Classification of Visual Strategies in Physics Vector Field Problem-solving

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Abstract: In this study, we taught 20 physics students two different visual strategies to graphically interpret the physical meaning of vector field divergence. Using eye-tracking technology, we recorded students' eye-movement behavior of both strategies when they were engaged in graphical vector field representations. From the eye-tracking data we extracted the number of fixations and saccadic direction and proposed a linear SVM model to classify strategies of problem-solving in the vector field domain. The results show different gaze patterns for the two strategies, and the influence of vector flow orientation on gaze-patterns. A high accuracy of $81.2\%(0.11\%)$ has been achieved by testing the algorithm using cross-validation, i.e. that the algorithm is able to predict the strategy the student applies to judge the divergence of a vector field. The results provide guiding tools for learning-effective instruction design and teachers gain benefit from monitoring the students' non-verbal level of performance and fluency using each strategy. Apart from that, students would receive the objective feedback on their progress of learning.

1 INTRODUCTION

Learning of fundamental science such as physics is crucial to develop the students' skills which is considered as an important gateway to their future employment and life opportunities. Education aims to provide an effective system to enhance the quality of learning. Such system needs to investigate how the students explore, perceive, process, and interpret a different kind of information. In the physics education, the visual system plays the most important role to explore and capture the information from different sort of representations. Many problems in upper division physics and other scientific disciplines require students to relate abstract concepts to multiple external representations (MERS), including diagrams, equations, graphics or data tables. From an educational perspective, it is well known that multiple representations of abstract concepts have the potential to substantially promote learning (Meltzer, 2005). However, the acquisition of this skill requires instructional support, especially when concepts become more sophisticated. In this context, a great deal of uncertainty rises when the eyes scan the representations for visually informative clues. This fact roots in the individual differences, education background, ethnicity, culture, environment, and many other factors. Moreover, in many physics concepts, the myriad ways of visual strategies are available for interacting with the different types of representations(Mozaffari et al., 2016a; Mozaffari et al., 2016b; Mozaffari et al., 2018a). However, some physics domains have distinct visual strategy rules. One example which has recently been studied consists of the visual interpretation of two-dimensional vector field plots with respect to divergence (Maries and Singh, 2013; Bollen et al., 2016; Ishimaru et al., 2016; Klein et al., 2018; Klein et al., 2019).

When students are instructed with multiple representations, eye tracking offers unique possibilities to track the students' processing of text, equations, diagrams, etc. it gives the opportunity to analyze and computationally model the eye-gaze data in order to objectively evaluate the students visually cognitive performance. This objective evaluation helps to improve the quality of learning in that particular physics domain. The educators could assess the students' non-verbal cognitive performance and on another hand, the students benefit from the feedback on their gaze-driven approach. Despite the vast attention paid

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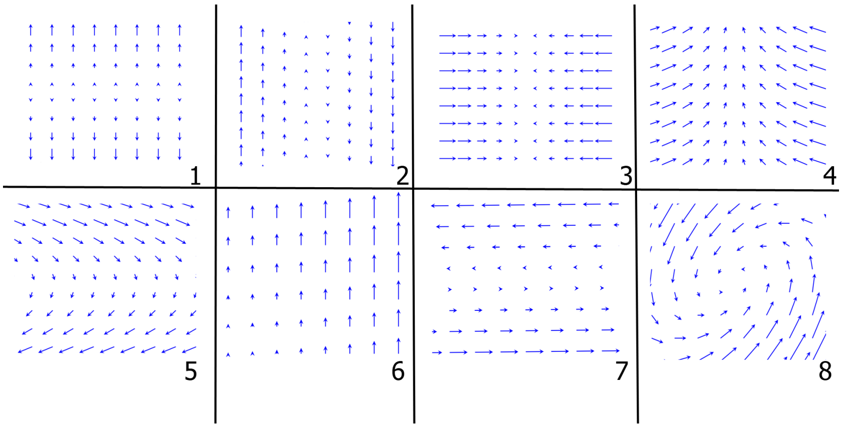

Figure 1: Vector field plot representations used in this study.

