

# Light Field Scattering in Participating Media

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**Abstract:** In this paper, we propose a representation of the light scattering in participating media, which can represent all order light scattering simply. To achieve the model, we focus on the light field in the participating media, and it is shown that the convolution of the light field can describe the attenuation of the light rays and scattering of them. By analyzing the convolution kernel, we derive a simple kernel that represents all order light scattering. Also, we introduce the estimation method of the characteristics of the participating media based on our proposed model. Several experimental results show that our proposed model can describe light scattering more appropriately than existing models.

## 1 INTRODUCTION

In recent years, image sensors such as cameras are one of the most important devices to obtain scene information, and they ordinary utilized for various applications, e.g., 3D measurement, object recognition, and so on. In ordinary cases, we assume that the camera obtains ‘clear’ information from the scene. However, images taken in the outdoor scenes are often disturbed by participating media such as fogs, smokes, and so on. Figure 1 shows example images taken in the participating media. The figure shows that the fogs and smokes disturb appropriate imaging in participating media. Furthermore, water also disturbs imaging if we want to take underwater images. The effect of the participating media disturbs various applications based on the camera images such as autonomous driving and driving assistant systems. In addition, 3D sensors, e.g., LiDAR, also cannot obtain an accurate distance in participating media because several sensors in the system also cannot get the appropriate data as same as the cameras.

In order to solve the problem, various kinds of methods are studied(Narasimhan et al., 2006; He et al., 2011; Kitano et al., 2017; Naik et al., 2015;

Satat et al., 2018). To solve the problem essentially, we need to use the accurate light scattering model to describe the phenomena in the participating media. In the field of computer graphics, several models are used to describe the light behavior accurately(Pharr et al., 2016). In tradition, light behavior is classified into single scattering and multiple scattering, and several models are proposed for each scattering. Although the models can describe the light scattering in the participating media in limited case, there are lots of situations which cannot be explained by these models. In order to describe the light scattering appropriately, light ray tracing techniques and the photon mapping techniques are utilized. Although they provide more realistic results, they are not suitable for analyzing images because they require a high computational cost. In this research, we propose a light scattering description model that can explain the single scattering as well as multiple scattering in low computational cost. In addition, we also show the participating media analysis method based on the proposed model. In this light ray explanation, we focus on the 5D light field in the scattering media, and we show that light scattering can be described efficiently and effectively using the light field.



Figure 1: Example images taken in participating media, fogs, smokes, and water.

## 2 LIGHT SCATTERING IN PARTICIPATING MEDIA

### 2.1 Participating Media and Light Transport Equation

We first explain the characteristics of participating media (Mukaigawa et al., 2010). The participating media includes lots of micro-particles, and the particles affect input light rays. For example, fog and smoke are representatives of the participating media, and they consist of particles such as water and dust. The particles scatter light rays, and thus, observed images in the media become unclear, as shown in Fig.1.

The effect of the media on the light rays are classified into light attenuation, in-scattering, and out-scattering. These effects are described by light transport equations (LTE). Let us consider the case when a light ray  $L(x, \omega)$  passes through the media at a point  $x$  directed to  $\omega$ . Under the assumption that  $dL(x, \omega)$  denote the change of the light ray, the effect of the participating media is described as follows:

$$L'(x, \omega) = L(x, \omega) + dL(x, \omega) \quad (1)$$

where  $L'$  is an affected light ray by the media.

The  $dL(x, \omega)$  consists in attenuation term, in-scattering term and out-scattering term. First, the attenuation describes light absorbing by the media, and it is described as follows:

$$dL(x, \omega) = -\sigma_a(x)L(x, \omega)ds \quad (2)$$

where  $\sigma_a$  is absorption coefficient and  $ds$  shows thickness of the media. Second, several light rays contact the particles and reflected in directions other than  $\omega$ . Therefore, light rays directed to  $\omega$  is attenuated, and it is described as follows:

$$dL(x, \omega) = -\sigma_s(x)L(x, \omega)ds \quad (3)$$

where  $\sigma_s$  denotes a scattering coefficient at  $x$ . Finally, scattered light rays from any directions other than  $\omega$  are added to light rays as out-scattering. The out-scattering is described as follows:

$$dL(x, \omega) = \sigma_s(x) \int_{\Omega} Lp(x, \omega', \omega)L(x, \omega')d\omega' ds \quad (4)$$

where  $p$  denote phase function, and it describes the probability that the light ray from  $\omega'$  is reflected direction  $\omega$ .

