# Power System Optimization Problems: Game Theory Applications

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Abstract: We look at the conflict situation in power system problems from an optimization perspective and use Game Theory (GT) concepts for modelling and solving the problem. In order to model the conflicts effectively, we first identify the players, the optimizing quantity and the optimizing platform. This paper details two power system problems and present a case study. We also identify two more areas where the same principles may be applied. Though our work focuses on Cooperative Game Theory (CGT), an extension to the Non-Cooperative Game Theory (NGT) platform is possible. Since GT is more relevant to a market structure, we use market engineering principles including multilateral trades, differential pricing, inverse elasticity rule, graph theoretical allocation, etc. as tools for organizing the optimization process. A useful addition for inducing stability in the decision making process is the concept of 'Power Vectors' borrowed from sports and game parlance for ceding players. Results are encouraging, with a transmission loss reduction of more than 70% in a five bus and 40% in a 24 bus system. We conclude that both versions of GT, the CGT and NGT are powerful tools for optimization in a practical scenario with conflicts and contradictory incentives.

## 1 INTRODUCTION

Many power system problems are conflict ridden requiring application of Game Theory (GT) concepts to mathematically model the complexities and optimize the 'live' variables. Conflict situations warrant the need to compete or cooperate/ negotiate and accordingly choose strategies to maximize benefits via a rational decision making. In GT, the decision variables contributing to the benefits are 'live' and evolve continuously based on strategies. This decision evolution procedure in a Cooperative Game Theory (CGT) model uses coalitions of players to maximize social welfare. Coalitions to reap more gains and share benefits are outcomes of shared information. However striking a discord and falling out of coalitions before the grand coalition is formed is a disruptive eventuality to be addressed in such scenarios. If decision making is simultaneous, without sharing of information, the operational structure is non-cooperative (NGT) and the force-majeure is the competition embedded in the game.

A distinctive feature of CGT is the 'characteristic function' which is maximized via formation of coalitions. Since the objective is conflict resolution, all perceived road-blocks or desirables may be used to embellish the characteristic function. Moreover, the strategies focus on maintenance of accord among coalition partners via an acceptable pay-off or sharing of benefits. This is to prevent the partners leaving the coalition for greener pastures at any point of time. Similarly for pay-offs in NGT, the strategies opted by the players are with the objective of achieving equilibrium, also called the Nash equilibrium. This is a 'minimax' solution, since each player minimizes the maximum payoff possible for other players if the game is zero-sum; they simultaneously minimize their own maximum loss. Thus situations with many complex options and each with several outcomes are most suitable for GT based modelling. The assumption and/or the hitch is that only rational decisions are taken by the players and nothing is left to chance.So, GT concepts can be applied to model engineering problems with conflicts which can be resolved by adopting strategies that optimize the 'live' design variables.

On examining reported works on applications of GT concepts to power system problems, it is noted problems in transmission expansion, loss allocation, demand/ response management, pricing, smart grid applications, renewable energy sizing, distribution networks, etc. are prominent. (Sore et.al 2006, Zhu et.al 2012, Kreyac et.al 2013, Mediwaththe, 2017, Mekontso et.al, 2019, He et.al 2019, Chen et.al 2017, Keren 2017, Contreras 1997) In all cases, two influencing factors observed are: Elasticity of demand versus price and incentive based commerce. Both factors make GT option possible in complex power system parlance. Elasticity is proposed to be designed and injected into the 4 problems presented here. And a differential pricing structure for power system variables like energy, demand blocks, open access of transmission corridors, etc is adopted to stimulate the incentive/ disincentive ridden commerce. Thus the four problems are appropriately modified and attendant issues addressed as follows.

# 2 GAME THEORY IN POWER SYSTEM OPTIMIZATION

Some pertinent questions on GT based optimization are raised here to define the power system problems presented, model them and identify the appropriate solution approaches. The conflicts associated with specific problems give the impetus for the approach.

- How can the conflicts be represented to apply GT concepts to arrive at workable solutions?
- How do the conflicts suggest theplayers?
- What would be the decision/ design variables and how do they evolve and stages thereof?
- What are the strategies that influence the evolution of the best of solutions?
- What is the level of information availability and sharing, for designing strategy sets?
- How is the characteristic function formulated?
- What is the outcomeof sharing the benefits?