eye-tracking research community to the data analysis, fewer studies address the classification of eye-movements patterns in education (Holmqvist et al., 2011). Thus, the modeling of the strategic-based eye movement behavior sounds promising to increase the learning quality in the physics domain. This research investigated student's visual understanding of vector field plots, which are an important tool for learning theoretical physics and which occur in the introductory and upper-division university physics curricula. Prior research has shown that most students and even graduates fail to connect the concept of divergence to graphical vector field representations (Pepper et al., 2012; Bollen et al., 2016). Basically, there are two equivalent but yet different approaches to this problem, requiring different visual strategies; integral and differential approaches. The contribution of this paper is two-fold: Firstly, it explores and investigates the context-related eye-tracking features. Secondly, presents and evaluates a supervised model of the two approaches upon the derived features. Before presenting the methodology and the data collection phase, the brief explanation of eye-tracking basics and the subject content are provided. They are required to understand the underlying methodological approach.

1.1 Eye-tracking

Eye-tracking is a non-intrusive method to obtain information about visual attention and cognitive processing. The most often used eye-tracking features are derived from fixations (relatively long periods, usually lasting between 100 and 600 ms, in which the eye is almost still) and saccades (very fast eye shifts between fixations, lasting less than 100 ms): Fixation duration, number of fixations, and saccade length. According to the theory of long-term working memory, the information-reduction hypothesis, and the holistic model of image perception, these three measures are associated with information processing, selective attention allocation, and visual span, respectively (Gegenfurtner et al., 2011). Even though these measures are fundamental to general eye-tracking methodology, they are not enough to evaluate spatio-temporal gaze patterns. In the Methodology section, the more sophisticated eye-tracking features are introduced.

1.2 Subject Background

In simplified cases that we consider in this study, students must decide whether vector fields have zero or non-zero divergence. The vector fields used in this study are displayed in Fig. 2. This task is challenging even for graduate students (Maries and Singh, 2013) and requires the interpretation of the mathematical definition and its application to the vector field. To judge whether a vector field has zero or non-zero divergence, one of the following two conditions can be used. The differential strategy (DS):

$$\mathrm{div}\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} \tag{1}$$

Or the integral strategy (IS):

$$\mathrm{div}\vec{F} = \lim_{V \to 0} \frac{1}{V} \int_{\partial V} \vec{F} \cdot d\vec{n}. \tag{2}$$

Application of Eq. 1 to a graphical vector field plot means that one must inspect the change of the x-component of the field in the x-direction and change of the y-component in y-direction (see Fig. 2 center). Vividly speaking, we must perform horizontal and vertical eye movements to judge the change of the vector field in horizontal and vertical directions, respectively, making this visual task perfectly suitable for eye-tracking methodology.

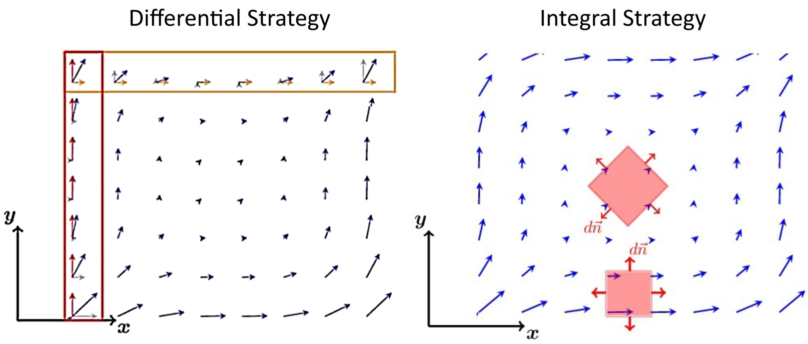

Figure 2: Graphical representation of a two-dimensional vector field. The left panels displays the procedure of the differential strategy. To judge whether y-component of the vector field (red arrows) changes in y-direction the students need to focus on the red box. Similarly, in order to judge whether x-component of the vector field (yellow arrows) changes in x-direction the students only need to focus on the yellow box. Combining both information, the students are able to conclude on the divergence of the vector field. The right panel shows the procedure of the integral strategy. To determine the divergence, the students need to quantify the arrows entering and exiting certain areas (here red squares).

In contrast, application of Eq. 1 to a graphical vector field plot means that we determine the divergence using the flux through the boundary ∂V of a test volume V in the field (or test areas in the two-dimensional case), see Fig.2 right. For instance, qualitative reasoning is simple if the outer surface normal $d\vec{n}$ is either parallel or perpendicular to the field vector $\vec{F}$ (e.g., cuboids or spheres in 3D and rectangles or circles in 2D). When students use this strategy, we expect a higher number of fixations, longer fixation duration and shorter saccade lengths as compared to the application of Eq. 1 to the field(Klein et al., 2018).