Several functions are proposed as phase function  $p$ . In this paper, we utilize the Henyei-Greenstein phase function defined as follows:

$$p(\theta) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g \cos \theta)^{2/3}} \quad (5)$$

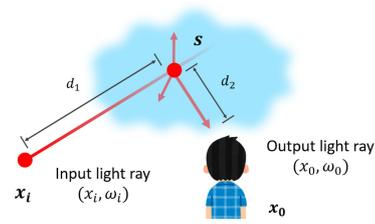


Figure 2: Single scattering in the participating media.

where  $\theta = \arccos(\omega, \omega')$ , and  $g$  ( $-1 \leq g \leq 1$ ) determine directional characteristics of the scattering.

By these three equations, Eq.(1) is rewritten as follows:

$$L'(x, \omega) = L(x, \omega) - \sigma_a(x)L(x, \omega)ds - \sigma_s(x)L(x, \omega)ds + \sigma_s \int_{\Omega} p(x, \omega', \omega)L(x, \omega')d\omega' ds \quad (6)$$

By solving the LTE, the status of the light ray in scattering media can be described. However, the derivation of an analytical solution to the equation is difficult because of the out-scattering term. Therefore, several algorithms based on numerical analysis are utilized in the field of computer graphics (Pharr et al., 2016). However, they require lots of computational costs, and it is not easy to utilize them for analysis of the media.

### 2.2 Single Scattering Model

In order to relax the computational complexity, the approximated model is utilized generally. Under the assumption that the density of the micro-particles is low, a single scattering model is used for describing the light scattering. In this case, we assume that the light rays are scattered (reflected) just one time by micro-particles in the media. Under this assumption, routes of the light rays are constrained completely by an input light ray  $L(x_i, \omega_i)$  and an output light ray  $L(x_o, \omega_o)$  as shown in Fig.2.

In this case,  $L'(x_o, \omega_o)$  is described as follows:

$$L'(x_o, \omega_o) = \sigma_s p(\theta) e^{-(\sigma_a + \sigma_s)(d_1 + d_2)} L(x_i, \omega_i) \quad (7)$$

where  $d_1$  and  $d_2$  denote distance from  $x_i$  to  $s$  and  $s$  to  $x_o$ , respectively. The  $\theta$  denotes an angle between  $\omega_i$  and  $\omega_o$ .

The computation of the single scattering model is straightforward. In addition, this model is approximately valid for shallow participating media. Therefore, the model is often utilized for analysis of the scattering media.

### 2.3 Multiple Scattering Model

We next consider the case when the density of the media is very high. In this case, multiple scatter-

ing model is utilized for describing the light scattering. In this model, it is assumed that the light rays are scattered repeatedly, and then, the directionality of the light ray is lost. Therefore, light rays into the dense participating media are distributed in any direction evenly. By using this model, light scattering by dense media such as milk is appropriately described.

## 2.4 Low-order Scattering

By using the two models, light scattering in the participating media is described. However, general scenes often include more complicated scattering. Mukaigawa et al.(Mukaigawa et al., 2010) indicate light rays input to the participating media scattered a few times, and then most scenes include not single scattering but low-order scattering. Although the method which separates each order scattering is proposed by them, the method requires several numbers of images because the method uses high-frequent pattern projection(Nayar et al., 2006) to separate the light rays. Therefore, it is difficult to apply the method for ordinary image analysis.