Optimum power flow in corridors and allocation of transmission price are modelled here. Two recent reported works, power system islanding and then restoration, both with conflicting requirements, are presented as amenable to GT applied model with evolved solutions based on incentives and disincentives. The conflict modelling, both in the CGT and NGT platforms, are projected here.

## 2.1 Optimizing Power Flow in the Power Corridors

Some conflicts to be resolved in restructured power markets requiring modelling via GT concepts follow

1. Due to uneven generation and usage pattern in grids, quantum of power moved over transmission lines is large. In some corridors power shuttling is of the order of 3-5 times the total transactions, causing unacceptable congestion, voltage drops and power losses in lines unless optimal transactions are made.
2. An otherwise useful Optimal Power Flow (OPF) analysis tool has little significance in an electricity market which has distributed and closely guarded information. Also,OPF focuses on influence of generators on energy prices to derive line impacts. These impacts as feedback inputs cannot target control of abuse of the power lines. This is especially true due to distributed ownership of generation and transmission assets and their conflicting incentives in an electricity market.
3. In an electricity market, generating companies (GENCOS) do not reveal sales data and capture maximum power portfolios resorting to even profit cuts from energy prices. Then more distribution companies (DISCOS) buy cheaper energy leading to congestion and other problems on the network; thus the end users make profits while the afflicted party, i.e. the Transmission Provider (TP) provides access for both use and abuse of the network.
4. However, power flows obey Kirchhoff's laws only and no contractual laws. While in the erstwhile system, roles and responsibilities and a centralized authority were assigned, an electricity market is 'free-for-all. Then TP is the only entity who can exercise control on the 'runaway' on lines. Then TP should optimize trades of the GENCOs and conduct least loss iteration. However, too many rules of the road like curtailment, loading vector etc. cannot be laid down by the TP since competition is hampered, the reason for the development of electricity markets. Thus, though optimization has applicability as a tool, the implementation of its findings needs other market engineering tools as in GT concepts.

### 2.1.1 GT Modelling of Power Transactions

Some choices are made to model the problem in the GT platform. (Varaiya 1997, Sudha 2011)

1. A multilateral market structure is chosen over the pool. Thus a cartel, prone to dangerous power and market games and commercial considerations outweighing engineering requirements is averted. Since no reliable cost-benefit data is revealed by the market agents a transaction model independent of the economic data of the end-users is the best.
2. The distinctive entities are thus the GENCOs, DISCOs and the central TP. The GENCOs have influence over determining the Energy Charges (EC). The TP is given the prerogative over the construct of the Transmission Service Charge (TSC). Finally the choice of determining who to buy from and in what ratio will lie with the DISCOs.
3. To implement least loss formulation, an appeal to the selfish profit motive of the end-users is the game

plan. This encompasses them in a game to collude or cooperate so as to minimize the impact on the grid. In cooperative games, modus operandi includes both collusion and cooperation and so is inferred as the best choice. This is because collusion is not detrimental to the ultimate aim of least loss and other impacts on lines as evinced by the TP, but serves only to increase the cohesion between the cooperating agents. Another reason for choosing CGT is that GT is one of the strongest tools of market engineering, of which CGT uses both the nuances of coalition- threat (when with others) and promises (when with oneself), leading to more stable coalitions. If maximum benefits are given to the grand coalition, when all loads are transacted, then the combinatorial process of coalition formation is faster and this thought process is employed.

4. DISCOs play the game because they can form best coalitions, being privy to all local information. This helps to identify partners causing counter-flows and yet enhance their trades in a multilateral set-up.

5. Four phases of CGT are viewed here in a multilateral trade structure. 1. DISCOs derive Local information and based on merits become decision authorities. 2. TP computes and broadcasts Central information vital for the next phase using the communicated trades. 3. In the Common Information Derivation and Negotiation phase DISCOs divulge information conducive to trades with some trades dropped, some increased, exchanged or even shared for individual, coalitional and group benefits. In this manner a set of stable coalitions or the grand coalition are formed in Phase 4 and the result is committed and accordingly scheduled.

The TSC is next designed as the characteristic function, which dictates decisions and negotiations.

### 2.1.2 TSC - Characteristic Function

The design of the TSC answers three questions:

1. What is the benefit of cooperating in coalitions?
2. In an asymmetric environment how do the agents locate reliable / fruitful partnerships?
3. How can agents ascertain that what is bought is what is got since electricity is fungible?