2 METHODOLOGY

This section provides the underlying methodology used in this paper. First, the preprocessing step is introduced. Second, the approach to investigate the implication of vector field orientation is presented. Then, the feature extraction procedure followed by the classification approach has been presented.

2.1 Preprocessing

We proposed an approach to cluster the distributed gaze-driven sequences in the vector field plot. The saccadic codification and transition matrix calculation are also explained.

2.1.1 Attentive Region Clustering

It is inevitable having noise and outliers in an eye-tracking experiment. Besides, depending on an experiment design, not all of the collected gaze patterns are relevant to the research. For instance, in this study, we ignored the fixations outside of the desired Area of Interest (AOI), which is the area of the vector field (see Fig. 1). Furthermore, in particular, this algorithm tends to find attentive region(s) in the vector fields 1—8 for both DS and IS. The inattentive regions could bias the dispersion-based features. Hence, we propose a novel approach, here termed Attentive Region Clustering Algorithm (ARCA), to cluster the attentive region(s) inside the vector field. The Attentive Region Clustering Algorithm is presented in Algorithm 1.

Identifying the visual strategies instructed in this study, the ARCA (Algorithm 1) provides the fixations during problem-solving whereas inattentive fixations are mostly scattered broadly inside the vector representation. Using the filtered fixations, in the next section, we propose a similarity measure to evaluate divergence of the vector field representations.

2.1.2 Saccadic Codifications

The visual strategies indicated in Fig. 2 offer either axis-wise evaluation in the differential problem or observing the vector flow through an arbitrary rectangle inside the vector field. Hence, eye movement directions during the problem-solving task reveal the quality of visual approaches made by the students based on the instructions. Selection of the axis side left or right, and up or down is optional to solve differential (DS) problems. It is somehow similar to integral (IS) tasks' visual strategy where the position of the rectangle with any rotation inside the vector field representation is flexible. Hence, we group the saccades into X for the x-axis, Y for the y-axis, M for diagonal, and N for anti-diagonal directions. To group saccades into X, Y, M, and N, the *absolute saccadic angularity* is used for the labeling. In this sense, all saccades within in the angular range of $337.5° - 22.5°$ com-

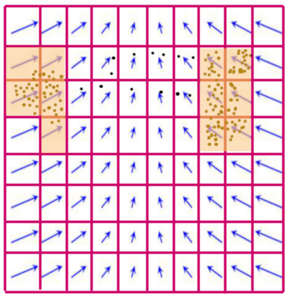

Figure 3: Illustration of the identification of tiles (yellow) which are regions within the 10 $\times$ 10 grid which received more fixations (black dots) then the threshold (here 3 fixations).

Algorithm 1: Attentive Region Clustering.

Result: Write here the result
calculate $fixations$ from raw gaze samples ;
define AOI in the stimuli as the representative region for the vector field area;
omit the fixations out of the AOI box;
$threshold$ = **3**$fixations$;
for $strategies : (DS, IS)$ **do**
  for $participants : [1:20]$ **do**
    for $stimulus : [V1:V8]$ **do**
      M = split AOI into 10x10 grid;
      create attention map: calculate the fixation population on M $tiles$;
      for $tile : M[1:10, 1:10]$ **do**
        if $|tile| < threshold$ **then**
          discard $tile$;
        end
        label-connected tiles: finding islands in M;
        intersect fixations of AOI area with M;
      end
    end
  end
end

bined with those from $157.5° - 202.5°$ are labeled as **X**, the ones from $22.5° - 67.5°$ combined with those from $202.5° - 247.5°$ are labeled as **M**, the ones from $67.5° - 112.5°$ combined with those from $253.5° - 297.5°$ are labeled as **Y** and the ones from $22.5° - 67.5°$ combined with those from $202.5° - 247.5°$ are labeled as **N**.

2.1.3 Calculate Transition Matrix

The sequence of saccades constructs a string for each trial. The elements of the string are associated with the corresponding saccadic labels. Consequently, 320 sequences from 320 trials were constructed. These sequences are fundamental to calculate the transition matrices. The transition matrices are used for transitional analysis, feature extraction, and classification.