## 3 LIGHT FIELD SCATTERING MODEL

### 3.1 Light Field Scattering

In order to describe the light scattering not only single and multiple scattering but also any order scattering in the participating media, we focus on the light field in the media. As described in Eq.(1), LTE describes the relationship between input light rays and output rays. Therefore, we describe the light scattering by light ray transition in the light field. By using this proposed description, light scattering can be explained efficiently. In addition, this description achieves describing single scattering, low-order scattering, and multiple scattering in the same manner.

In this manner, we separately consider input light rays to a point and output light rays from the point. By combining the input light rays and output light rays, we achieve a description of the light scattering efficiently.

### 3.2 Input Light Rays

We first consider input light rays to a point in the scene. Let  $L(\mathbf{x}, \omega)$  denote a light ray at a point  $\mathbf{x}$  directed to  $\omega$ . Under the assumption that light rays go straight in scattering media other than reflection

by particles, integrated input light ray  $L'(\mathbf{x}, \omega)$  to the point  $\mathbf{x}$  directed to  $\omega$  are computed as follows:

$$L'(\mathbf{x}, \omega) = \int_d e^{\sigma_t d} L(\mathbf{x} + d\omega, \omega) dd \quad (8)$$

Let  $g(d)$  denote a function which describe light attenuation and it is defined as follows:

$$g(d) = \begin{cases} e^{-\sigma_t d} & \text{if } d \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

By using the  $g$ , Eq.(8) is rewritten as follows:

$$L'(\mathbf{x}, \omega) = \int_d g(d) L(\mathbf{x} + d\omega, \omega) dd \quad (10)$$

This is convolution of  $L(\mathbf{x}, \omega)$  by a convolution kernel  $g$ . Therefore, integration of the input light ray can be computed by just convolution of the light field.

### 3.3 Output Light Rays

We next consider output light rays scattered by particles. The integrated input light rays  $L'$  are scattered by the particles based on a phase function  $p$ . In this scattering, we focus on an output light ray  $L''(\mathbf{x}, \omega')$  directed to  $\omega'$ . Under the assumption that the phase function is isotropic, the scattering is computed as follows:

$$L''(\mathbf{x}, \omega') = \sigma_s \int_{\omega} p(\omega, \omega') L'(\mathbf{x}, \omega + \omega') d\omega \quad (11)$$

In this case, the phase function  $p$  can be regarded as a convolution kernel. Therefore, the light scattering by the particles is also described by just convolution.

Note that  $\omega + \omega'$  in Eq.(11) is not correct since this convolution is done on a sphere. The effective convolution and fourier transformation on the sphere can be done by using spherical harmonics(Su and Grauman, 2017). Therefore, we simply describe this convolution by just  $\omega + \omega'$  for convenience.

### 3.4 N-th Order Scattering Description

As described in the previous sections, integration of input light rays and scattering of the input light rays are described by just convolution. By integrating both of convolutions, light scattering in participating media is described as follows:

$$L'(\mathbf{x}, \omega) = \sigma_s g * s * L(\mathbf{x}, \omega) = \sigma_s k * L(\mathbf{x}, \omega) \quad (12)$$

where  $'*$ ' denotes convolution and  $k = g * s$  is the integrated kernel.

By convolution  $\sigma_s k$ , the input light field  $L$  is updated to the first-order scattering light  $L'$ . Since the scattered light is also scattered by media, again and

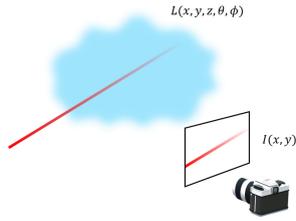


Figure 3: 2D image as a slice of the light field.

again,  $N$ -th order scattering light  $L^N$  is described as follows:

$$L^N(\mathbf{x}, \omega) = \sigma_s k * L^{N-1}(\mathbf{x}, \omega) \quad (13)$$

where  $L^0 = L$  and  $L' = L^1$ . This equation indicates that  $\sigma_s L^N$  is scattered by the media, and the rest  $(1 - \sigma_s)L^N$  is observed directly. Therefore, the observed light field  $L_a$ , including all scattering component, is described as follows:

$$L_a = (1 - \sigma_s) \sum_{j=0}^{\infty} L^j \quad (14)$$

In this equation, we derive  $N$ -th order light scattering by image convolution.

