An Optimization Method for a Multi-day Distribution Problem with Shortage Supplies

Netiphan Amphaiphan[1] and Wasakorn Laesanklang[1,2]

[1] *Department of Mathematics, Faculty of Science, Mahidol University, Rama VI Road, Ratchathewi, Bangkok, 10400, Thailand*

[2] *Centre of Excellence in Mathematics, CHE, Thailand*

netiphan.amp@student.mahidol.ac.th, Wasakorn.lae@mahidol.edu

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Abstract: We investigated a multi-day distribution problem while supplies are limited. This scenario can be found in post-natural disasters or economic crisis such as floods, earthquakes, palm oil shortage crisis, etc. The objective function of this problem is to minimize total traveling distance, unsatisfied cost, and variance of supply delivery proportion. In order to solve this multi-day problem optimally, it requires large computing memory and takes a long computational time. Therefore, we divided these large problems into multiple daily sub-problems and solved the sub-problems with the exact method. The sub-problems were solved sequentially for which the prior daily sub-problem is to be tackle first and the following daily sub-problems are defined based on the prior daily sub-problem solution. Changes were applied to update demands and to adjust delivery priority. There are three delivery priority setups proposing in this paper. Also, we present an experiment using the three proposed methods to solve modified Solomon's vehicle routing problem datasets which extended a single period vehicle routing problem with time windows to be seven-day routing problems.

1 INTRODUCTION

This paper proposes an exact-heuristic hybrid method to solve a multi-day distribution problem with shortage supplies. Normally, supply distribution in normal situation is an operation which has been studied for many years. The problem has been shown in many paper and text books such as N-location distribution problem (Karmarkar and Patel, 1977) and production distribution problem (Boudia et al., 2007). However, there are situations where supplies are limited in which a normal distribution operation is not practical because there are too many demands that cannot be filled. Supplies shortages are not normally occur in high or medium income countries. Although, emergency and disaster situations may cause the supply shortages. For example, in 2011, Thailand encountered an extreme flood triggered by Tropical Storm Nock-Ten. There were 13.6 million people stuck in the disaster area. Thai government transformed government buildings, schools, university buildings to be temporally shelters because majority of houses were unable to occupied. People in these relieve care needed food, medicine and other supplies so that they could survive the day. Meanwhile, these commodities could not reproduce in the disaster area, they were transported from other part of Thailand to a distribution center. The operation at distribution center was indeed challenging as it must fill demands in remote area, at which might not have road access. In some case, the operation may have a limitation due to the number of vehicles and their capacity rather than a limitation of supplies. Recently, in 2019, there was another tropical storm hit the northeastern of Thailand. There were reports of insufficient relieve goods and distribution problem in post-disaster event. This has shown that there were no efficient plan to deal with the distribution problem with limited supplies in Thailand.

An example of the real-life supply shortages were the case when Thailand faced the palm oil crisis in 2011. Palm oil production was decrease and there were no supplies in the market. An immediate policy, which was order from the Thai government to limit palm oil purchase per transaction to three liters. The policy applies in order to prevent consumer to stockpile palm oil. Similarly, distributor cannot apply normal distribution operation because the best possible solution is to have the palm oil products to sell near distribution center. This plan will indeed effect customers who live very far from the distribution center that they cannot have excess to the product in a reasonable price. Therefore, these rare events needs a special operation plan.

This paper proposes a method to find supply distribution plan especially when all demands cannot be fulfilled. The solution of this problem gives a plan to distribute supplies with one vehicle mode in multiple periods. The plan should ration supplies equally among the demand points. We present a mathematical programming model for a multi-day distribution problem with shortage supplies. However, this model cannot be solved as our sample problems are very computationally expensive. Therefore, we implement a method utilizing a mixed integer programming model to tackle this problem by solving the problem in a daily basis and to have demand information passing to the next day problem to balance supplies distribution. The goal is to generate routing plans for the whole time period and to have an equal supply distribution.

The remaining of the paper is organized as follows. We discuss related research in Section 2. Section 3 describes the problem and its assumptions. It also present our weekly quadratic programming model. Section 4 gives a daily problem and how our plan is recalculated. Section 5 describes the dataset used in this work and the results from our method. Section 6 presents our conclusions.