A transition matrix is 4×4 2-D array as an alternative representation of the corresponding sequence. For instance, Table 1 presents an exemplary transition matrix for the sequence "**XXXMYYYNYX**".

2.2 Implication Vector Field Orientation

Vector field flows in the different directions as indicated in Fig. 1. As explained in section 2.1.2, because of the freedom of choice in direction and area in both visual strategies, it raises question about the impact of vector flow on students decision about the area and processing direction. Due to the mentioned freedom, the higher level of uncertainty is expected. As a measure of uncertainty, we investigated Jensen-Shannon divergence(JSD) in the transition matrices. JSD is an Shannon Entropy-based method to measure the similarity between two probability distributions (Holmqvist et al., 2011). It is an extended version of Kullback-Leibler divergence(KLD). As KLD fails to fulfill the triangle inequality which leads to asymmetric results, we preferred to apply the JSD (Holmqvist et al., 2011) for our purpose. The JSD results range in [0..1] intervals where the higher value indicates the larger the divergence. The Kullback-Leibler divergence and Jensen-Shannon divergence are defined as following:

$$KLD(p||q) = \sum_X p(x)\log_2\left(\frac{p(x)}{q(x)}\right) \tag{3}$$

$$JSD(p(x)||q(x) = \frac{KLD(p(x)||\frac{p(x)+q(x)}{2}) + KLD(q(x)||\frac{p(x)+q(x)}{2})}{2} \tag{4}$$

Table 1: An exemplary transition matrix for the sequence "XXXMYYYNYX". For example, $X \longrightarrow X$ transition two times happened in the beginning of the sequence.

	X	Y	M	N
X	2	0	1	0
Y	1	2	0	1
M	0	1	0	0
N	0	1	0	0

Input data—$p(x)$ and $q(x)$—have the form of probability density functions, i. e., the normalized transition matrices.

Separated for both IS and DS strategies, for all we calculated the normalized transition matrix of each participant. Then for each group (IS and DS), the JSD similarity measure applied.

2.3 Feature Extraction

Appropriate feature selection is a highly important stage to construct a robust machine learning model. Both dispersion-based and sequential-based features are necessary to build a robust model for classification of the visual strategies defined in section 1.2. In this study, we calculated the following features for each trial ($N = 320$) to build up our model:

2.3.1 Stationary Entropy (H_s)

Entropy is a measure in information theory to describe the information in a variable in terms of ordering. This measure is called Shannon entropy and it is defined as:

$$H_s = -\sum_i p(r_i)\log_2(r_i), r_i > 0 \quad (5)$$

where H_s is the stationary entropy in bits and $p(r_i)$ is the proportion of saccadic label r_i. r_i replaced with 1e$-$9 in case $r_i = 0$. We normalized the stationary entropy by dividing the result with the maximum possible entropy. In our case, four labels construct the sequence. Therefore, $\sum_{i=1}^{4}\frac{1}{4}\log_2(\frac{1}{4}) = 2.0$, is the maximum possible bits. Hence, all stationary entropy results divided by 2.0.

2.3.2 Transition Entropy (H_t)

The entropy can be calculated for a transition matrix (Mozaffari et al., 2016b; Krejtz et al., 2014). The lowest possible value is zero when there is no uncertainty about what type of transition will occur. The maximum value for entropy is when all the cells in the transition matrix carry different values.

$$H_t = -\sum_{i,j} p(r_i)\sum p_{ij}\log_2(p_{ij}), r_i \& p_{ij} > 0 \quad (6)$$

r_i and p_{ij} replaced with 1e$-$9 in case of 0.

where H_t is the transition entropy, $p(r_i)$ is the proportion of saccadic label r_i, and p_{ij} is the value of normalized transition matrix in row i and column j.

2.3.3 Relative Saccade Angularity (A_r)

The average of relative saccade angles shows the tendency of the students to drive their visual attention to the same direction. Relative saccade angularity is defined as the angle between a saccade and the previous saccade[1].

2.3.4 Fixation Duration (F_d)

Fixation duration (Holmqvist et al., 2011) is a classical metric in eye-tracking research. It is a dispersion-based measure indicating the density of visual attention.

2.3.5 Attention Score (F_s)

We calculate the attention score(F_s) by dividing the number of fixations in the attentive region calculated with ARCA by all the fixations in the AOI. This measure approximates the focus on the instructed strategy.