TSC is designed (Sudha 2011) considering some entity interactions andbenefits of co-operating and honouring transactionsas the characteristic function.

a. Instead of passively providing open access, the TP manipulates the situation such that each DISCO is forced to compare TSC with EC and arrive at a compromise solution. The design should penalize all unacceptable transactions and be the lowest for a least loss formulation for a set of loads at any time.

b. Since the DISCOS and GENCOs have contradictory intentions and strategies only DISCOs are chosen to play. On comparing EC and TSC, the DISCOs shift their contracts to more profitable GENCOs. Since GENCOs do not control both EC and TSC, market is not skewed. DISCOs are accountable for the loss and play to reduce it.

c. Market engineering principles are beacons in the design of prices since perception exists that electricity sector has little elasticity. Experimenting with differential pricing and penalized deviations from least loss condition, elasticity of transmission and an empowered design of TSC was achieved.

The features in the TSC enable the DISCOS and their coalitions to identify scope for partnerships with an understanding on how to share the TSC. In an information asymmetric market only a few sources are reliable. Therefore the concept of power vectors from graph theory has been developed and put to use in three phases of the game. The main idea of power vector is that a node derives maximum power from subservient nodes and their chain of successors. Using power vectors, if DISCO A say appraises DISCO B as having a high strength it means that B makes a good candidate for causing counter-flows in relevant lines via B’s trades. Also, the benefits for agent A continue to increase if B’s trades enhance further. But the appreciation needs to be mutual for any meaningful dialogues between the two agents in the common information phase.

In short the players of the CGT based game are DISCOs; the design variables are the trades with minimized TSC. Strategies of DISCOs identify suitable coalitions causing counter-flows in lines. Information asymmetry is resolved through coalition derived information. Stability for the solution is ensured via power vector governed pay-off design. The TSC, as the bone of contention gives the design of payoff vector for forming coalitions. If feasible, it must lie in the solution space of the game.

## 2.2 TSC Sharing Problem

To match power allocated by GENCOs with trade contracts, fungible nature of electricity is a deterrent. In a trade based market, it is distressing that after all the haggling and negotiations between DISCOs and GENCOs, power flow from a specific generator to a precise load is affected by other trades on the grid, even gaming. In short in an electricity market with an assembly of trades, what is bought is not what is got; negotiation entered into by the agents is based on pseudo- trades. Since all trades mix on the grid system, contracts exist even if there is nil or minimal allocation from a generator and impact on the lines still have to be accounted for. Game and Graph theory are used to resolve such ambiguities.

### 2.2.1 CGT and Transmission Pricing

CGT comes hand in phase3; when two DISCOs or a conglomerate consider further mergers, after resolution using graph theoretical allocation. The resource allotment enables the partners to negotiate further, contract more profitable trades at lower grid impact and by cooperating, a lower TSC. Moreover, divergence from a common understanding is tackled using Ramsey pricing rule. Thus, via best trades at every coalitional step, the grand coalition is reached.

Diametrically opposite to the existing power system, the chain of events in the electricity markets leading to power flow starts from the other end of the spectrum i.e. trades are contracted and thereafter there is a confluence of trades on the transmission lines. So, first demand and supply is visualized as a set of trades, then assembled via a combinatorial process. Trades merge with an eye on TSC, two at a time in the first iteration, all the time going for the least loss trades. In subsequent iterations the fused trades continue to coalesce till the grand coalition or a set of stable coalitions is reached. The attainment of the solution, i.e. the payoff vector is the proof of the feasibility of the proposed model. Of the several techniques, the marginal vector, if it can be obtained is the stable and unique solution and is strived for.

