory (bottom) Tp ≥ Ti.

First, we determine the area under curve of figure 1a. It is worth noting that the inventory level of the raw material is used at the production rate P. Hence, the slope A = -P.

The area under curve of figure 1a is given by:

$$A_{triangle} = \frac{1}{2} T_{pr} Q$$

To find the area under curve of figure 1b, we note that during the production period, finished items are produced at a rate of P and consumed at a rate of $D = D_p + D_i$. To avoid shortages, it is assumed that $P > D$.

Thus, finished product inventory is accumulated at a rate of $P - (D_p+D_i)$ until a maximum inventory level Q_{max} is reached the end of the production period. Hence, the slope $B = P - (D_p+D_i)$ and

$$Qmax = T_{pr}[P - (D_p+D_i)].$$

First the arear under the green curve is determined as follows:

$$A_{triangle} = \frac{b \times h}{2}$$

where $b = T_{pr}$, $h = Qmax$, and the slope of the hypotenuse is $P - (D_p+D_i)$. Hence,

$$A_{triangle} = \frac{1}{2} Tpr^2 [P - (D_p+D_i)]$$

Next, the area under the red curve is determined as follows. Note that, at the end of the production period and until time T_1, the finished items inventory is depleted at a rate of $-D = -(D_p+D_i)$. Hence,

$$A_{trapezoid} = \frac{(b_1+b_2) \times h}{2}$$

$$b_1 = T_{pr}[P - (D_p+D_i)]$$

$$h = T_1 - T_{pr}$$

But the slope of the hypotenuse is $C = -(D_p+D_i)$ so that the equation of that line would be:

$$Q = -(D_p+D_i)t + Q_o.$$

At $t = T_{pr}$, $Q = Qmax = T_{pr}[P - (D_p+D_i)]$. Hence,

$$T_{pr}[P - (D_p+D_i)] = -(D_p+D_i)\, T_{pr} + Q_o$$

Thus $Q_o = T_{pr}P$, and hence $Q = -(D_p+D_i)\, t + T_{pr}P$. When $t = T_1$, $Q = -(D_p+D_i)\, T_1 + T_{pr}P$. Thus,

$$A_{trapezoid} = \frac{1}{2} \left(T_{pr}(P-(D_p+D_i)) + T_{pr}P - (D_p+D_i)T_1\right)(T_1 - T_{pr})$$

Finally, the area under the blue curve is calculated. Note that from time T_1 until the end of the inventory period, at time T_2, only perfect quality finished items are left in inventory. These items are depleted at a rate $-D_p$ so that the slope is $-D_p$. Thus,

$$A_{triangle} = \frac{1}{2} \left(T_{pr}P - (D_p+D_i)\, T_1\right)(T_2 - T_1).$$

Therefore,

$$TC(Q) = K_o + K_s + CQ + C_pQ + C_sQ + \frac{1}{2} C_{hr} \times (T_{pr}Q) + \frac{1}{2} C_{hf} \times \left[T_{pr}^2 \left(P - (D_p + D_i)\right) + \left(T_{pr}[P - (D_p + D_i)] + T_{pr}P - T_1 (D_p + D_i)\right)(T_1 - T_{pr}) + \left(T_{pr}P - T_1 (D_p + D_i)\right)(T_2 - T_1)\right] \tag{4}$$

Since $T_p > T_i$, $T_1 = \min\{T_p, T_i\} = T_i$ and $T_2 = \max\{T_p, T_i\} = T_p$. Substituting $T_{pr} = \frac{Q}{P}$, $T_1 = T_i = \frac{(1-q)Q}{D_i}$, and $T_2 = T_p = \frac{qQ}{D_p}$, the TC(Q) function becomes:

$$TC(Q) = K_o + K_s + CQ + C_pQ + C_sQ + \frac{1}{2}C_{hr} \times \left(\frac{Q^2}{P}\right) + \frac{1}{2}C_{hf}\times\left[\left(\frac{Q}{P}\right)^2 \left(P-(D_p+D_i)\right) + \left(\left(\frac{Q}{P}\right)[P-(D_p+D_i)] + Q - \left(\frac{(1-q)Q}{D_i}\right)(D_p+D_i)\right)\left(\frac{(1-q)Q}{D_i} - \frac{Q}{P}\right) + \left(Q - \left(\frac{(1-q)Q}{D_i}\right)(D_p+D_i)\right)\left(\frac{qQ}{D_p} - \frac{(1-q)Q}{D_i}\right)\right] \quad (5)$$