2.3.6 Direction Rank Entropy (H_d)

In section 2.2, the freedom of selecting areas and directions in the vector field representation has already been explained. However, tracing one direction, e. g. moving the visual attention simply up and down yield the same saccadic code (Y). Discriminating IS and DS visual strategies requires information about opposite direction movements. The procedure to calculate H_d is the following:

1. In each sequence, directions to the left, down, down-left, and down-right weighted with -1 and the rest with $+1$. For instance, label X in the left directions becomes -X and +X Vice versa.
2. Then we add all the labels and get the absolute weight for each label. For example, -2X, -4Y, and 2M yields [2, 4, 2, 0].

[1]Absolute saccade angularity defines as the angle between the saccade and horizon

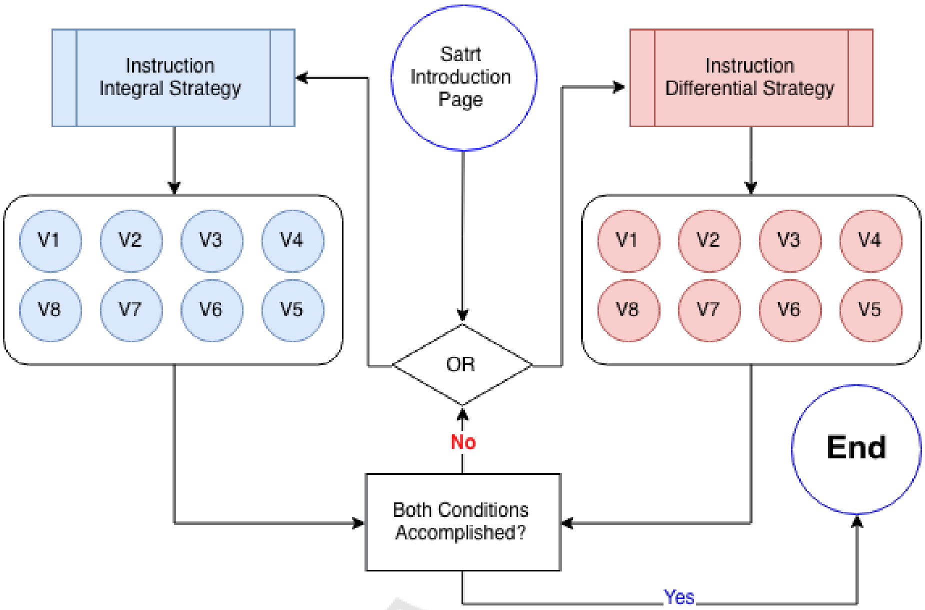

Figure 4: The procedure of the experiment. The students started with an introduction about the concept of Vector Divergence Problem displayed on a computer screen. Then, students started with the DS strategy (see Fig. 2 left panel) or with the IS strategy (see Fig. 2 right panel). Both instructions, DS and IS, covered 250 words and included a step-by-step description with visual cues about the application. In each instruction period, students applied the prevailing strategy to eight vector fields (V1—V8) 1 which were presented one after another.

3. Divide the weighted vector with the length of the sequence.
4. Normalize the weighted vector with the *l1* norm.
5. Replace zero values with 1e−9.
6. Calculate entropy of the weighted vector using Eq. 5.

2.3.7 Attentive Cluster Numbers (C_r)

The attentive region acquired by ARCA could distinguish the visual strategies. To elaborate more, students may like to assume multiple rectangles in the vector field area to solve the integral problem. Therefore, in the IS, the number of attentive regions is relatively higher ($mean = 2.65, std = 0.36$) compared to the axis-based strategy for DS ($mean = 1.71, std = 0.23$).

2.4 Classification

In order to classify two visual strategies in the VFD domain (IS and DS), in this stage of the research, the Support Vector Machine (SVM) was employed to build up the binary classifier. Using 10-fold cross validation, 65% of data was selected randomly for training and the rest was used for testing. Also, the best-tuned parameters (C, γ, and *kernel*) were selected by performing grid search accompanied with the cross-validation. The model trained with the feature vector presented in section 2.3. Investigating other machine learning models left for the future work.

3 DATA COLLECTION

This section demonstrates the data collection phase. The participants, the study design, as well as the experiment's procedure, are presented here.