### 2.2.2 Stable Coalition for Sharing TSC

1. The construct of TSC and the elastic curves lend convexity and superadditivity (Herings et.al 2006, 2007) to the game, resulting in an economically feasible payoff vector existing in the solution space. Convexity implies that more benefits accrue as more agents join the coalition. Hence, the TP reveals the maximum benefit only, which a grand coalition alone can earn. At intermediate steps the perceived minimum of TSC is used for evaluating the coalitions. ThusTP dispatches all loads or assembles all the best trades contracted i.e. schedule generators to meet all loads and losses.
2. Permutational convexity in a game implies incentives for including more, higher ranked players in the coalitions for a specific ranked permutation. Also with a permutationally consistent power vector (rank order echoes the power vector) the socially stable solution core has the marginal vector, a highly desirable unique payoff vector or solution. Social, technical, commercial, considerations can rank the players and a combination of these is proposed as a power vector, also for checking consistency.
3. A permutationally convex game is designed with a compatible power vector to ensure the presence of the sole marginal vector in the socially stable core. (Permutational compatibility ensures that no agent can hijack the game in her favour.) The locally computable power vectors and the derivation of hierarchy proposed are workable and compatible with all the steps taken so far and hence the solution is rationally true and acceptable.
4. Certainly, though not unique, a valid solution space is realized. It was checked for balancedness using power vectors of coalitions that sustain these payoff vectors. All these steps assure a socially stable core which is needed for stability an absolute necessity to prevent anarchy on the network.

### 2.2.3 TSC and Power Vectors

For a network with $n$ nodes, $L$ lines, line flow $z$, and line loss $q$, if weights for penalizing loss, sum of power flowing in all lines and flow in congested lines are $a$ (\$/MW$^2$h), $b$ and $d$ (\$/MWh) respectively and embedded cost is $c$in (**\$**/hr.), then the price function *p(q)*in **\$**/hr is

$$p(q) = aq^2 + \mathrm{b} \sum_L z + \sum_{congested} z + \mathrm{c} \tag{1}$$

To measure power of players (Herings et.al 2001) ascribe to a node, power, from both the number as also power of its successor. Let $\mathcal{A}$ be the collection of irreflexive digraphs on the vertex set $N = \{1, 2, \ldots, n\}$ with $(i, j) \in N \times N$ denoting the arc $\vec{ij}$ , $((i,j)$ ϵ **A** (node $i$ dominates $j$). The positional power function is the function $f^p : \mathcal{A} \to R^n$ which maps each $A \in \mathcal{A}$ to

$$f^p(A) = \frac{1}{n}(I - \frac{1}{n}T^A)^{-1} s^A \tag{2}$$

Here $T^A$ is the adjacency matrix of $A$, with the $ij^{\text{th}}$ entry $t^A_{ij} = 1$ if $(i, j)$ is an arc of $A$ and 0 otherwise; $s^A$ is the score vector giving the number of successors of each node. The TSC is used as the characteristic function and power vectors are used to form coalitions and also to design the socially stable core as shown in the case study in section 3.

## 2.3 Islanding of a Power System

The problem of optimized islanding of power system using GT concepts, on fault clearance is briefed.

1. The conflicts to be addressed are: number and extent of islands, coherency of generators within, and the power corridors to be evacuated, priority of loads.

2. The players are the GENCOs (of islands based on geographical proximity/ parts thereof).
3. The characteristic function is to be derived as a quadratic, penalizing the most unwanted outcomes of islanding such that an elastic curve can be derived.
4. The phases of coalition formation based on a power vector design which shows the technical features of the consequence of islanding.
5. The solution with pay-off vectors.

Depending on the catastrophic event preceding it, the restoration process also is to be optimized.

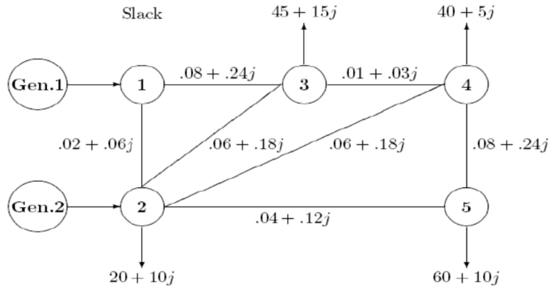

Figure 1: A 5 bus system.

## 2.3 Power System Restoration

1. All agents will seek priority in getting the power restored, because of the enormous economic and social implications with aligned conflicts.
2. The islanded areas and some privileged loads will be the players of this optimization game.
3. To use NGT and arrive at a Nash equilibrium.
4. The solution is to be iteratively inducted with the outcome of optimal restoration vector as pay-off.