After simplification, we have:

$$TC(Q) = K_o + K_s + CQ + C_pQ + C_sQ + \frac{1}{2}C_{hr} \times \left(\frac{Q^2}{P}\right) + \frac{1}{2}C_{hf} \times \left(\frac{Q^2}{D_i} - \frac{qQ^2}{D_i} - \frac{Q^2}{P} + \frac{qQ^2}{D_p} - \frac{(D_p+D_i)qQ^2}{D_pD_i} + \frac{(D_p+D_i)q^2Q^2}{D_pD_i}\right) \quad (6)$$

Next, the total revenue and total profit for this model are:

$$TR(Q) = S_pqQ + S_i(1-q)Q$$

$$TP(Q) = S_pqQ + S_i(1-q)Q - \left[K_o + K_s + CQ + C_pQ + C_sQ + \frac{1}{2}C_{hr} \times \left(\frac{Q^2}{P}\right) + \frac{1}{2}C_{hf} \times \left(\frac{Q^2}{D_i} - \frac{qQ^2}{D_p} - \frac{Q^2}{P} + \frac{qQ^2}{D_p} - \frac{(D_p+D_i)qQ^2}{D_pD_i} + \frac{(D_p+D_i)q^2Q^2}{D_pD_i}\right)\right] \quad (7)$$

The expected value of TP(Q) is

$$E[TP(Q)] = S_pQ\cdot E[q] + S_iQ\cdot E[1-q] - K_o - K_s - CQ - C_pQ - C_sQ - \frac{1}{2}C_{hr} \times \left(\frac{Q^2}{P}\right) - \frac{1}{2}C_{hf} \times \left(\frac{Q^2}{D_i} - \frac{Q^2}{P}\right) - \frac{1}{2}C_{hf} \times \left(\frac{Q^2}{D_p} - \frac{Q^2}{D_i} - \frac{(D_p+D_i)Q^2}{D_pD_i}\right)\cdot E[q] - \frac{1}{2}C_{hf} \times \left(\frac{(D_p+D_i)Q^2}{D_pD_i}\right)\cdot E[q^2]$$

Applying the renewal reward theorem, $E[TPU(Q)] = \frac{E[TP(Q)]}{E[T]}$, where $T = T_p = \frac{qQ}{D_p}$. Hence,

$$E[TPU(Q)] = \frac{S_pQ\cdot E[q] + S_iQ\cdot E[1-q] - K_o - K_s - CQ - C_pQ - C_sQ - \frac{1}{2}C_{hr}\times\left(\frac{Q^2}{P}\right) - \frac{1}{2}C_{hf}\times\left(\frac{Q^2}{D_i} - \frac{Q^2}{P}\right) - \frac{1}{2}C_{hf}\times\left(\frac{Q^2}{D_p} - \frac{Q^2}{D_i} - \frac{(D_p+D_i)Q^2}{D_pD_i}\right)\times E[q] - \frac{1}{2}C_{hf}\times\left(\frac{(D_p+D_i)Q^2}{D_pD_i}\right)\cdot E[q^2]}{\frac{Q}{D_p}\cdot E[q]}$$

Setting the derivative of the above expression equal to zero and solving for Q, we get:

$$Q^* = \sqrt{\frac{K_o+K_s}{\frac{1}{2}C_{hr}\times\left(\frac{1}{P}\right) + \frac{1}{2}C_{hf}\times\left(\frac{1}{D_i} - \frac{1}{P}\right) + \frac{1}{2}C_{hf}\times\left(\frac{1}{D_p} - \frac{1}{D_i} - \frac{(D_p+D_i)}{D_pD_i}\right)\cdot E[q] + \frac{1}{2}C_{hf}\times\left(\frac{(D_p+D_i)}{D_pD_i}\right)\cdot E[q^2]}} \quad (8)$$