3.1 Participants

Twenty major physics students of the University of ANONYMOUS (15 male, 5 female) aged 19–24 (average 20.6 years) took part in the experiment. All participants were about to attend an introductory electromagnetism course and had successfully completed two mechanics lectures (calculus-based mechanics and experimental physics). Divergence has been introduced in both mechanics lectures and has also been recapitulated in the electromagnetism course before the experiment was conducted. Participation was voluntary, took 30 min in total (survey and experiment), and was compensated with 10$.

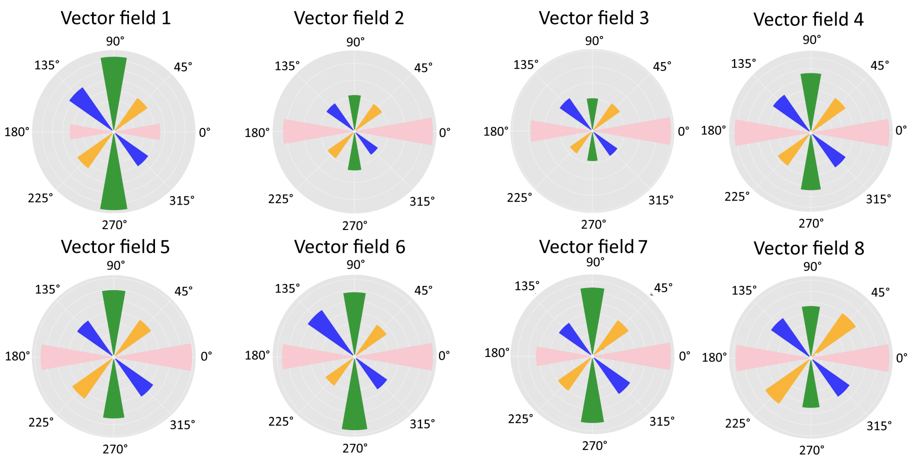

Figure 5: The saccadic direction when students applying the IS to judge whether a vector field has zero or non-zero divergence. The colors indicate the angular interval labeled with X (light red) indicating horizontal saccades, Y (green) meaning vertical saccades, M (yellow) diagonal saccades and N (blue) anti-diagonal saccades. Exploring diagonally and anti-diagonally is more evident in this approach compared to DS.

3.2 Study Design and Material

The students started with an introduction to the concept of vector field divergence displayed on a computer screen. The sequence of the experiment is illustrated in Fig. 4. All students started with strategy 1 (Figure. 2 left); the derivative strategy, DS) or strategy 2 (Figure. 2 right; the integral strategy, IS). Both instructions, DS and IS; covered 250 words (1 textbook page), respectively, and included a step-by-step description with visual cues about the application (worked-out example). In each instruction period, students applied the prevailing strategy to eight vector fields shown in Fig. 1 which were presented one after another. The vector fields used in the study are presented in Fig. 1. Students did not receive any feedback after completing a VDP and were unable to revisit the instruction page.

3.3 Eye-tracking Procedures

We obtained gaze data for all twenty students using a Tobii X3-120 [2] eye-tracker installed on a 24" LCD screen with an aspect ratio of 16:9 as they worked with the VDP. All students had normal or corrected-to-normal vision. The device has an accuracy of 0.4 degrees[3] and allows a relatively high freedom of head movement. The sampling frequency was 120 Hz. Gaze recording was accomplished using the Tobii Pro Studio [4]. The eye-tracking measures, including fixations and saccades, are calculated with the anonymous[5] library written in Python.

4 RESULTS

4.0.1 Vector Orientation Implication

Fig. 5 and 6 show the saccadic directions preferred by students in integral and differential approaches, respectively. The axis-wise tendency is more evident in the DS. However, horizontal or vertical movements are varied in different movements. The similarity measure based on Jensen-Shannon divergence for the vector flows is presented in Figure 7. For the purpose of the significant test, we added pairwise results of within JSD similarity measure in a dedicated array for each vector of DS and IS. The results suggest in some vector field—but not all— that there is a tendency to choose various options of selecting the area or saccadic direction. This could be interpreted as dynamically fitting the strategy to the vector flows in order to get the right answer. For the purpose of the significant test, we added pairwise results of within JSD similarity measure in a dedicated array for each vector of DS and IS. The one-way ANOVA significant test shows the significant differences between some vector fields as shown in Tab. 2.