Note that the image convolution can be described by just element multiplication in the frequency domain. In addition, the techniques of the fast Fourier transformation require low computational cost for image convolution. Therefore, the light scattering representation based on image convolution can be achieved by just low computational cost.

## 4 PARTICIPATING MEDIA CHARACTERISTICS ESTIMATION

In this section, we describe an estimation method for the participating media characteristics based on a light scattering model described in the previous section. Under the assumption that the participating media is uniform, an attenuation coefficient  $\sigma_a$ , symmetry factor  $g$  in a phase function, and a scattering coefficient  $\sigma_s$  determine the characteristics in this model. Therefore, we need to estimate the three parameters.

In ordinary cases, we cannot obtain the status of the light field directly because conventional sensors cannot observe a 5D light field. Therefore, we need to estimate the parameters from a 2D image, which is a slice of the 5D light field, as shown in Fig.3.

Let an image  $I(x, y)$  denote a slice of the light field  $L$  by a direction  $\omega_o$  to a camera and a plane in the scene. Under the assumption that input light ray  $L_i$  is known, we define an evaluation function  $E$  as follows:

$$E = \|I(x, y) - \mathcal{S}(k(\sigma_t, \sigma_s, g)L_i)\|^2 \quad (15)$$

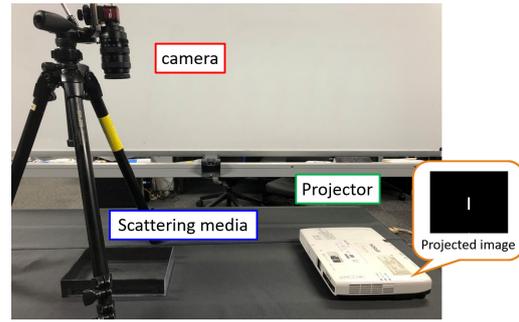


Figure 4: Experimental environment.

where  $k(\sigma_t, \sigma_s, g)$  is a convolution kernel determined by the parameters, and  $\mathcal{S}$  denotes slicing of the light field correspond to the input image. Values that minimize the  $E$  are suitable parameters to describe the characteristics of the participating media.

## 5 EXPERIMENTAL RESULTS

### 5.1 Environment

In this section, we show several experimental results by our proposed method. In this experiment, we combined a few amounts of white ink and water. The combined water was used as the participating media. The white water filled a water tank, and the tank was observed from the upper side, as shown in Fig.4. In this environment, light rays were input to the tank by a projector. The relative position between a camera and a projector was calibrated beforehand, and then the input light ray is controlled and known. An Example of the input image is shown in Fig.5. In order to help the understanding, the grayscale images are colored, as shown in the right image. In these images, densities of the white ink were changed to survey our proposed model can describe the different density participating media. Not only our proposed model but also a single scattering model was used for the estimation of the parameters for comparison.

### 5.2 Results

Figure 6 shows input images and estimated results. In this results, result images were synthesized from input light rays and estimated parameters by our proposed method. For comparison, images including 1st order scattering (single scattering) and until several order scatterings are shown, respectively. RMSE between an input image and a reconstructed image is shown below of each image.

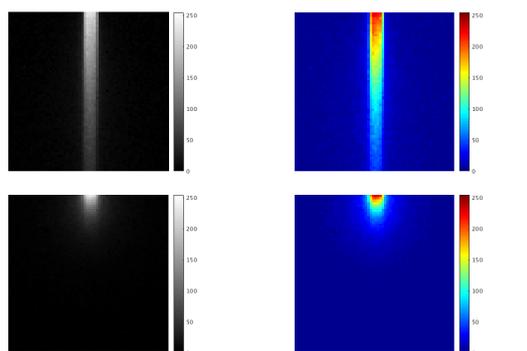


Figure 5: Examples of observed images. The left images show the original grayscale images, and the right image shows the colored images. In the upper image, the density of the participating media is high, and it is low in the lower images.