# 3 A CASE STUDY

A case study for a 5 bus system for the first two problems in optimization is given below. Briefly the following phases are explained:

1. A set of .multilateral trades are derived from the problem using graph theoretical allocation (Table 1) (Wu, 2000, Varaiya, 1999, Penh et.al 2002).
2. Next is the local phase computations and derivation of all required information by all DISCOs. (Sample cases of 2 DISCOs deriving power vectors for their own trades and other useful data for negotiations are given in Table 2, 4).
3. The coalition move in the direction of data related to optimal trades using power vectors (Table 2, 3, 4,).
4. Optimal trades are derived & scheduled and committed at the best total TSC (Table 4).
5. Sharing of the benefits in the final phase, which proves the feasibility of the method and is reflected in the final core is derived. (Fig. 3). (Appendix).

Table 1: Division of power demand and flows into trades.

| Disco on buses No: (Demand) | Genco on bus 1 Load (MW) | Genco on bus 2 Load (MW) |
|---|---|---|
| 2 (20MW) | 13.623 | 6.376 |
| 3 (45MW) | 39.544 | 5.456 |
| 4 (40MW) | 30.463 | 9.536 |
| 5 (60MW) | 41.374 | 18.628 |
| Total load | 129.74 | 40 |
| Line loss | 4.77MW | |
| Sum of power flow in all lines | 262.6 MW | |

Table 2: Local Information computation: Power Vectors.

| | Disco2 buying 20MW from Genco1 | | | Disco3 buying 45MW from Genco4 | | |
|---|---|---|---|---|---|---|
| Bus No: | Lines with $t_{ij}=1$ | $s^A$ | Power vector | Lines with $t_{ij}=1$ | $s^A$ | Power vector |
| Ref | 0-1 | 1 | .2401 | 0-4 | 1 | .2736 |
| 1 | 12,1-3 | 2 | .4402 | 1-3 | 1 | .2021 |
| 2 | 2-0 | 1 | .2067 | 1-3,2-3 | 2 | .4024 |
| 3 | 2-3,3-4 | 2 | .4347 | 3-0 | 1 | .2123 |
| 4 | 2-4,4-5 | 2 | .4013 | From 4 | 3 | .6414 |
| 5 | 2-5 | 1 | .2012 | 2-5 | 1 | .2338 |

Table 3: Coalition Formation using Common Information.

| Bus | | Power Vector | Graph theoretical allocation of load | | |
|---|---|---|---|---|---|
| Ref | | 0.5187 | Load | Gen. 1 | Gen. 4 |
| coalition {2,3} | 1 | 0.4528 | 20MW | 13.322 | 6.678MW |
| | 2 | 0.4620 | 45MW | 5.886MW | 39.11MW |
| | 3 | 0.2532 | 40MW | 0 | 40MW |
| | 4 | 0.6598 | Loss | .231MW | - |
| | 5 | 0.2437 | Total | 19.439 | 85.798 |

Table 4: Optimal trades as derived in the Central Information Derivation Phase.

| Optimal Trades in MW | | |
|---|---|---|
| bus(demand) | Load- Genco 1 | Load- Genco 4 |
| 2(20) | 13.846 | 6.154 |
| 3 (45) | 5.002 | 39.996 |
| 4 (40) | 0 | 40 |
| 5 (60) | 23.654 | 36.546 |
| Total | 44.102 | 122.696 |
| Loss | 1.6 MW | |
| Sum of power flow in all lines: 163.7 MW | | |

# 4 CONCLUSIONS

It is mooted that game theory offers a very suitable platform to model complex situations in power system optimization problems. The whole idea of GT concepts encourages choices and hence is fertile ground for having different perspectives for deciding the players of the game. The benefits nor its maximization process is narrow framed and offers plenty of research opportunities. The three phases of CGT was demonstrated to successfully coordinate multilateral trades using two tools, a suitable TSC and power vector and that the Socially Stable game is instrumental in ensuring stable trades. The case studies on 5 bus and 24 bus (not shown here) power systems reveal the following advantages.

1. In a 5 bus, 169.74 MW demand system with a loss of 4.44 MW and a total power shuttling over the lines of 262.6 MW is optimized to a power system with 1.6 MW loss and a total power of 163.7 MW shuttling on the lines.
2. A 24 bus system with a demand of 1219 MW, and 36.355 MW loss optimizes to 15.43 MW loss and power shuttling dropping from 3825 to 2805 MW via GT concepts.

All contributions to the process are based on market engineering techniques which are more applicable, suitable and acceptable.

In Figure 2 is given one such contribution where a coalition based optimization is visualized as a step in the negotiation phase.