2.3 The Case Tp < Ti

In this case, we assume that the inventory cycle for imperfect quality items is longer than that of perfect quality items. Thus, the inventory cycle terminates when the perfect quality items are depleted. We assume that the remaining imperfect quality items are sold in one batch at a lower price of S_d, where $S_p > S_i > S_d$. Figures 2a and 2b depict the inventory levels of the raw material and the finished product.

In this case, $T_1 = \min\{T_p, T_i\} = T_p = \frac{(1-q)Q}{D_i}$. Similar steps followed in the previous case result in:

$$TC(Q) = K_o + K_s + CQ + C_pQ + C_sQ + \frac{1}{2}C_{hr} \times \left(\frac{Q^2}{P}\right) + \frac{1}{2}C_{hf}\times\left[\left(\frac{Q}{P}\right)^2 \left(P-(D_p+D_i)\right) + \left(\left(\frac{Q}{P}\right)[P-(D_p+D_i)] + Q - \left(\frac{(1-q)Q}{D_i}\right)(D_p+D_i)\right)\left(\frac{(1-q)Q}{D_i} - \frac{Q}{P}\right)\right] \quad (9)$$

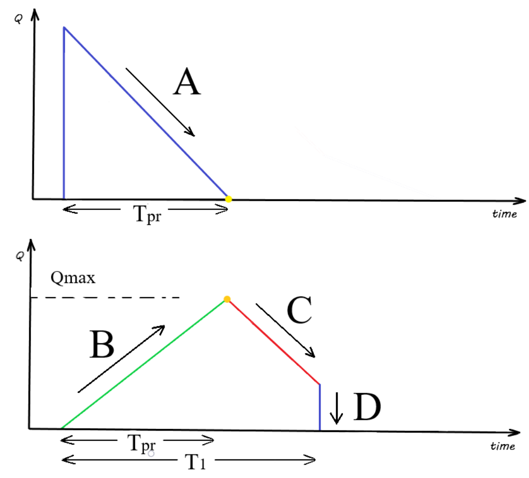

Figure 2a: Raw material inventory (top) Figure 2b: Finished product inventory (bottom) Tp < Ti.

After T_1, the remaining products are of imperfect quality and are to be sold in a single batch at an even lower selling price S_d. Thus, the amount of remaining finished products is $T_{pr}P - (D_p+D_i)\,T_1$. Hence

$$\mathrm{TR(Q)} = \mathrm{S_p qQ} + \mathrm{S_i(qQ)}\left(\frac{\mathrm{D_p}}{\mathrm{D_i}}\right) + \mathrm{S_d}\left[\mathrm{Q} - \left(\frac{\mathrm{qQ}}{\mathrm{D_p}}\right)(\mathrm{D_p}+\mathrm{D_i})\right] \tag{10}$$

The total profit is simply TP(Q) = TR(Q) − TC(Q)

Hence TP(Q) is:

$$\mathrm{TP(Q)}=\mathrm{S_p qQ}+\mathrm{S_i(qQ)}\left(\frac{\mathrm{D_p}}{\mathrm{D_i}}\right) + \mathrm{S_d}\left[\mathrm{Q} - \left(\frac{\mathrm{qQ}}{\mathrm{D_p}}\right)(\mathrm{D_p}+\mathrm{D_i})\right] - \mathrm{K_o} - \mathrm{K_s} - \mathrm{CQ} - \mathrm{C_pQ} - \mathrm{C_sQ} - \frac{1}{2}\mathrm{C_{hr}} \times \left(\frac{\mathrm{Q^2}}{\mathrm{P}}\right) - \frac{1}{2}\mathrm{C_{hf}} \times \left[\left(\frac{\mathrm{Q}}{\mathrm{P}}\right)^2 (\mathrm{P} - (\mathrm{D_p}+\mathrm{D_i})) + \left(\left(\frac{\mathrm{Q}}{\mathrm{P}}\right)[\mathrm{P} - (\mathrm{D_p}+\mathrm{D_i})] + \mathrm{Q} - \left(\frac{\mathrm{(1\text{-}q)Q}}{\mathrm{D_i}}\right)(\mathrm{D_p}+\mathrm{D_i})\right)\left(\frac{\mathrm{(1\text{-}q)Q}}{\mathrm{D_i}} - \frac{\mathrm{Q}}{\mathrm{P}}\right)\right] \tag{11}$$