[2]https://www.tobiipro.com/product-listing/tobii-pro-x3-120/

[3]reported by the manufacturer

[4]https://www.tobiipro.com/product-listing/tobii-pro-studio/

[5]Due to the blind review will be added for the camera-ready version

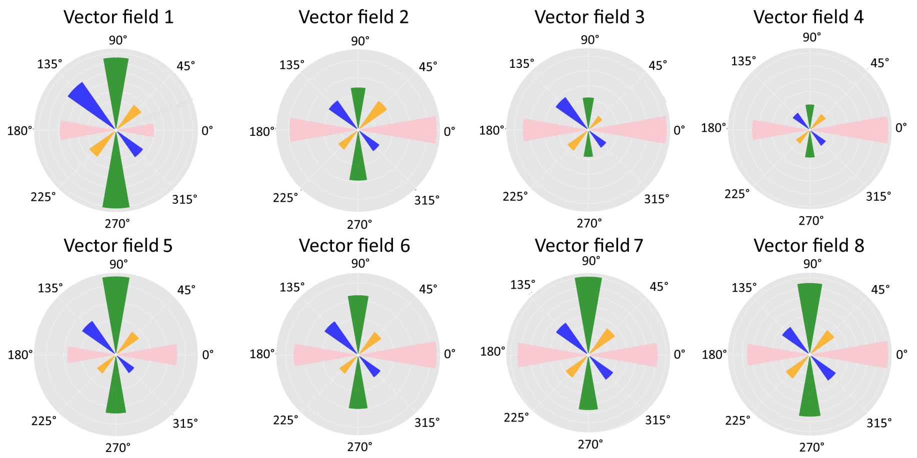

Figure 6: The saccadic direction when students applying the DS to judge whether a vector field has zero or non-zero divergence. The colors indicate the angular interval labeled with X (light red) indicating horizontal saccades, Y (green) meaning vertical saccades, M (yellow) diagonal saccades and N (blue) anti-diagonal saccades. Horizontal (X) and vertical eye-movements are more pronounced in this approach compared to the IS.

Table 2: Vector fields having significant differences in the one-way ANOVA test with a p-value<0.05 and $df = 189$.

Strategy	Vector Fields		F	p-value
DS	2	8	14.64	0.0001
DS	4	5	9.22	0.0025
DS	4	6	8.77	0.0032
DS	5	7	5.84	0.0160
DS	5	8	13.5	0.0002
DS	6	7	5.17	0.0235
DS	6	8	13.35	0.0002
IS	1	4	8.74	0.0033
IS	1	7	5.84	0.02
IS	2	4	8.12	0.004
IS	4	5	56.24	0
IS	6	7	18.96	0.0017
IS	7	8	109.85	0

4.0.2 Classification Results

The best model selected by grid search and cross validation is an SVM with linear kernel, $C = 10$, and $\gamma = 0.001$ for the vectors presented in section 2.3.

Table 3 indicates the precision and recall of the model on testing set. An accuracy of $81.2\%(0.11\%)$ has been achieved by the linear SVM model.

Considering on uncertainty in human behavior, in particular, eye movements, the results of the binary classification is promising to model experts gaze pattern in order to evaluate the real-world problem-solving task in an intelligent e-learning user interface. However, there is several room for improvement which is discussed in the next section.

5 DISCUSSION AND FUTURE WORK

Considering the Dreyfus model of skill acquisition perspective (Benner, 2004; Dreyfus, 2004; Dreyfus and Dreyfus, 1980), this contribution of this work is twofold: First, it helps to offer the novice learners by providing appropriate instruction needed for conceptual learning. The awareness of visual behavior of the students by monitoring of a particular problem-solving strategy might be highly beneficial for them. Although not all of the problems in physics are based on graphical representations such as the divergence of a vector field plot, it opens the door for an investigation how to address certain missed instructions. Thus, the second contribution is from another point of view: ”How to detect non-verbal patterns of the skilled per-