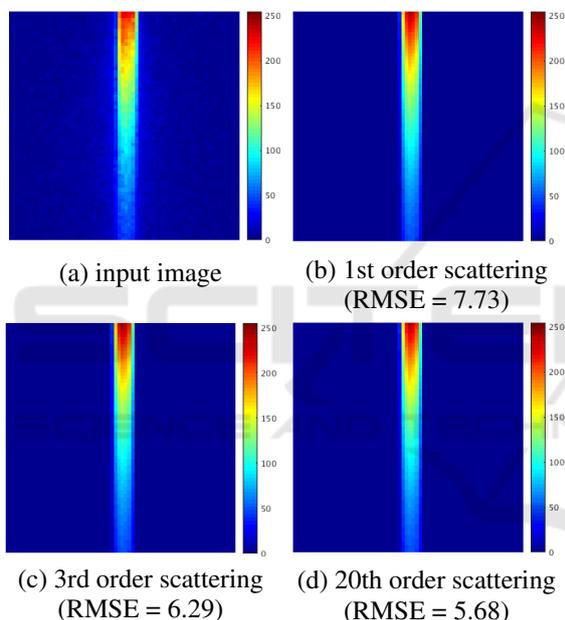


Figure 6: Reconstructed images and input images when the density of the participating media is low. Figure(a) shows an input image and (b)~ (d) show images including until 1st order, 3rd order and 20th order scatterings.

In this result, RMSE between the input image and the reconstructed image becomes smaller according to scattering order increasing. The result indicates that the input image includes not only lower light scattering but also higher-order scattering even if the density of the media is not so high. Our proposed method can describe the various order scatterings, and thus RMSE becomes lower when the scattering order becomes higher.

Figure 7 shows estimated results of the light scattering in denser participating media. In these results, RMSE becomes lower according to the scattering order as sim-

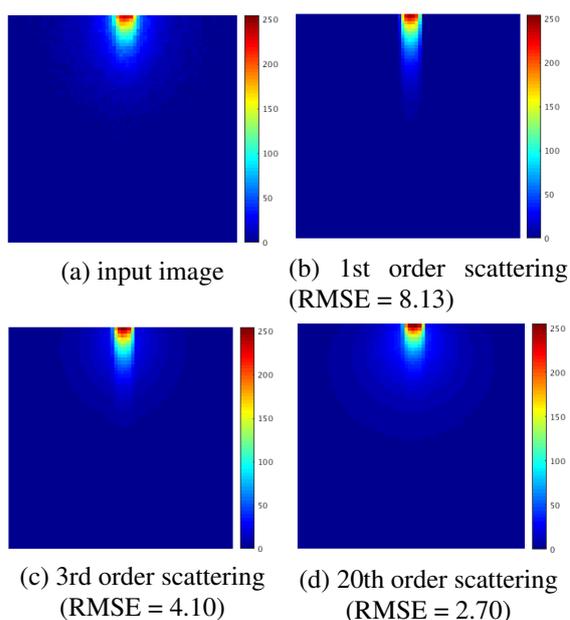


Figure 7: Reconstructed images and input images when the density of the participating media is high. Figure(a) shows an input image and (b)~ (d) show images including until 1st order, 3rd order and 20th order scatterings.

ilar to the previous results. The results indicate that our proposed method can describe not only shallow participating media, but also the dense media by using the same manner.

## 6 CONCLUSION

In this paper, we propose a light scattering model that can describe not only low-order scattering but also high-order scattering in participating media. In this model, we describe light attenuation and light scattering by the convolution of the light field. By using the kernel, light scattering in the participating media can be described accurately and efficiently. Based on the convolution description, we introduce the estimation method of characteristics of participating media. Several experimental results show that the proposed method can describe the light scattering in the participating media. Besides, our proposed method estimate the characteristics of the participating media. We will consider a more stable estimation method and also consider the case when the participating media is not uniform in future work.

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