The expected total profit is

$$\mathrm{E[TP(Q)]} = \mathrm{QE(q)}+\mathrm{S_i}\left(\frac{\mathrm{QD_p}}{\mathrm{D_i}}\right)\mathrm{E(q)} + \mathrm{S_dQ} - \mathrm{Sd}\left(\frac{\mathrm{Q}}{\mathrm{D_p}}\right)(\mathrm{D_p}+\mathrm{D_i})\,\mathrm{E(q)} - \mathrm{K_o} - \mathrm{K_s} - \mathrm{CQ} - \mathrm{C_pQ} - \mathrm{C_sQ} - \frac{1}{2}\mathrm{C_{hr}} \times \left(\frac{\mathrm{Q^2}}{\mathrm{P}}\right) - \frac{1}{2}\mathrm{C_{hf}} \times \left\{\left[\frac{2Q^2}{D_i} - \frac{Q^2 D}{D_i^2} - \frac{Q^2}{P}\right] + \left[\frac{2Q^2 D}{D_i^2} - \frac{2Q^2}{D_i}\right]\mathrm{E(q)} - \left[\frac{Q^2 D}{D_i^2}\right]\mathrm{E(q^2)}\right\} \tag{12}$$

Using $\mathrm{E[TPU(Q)]} = \frac{\mathrm{E[TP(Q)]}}{\mathrm{E[T]}}$ with $\mathrm{T} = \mathrm{T_p} = \frac{\mathrm{qQ}}{D_p}$, we have

$$\mathrm{E[TPU(Q)]} = \frac{\mathrm{S}_p\mathrm{QE(q)} + \mathrm{S}_i\left(\frac{\mathrm{QD_p}}{\mathrm{D_i}}\right)\mathrm{E(q)} + \mathrm{S}_d\mathrm{Q} - \mathrm{S}_d\left(\frac{\mathrm{Q}}{\mathrm{D_p}}\right)(\mathrm{D_p}+\mathrm{D_i})\mathrm{E(q)} - \mathrm{Ko} - \mathrm{Ks} - \mathrm{CQ} - \mathrm{CpQ} - \mathrm{CsQ} - \frac{1}{2}\mathrm{C}_{hr} \times \left(\frac{\mathrm{Q^2}}{\mathrm{P}}\right) - \frac{1}{2}\mathrm{C}_{hf}\,\mathrm{Q^2} \times \left\{\left[\frac{2}{D_i} - \frac{D}{D_i^2} - \frac{1}{P}\right] + \left[\frac{2D}{D_i^2} - \frac{2}{D_i}\right]\mathrm{E(q)} - \left[\frac{D}{D_i^2}\right]\mathrm{E(q^2)}\right\}}{\frac{\mathrm{Q}}{D_p}\cdot\mathrm{E[q]}}$$

The optimal order quantity is

$$\mathrm{Q^*} = \sqrt{\frac{\mathrm{K_o}+\mathrm{K_s}}{\frac{1}{2}\mathrm{C_{hr}}\times\left(\frac{1}{\mathrm{P}}\right) + \frac{1}{2}\mathrm{C_{hf}} \times \left\{\left(\frac{2}{\mathrm{D_i}} - \frac{(\mathrm{D_i}+\mathrm{D_p})}{\mathrm{D_i^2}} - \frac{1}{\mathrm{P}}\right) + \left(\frac{2(\mathrm{D_i}+\mathrm{D_p})}{\mathrm{D_i^2}} - \frac{2}{\mathrm{D_i}}\right)\cdot\mathrm{E[q]} - \left(\frac{(\mathrm{D_i}+\mathrm{D_p})}{\mathrm{D_i^2}}\right)\cdot\mathrm{E[q^2]}\right\}}} \tag{13}$$