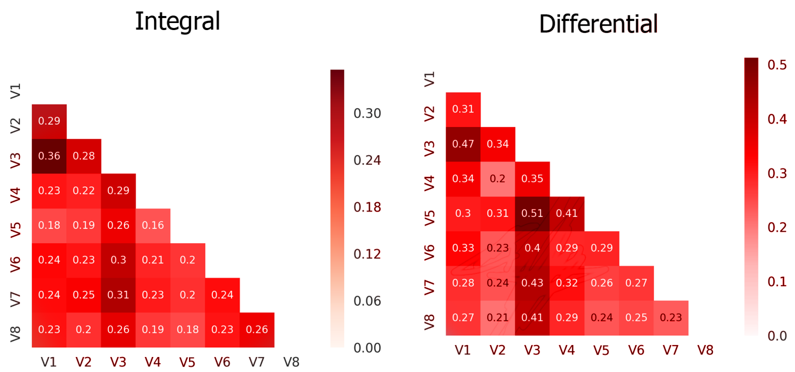

Figure 7: The Jensen-Shannon divergence in vector field representations.

Table 3: The score of the linear SVM model with $C = 10$ and $\gamma = 0.001$.

	precision	recall	f1-score	support
DS	0.89	0.70	0.78	59
IS	0.73	0.91	0.81	53
avg / total	0.81	0.81	0.79	112

sons and transfer it heuristically to the novices?". Fortunately, with the rising of deep learning in recent years, those associated visual patterns for solving and comprehending the related problem can be exploited and encoded with modern technologies (Kise et al., 2017). According to these concerns, we start to use a simple linear model to have proof for the future investigations of more sophisticated machine-learning techniques. The classification score provides an indication of the right path to pursue although there are some constraints on data in both terms of quality and quantity. Jenson-Shanon Divergence (JSD) score is intentionally used in this research because of its application to design an autoencoder for learning transfer. An autoencoder is an unsupervised type of artificial neural network used to learn efficient data codings. To have an efficient representation of data, the aim of autoencoders is to minimize the degree of uncertainty e.g., JSD. Hence, to give a picture for further research, the one-way ANOVA test is performed to statistically check the significance of the difference between the JSD score means of two independent groups, which are DS and IS. The classification results are promising to develop gaze-based pattern classification models. In this study, we trained linear SVM model for our purpose. However, other machine learning techniques can be investigated. For instance, Recurrent Neural Networks are highly suitable for sequential-based gaze data. The problem of deep networks is to have an adequate training set. Such an amount of data is very cumbersome to collect in the eye-tracking studies. Considering the rising interest in using generative models, the main idea of future work is to create a generative model from recorded data. Synthetic domain-specific eye movement data then can be used and evaluated with an appropriate deep network architecture (Mozaffari et al., 2018b).

The results also showed the effect of vector flow to form the strategy. Deeper data analysis in this regard leads to develop a more generic model in VDP. For instance, the correlation of the direction of the vector in the vector field plot with saccadic direction is worthy to explore. Conducting a none-instruction-based experiment with the same vector field stimuli is in our agenda of research. In particular, evaluating and quantifying the performance of students' task-related the gaze behavior (IS and DS) with the model has been achieved in this study. Furthermore, investigation of gaze-patterns in reading—reading speed, regression rate, and reading depth—and find the relations to the comprehensibility of problem-solving technique is another area of future work. The methods and ideas used in this research are plausible to apply in other domains of eye-tracking research. Finally, the concept idea of this paper connects machine learning and human-computer interaction (Ishimaru et al., 2016; Ishimaru et al., 2018) to develop an intelligent user interface to advance education in fundamental science such as physics and mathematics.

6 CONCLUSION

In this paper, we explored the eye-movement pattern of 20 students in instruction-based problem solving for integral and differential approaches in the domain of vector field divergence in physics. The results show that the flow orientations of the vectors have an influence on students attention areas in the vector field representation and on the pursue of different saccadic directions. Using a 10-fold cross-validation and grid search parameter we tune the Support Vector Machine in order to classify the visual strategies (DS and IS), a linear kernel SVM with $C = 10$, and $\gamma = 0.001$ has achieved an accuracy of $81.2\%(0.11\%)$. This means, that besides large individual variations in eye-gaze patterns among students, the algorithm is able to classify strategic gaze-patterns in a specific problem domain. On one hand, the results are helpful for improving the quality of learning and teaching since they provide a valid and detailed feedback for teachers on the effectiveness of their instructions to teach a certain strategy from monitoring the student's non-verbal performance. On the other hand, the algorithm may be used to give students an objective immediate feedback on their progress of learning.

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