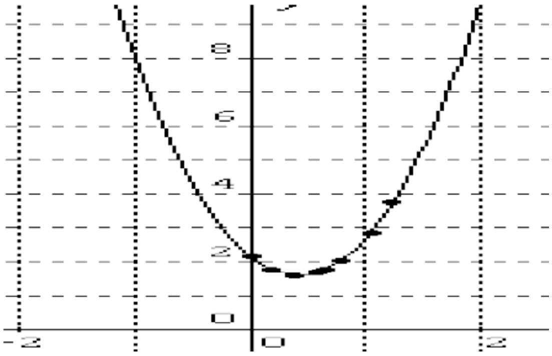

Figure 2: Least loss iteration by coalition {2,3}.

Another contribution is indicated in Figure 4 where a sample of a TSC designed in a novel manner such that the elastic nature is utilized by the DISCOs for least loss iteration.

The derivation and adaptation of such vectors at each stage of the GT based optimization is another contribution, especially since it has been imported from the sports and games field to cede players. Here, the powerful use is for deciding by the agents, initiating the trades, the best partner to obtain counter-flows and thus reduce TSC as the partnership deal between the coalition partners.

The inherent choice factor, its capacity to promote competition and scope for negotiation and extraction of hidden information, resolve the uncertainty factor in an information asymmetric complex scenario. In conclusion it can be said that the biggest engineering advantage of GT is that solution of the problem becomes a common agenda and a unifying force, even in a profit motivated milieu, where commercial considerations overrule engineering requirements.

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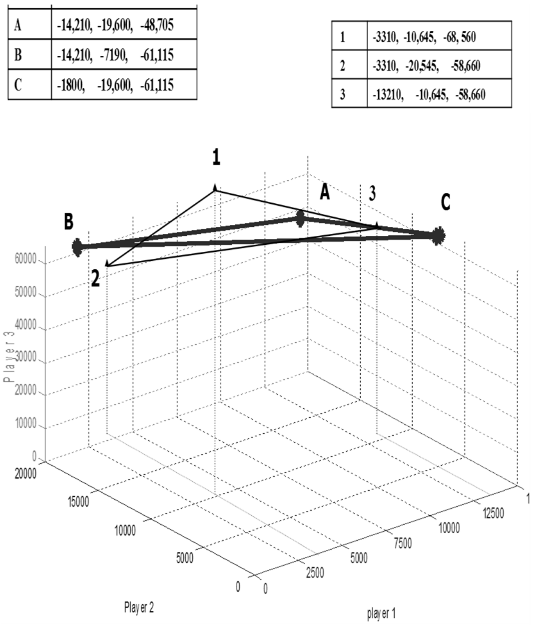
# APPENDIX



| | |
|---|---|
| A | -14,210, -19,600, -48,705 |
| B | -14,210, -7190, -61,115 |
| C | -1800, -19,600, -61,115 |

| | |
|---|---|
| 1 | -3310, -10,645, -68, 560 |
| 2 | -3310, -20,545, -58,660 |
| 3 | -13210, -10,645, -58,660 |

Figure 3: Socially Stable Core for a 5 bus TSC sharing game.

Table 5: Optimizing trades derived in, local and negotiation & common Information Derivation Phase.