3 NUMERICAL EXAMPLES

To illustrate the two models developed in section 2, consider the situation where a closet manufacturer sells wooden tables to retailers. The manufacturer orders wooden boards and assembles them as single door closets. The percentage of perfect quality boards ranges between [70% → 90%]. However, after screening, which is 0.03$ per item, it turns out that some boards are 180cm and others are 160cm of height. The 180cm boards are considered of perfect quality and the 160cm boards are the imperfect quality. The manufacturer can produce 300 perfect quality closets per day, and 100 imperfect quality closets per day, with a production cost of $10 per product. The purchasing cost is 4$ per wooden board, and the ordering cost is 1,000$, and the setup cost for production is $250. The retailers demand 100 units of 180cm closets per day, and 50 units of 160cm closets per day. Each 180cm closet costs the retailer 450$ and 300$ for the 160cm closets. The holding cost of raw material is 0.01$ per unit per day and for the finished products is 0.02$ per unit per day. Find the optimal number of wooden boards per order Q*, then find the total profit per cycle. $\mathrm{E[q]} = \mu = \frac{0.7+0.9}{2} = 0.8$ and $\mathrm{E[q^2]} = \mathrm{Var(q)} + (\mathrm{E[q]})^2 = 0.00367+0.64 = 0.64367$. Hence, $\mathrm{Q^*} = 4{,}541.6 \approx 4{,}542$ units and the total profit per unit time is $\mathrm{TPU(Q^*)} = \frac{\mathrm{TP(Q^*)}}{\mathrm{T}} = \frac{\mathrm{TP(Q^*)}}{\frac{\mathrm{qQ}}{\mathrm{D_p}}} = 53{,}013.2\$$.

Now suppose that the percentage of perfect quality boards ranges between [60% → 80%]. The manufacturer can produce 200 perfect quality tables per day, and 150 imperfect quality tables per day, with a production cost of $5 per product. The purchasing cost is 1$ per wooden board, and the ordering cost is 400$, and the setup cost for production is $100. The retailers demand 100 units of 3cm thick tables per day, and 50 units of 2cm tables per day. Each 3cm thick table costs the retailer 30$ and 20$ for the 2cm thick tables. The holding cost of raw material is 0.01$ per unit per day and for the finished products is 0.015$ per unit per day. It turned out that the 3cm thick tables are sold out before the 2cm thick tables. Assume that the manufacturer sells all the remaining 2cm thick tables at once at a discounted price of 15$. Then Q* = 3,504.7 ≈ 3,505 units.

4 CONCLUSION

A mathematical function was developed to introduce an EPQ model that uses both the perfect and imperfect quality items of raw material in the production of the finished product. The production process results in two types of finished products, perfect and imperfect. A continuous demand is assumed for both perfect and imperfect quality finished items. The mathematical model was formulated for two cases that depend on the length of the inventory cycles for the perfect and imperfect quality finished items.

The variability of holding cost is a crucial requirement, reflecting a realistic assumption in real-life situations. In fact, in many situations, the holding cost increases with longer storage periods, as the extended storage may require more sophisticated, and thus expensive, storage equipment and conditions. A case in point could be the storage of food, pharmaceutical products, and hazardous material that require certain storage and quality dimensions to avoid spoilage and/or risk.

The model was validated using a numerical example. A practical case study will better demonstrate real-life applications. The model should also take into consideration the stock type, holding time, holding cost, demand for perfect quality items, and demand for imperfect quality items. Moreover, the models should incorporate the pricing decisions for both types, as well as the factors influencing the demand for reach of the types.

Finally, future models should deal with a coordinated supply chain, consider time value of money, and incorporate the effect of emission tax.

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