| Buyer Disco | Seller Genco | Power Trade in MW | line loss | $\sum$z or $f_L$ | Z or $f_L$ in 1-3 | Z or $f_L$ in 4-5 |
|---|---|---|---|---|---|---|
| 2 | 1 | 20 | .0674 | 25.43 | 3.143 | .5714 |
| 2 | 4 | 20 | .0869 | 36.19 | 3.429 | 3.619 |
| 3 | 1 | 45 | .6017 | 93.86. | 16.71 | 5.143 |
| 3 | 4 | 45 | .1794 | 53.79 | 1.929 | 1.714 |
| 5 | 1 | 60 | 1.571 | 139.9 | 12.86 | 18.1 |
| 5 | 4 | 60 | 1.307 | 117.9 | 6.857 | 27.24 |
| {2,3} | 1 | 65 | .8954 | 108.4 | 19.86 | 4.571 |
| {2,3} | 1,4 | (20,45) | .2314 | 69.07 | 5.072 | 2.286 |
| {2,3) | 4 | 65 | .4417 | 82.81 | 13.29 | 1.524 |
| (2,3} | 4,1 | (20,45) | .3485 | 85.83 | 1.5 | 5.333 |
| {2,3}- (G1-13.32& 5.89 to 2&3), (G4-6.68 & 39.11 to 2&3) | | | .2312 | 69.02 | 4.812 | 2.406 |
| {2,5} | 1 | 80 | 2.016 | 158.7 | 16 | 18.67 |
| {2,5} | 1,4 | (20,60) | 1.43 | 132.5 | 3.714 | 27.89 |
| {2,5} | 4 | 80 | 1.567 | 132.9 | 9.429 | 21.71 |
| {2,5} | 4,1 | (20,60) | 1.742 | 146.9 | 10.29 | 30.86 |
| {2,5}- (G1-12.73&21.91 to 2&5) (G4-7.27 & 38.09 to 2 & 5) | | | 1.3622 | 124.1 | 1.095 | 25.58 |
| {3,5} | 1 | 105 | 3.099 | 194.5 | 29.57 | 12.95 |
| {3,5} | 4,1 | (45,60) | 1.622 | 150.6 | 14.79 | 19.81 |
| {3,5} | 1,4 | (45,60) | 1.415 | 145.3 | 9.857 | 22.1 |
| {3,5} | 4 | 105 | 1.651 | 154.1 | 4.929 | 28.95 |
| {3,5}-(G1-5 & 25.76 to 2 & 5 ) (G4-40 & 34.24 to 2 &5 ) | | | 1.3512 | 141.3 | 5.177 | 24.27 |
| {2,3,5}-(G1-13.85,5,23.45 to 2,3&5 ) (G4-6.15,40,36.55 to 2,3&5) | | | 1.604 | 163.7 | 5.607 | 26.1 |

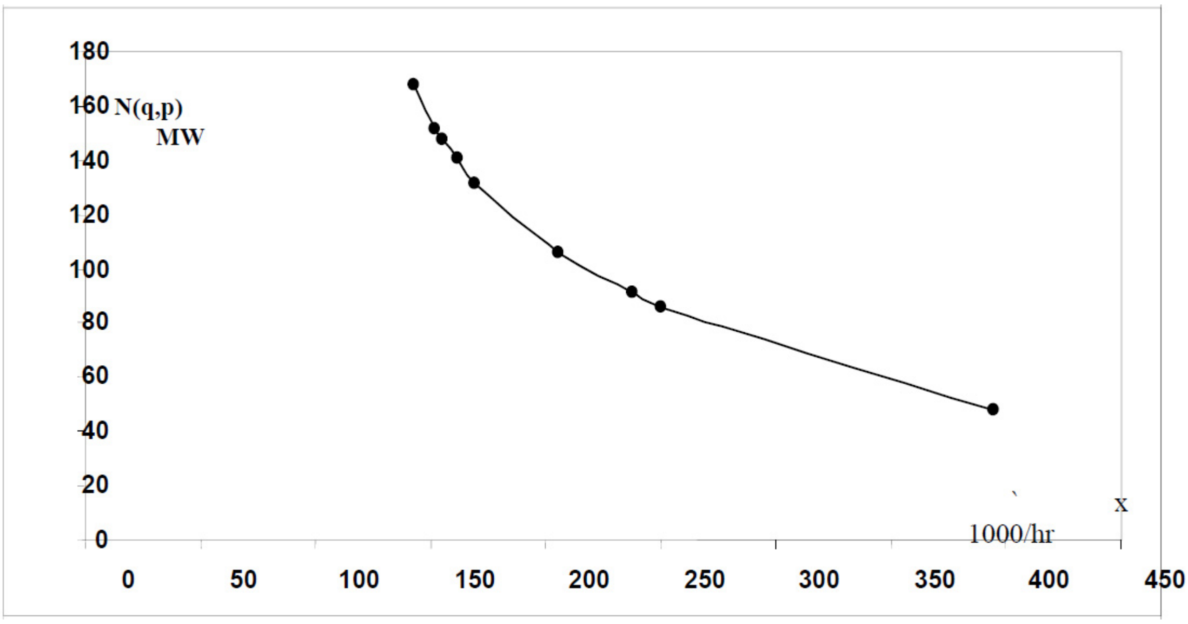


Figure 4: Elasticity curve- Demand Vs TSC for 165MW met by generation at bus 1&2.