2 LITERATURE REVIEW

There were several literature for transportation problem in post natural disasters. (Özdamar et al., 2004) proposed a model that generated an emergency logistic plan for natural disaster. They investigated a scenario when supplies were limited and there were prediction of demands. (Liu et al., 2007) studied large-scale emergency plans and proposed a mathematical model to minimize the total unsatisfied demands. (Afshar and Haghani, 2012) developed a comprehensive model for pick up and delivery schedule for natural disasters. (Hsueh et al., 2008) studied a vehicle routing problem for disasters with two-phase decisions, tactical decisions and routing decisions. (He and Zhang, 2016) determined priority allocation coefficient for post-disaster relief supplies using set theory. (Qin et al., 2017) investigated insufficient emergency supplies in the initial period right after a disaster event with uncertain demands of affected locations. This proposed solution method was split into two stages, determining the serviced demand point and optimizing the related factors such as a number of used vehicles, supplies and routes. (Liu et al., 2019) studied a location-routing problem for the early post-earthquake where relief goods were insufficient. The goal was to find a fair distribution plan using a demand loss function.

Emergencies and disasters mainly create short term insufficient supply situations. However, there were researches dealing with long term supply shortages such as a distribution problem with limited inventory (Chien et al., 1989) and blood supply chain in Thailand (Chaiwuttisak et al., 2014).

There were many ways to solve large-scale problems such as local search, neighborhood search, genetic algorithm, etc. Problem decomposition is a technique that can be applied to make a large problem become smaller sub-problem so that the solver find a solution faster. (Kim and Kim, 1999) decomposed a multi-period vehicle scheduling problem into *N* single-retailer problems to generate delivery schedule. (Cheng and Wang, 2009) investigated VRP with time window constraints using decomposition technique to reduce the problem size. (Laesanklang and Landa-Silva, 2017) also applied decomposition and repair methods for home healthcare planning to obtain a feasible solution.

There were a few research works in the literature tackled multi-period supplies shortage problem such as blood supply chain in Thailand (Chaiwuttisak et al., 2014) and multi-depot production planning (Parthanadee and Logendran, 2006). The studies on general multi-period VRP with shortage supplies are relatively rare. Our work investigates a multi-period VRP with supply shortages while splitting the problem into multiple daily sub-problems.

3 MULTI-DAY DISTRIBUTION WITH SHORTAGE SUPPLIES

This section describes the quadratic programming model for the multi-day distribution problem with shortage supplies. The problem is to generate a transportation plan over a multi-day period with supply shortage event. Therefore, solutions for this transportation plan should distribute the products equally to all demand nodes. This may differ from the normal distribution operation for which it is to find the minimum operation cost. The assumptions for the multi-day distribution problem with shortage supplies are listed below.

1. Transportation costs and travel times are proportional to travel distances. Also, transportation cost from any location i to j in all periods are identical.
2. Distance from location i to location j equals to

distance from location j to location i.

3. Each customer must be visited no more than once a day.
4. The total delivered commodities must not exceed vehicle capacity.
5. Each location must be visited no more than once per day but that visit may have not fill the demand in full.
6. All vehicles are homogeneous.
7. Demands are given for all locations for the whole time horizon. Although, demands may accumulate if the demand point has not been visited.

Ideally, to have the best possible solution, the multi-day problem should be solved as a whole. We may present this problem using a quadratic programming model.

3.1 Weekly Problem

As we mentioned in Section 1, we would like to find delivery plans that have equally supply distribution for every demand node. Therefore, the quadratic programming model was proposed for this problem.

Table 1 presents notations to be used in this paper. The notations can be grouped as sets, parameters, and variables.

The decision is to assign a set of vehicles V to make visit at a set of locations C at time period in a set of time T. Each vehicle $v \in V$ has maximum capacity q_v and each location $i \in C$ also has demands on day $t \in T$, represented by s_i^t. We considered a distribution with single depot so that c_0 and c_0' are artificial start and end nodes where the two artificial nodes represent one physical depot.

The problem is constructed based on graph where nodes or vertices are locations to visit and the edges are routes between the two locations. A variable $x_{i,l}^{v,t}$ is a binary decision variable in which $x_{i,l}^{v,t} = 1$ represents that a vehicle v must make a visit at location l after location i at day t, and $x_{i,l}^{v,t} = 0$ otherwise. An assignment to travel between location i and location j has corresponding distance $d_{i,j}$ and travel time $\alpha_{i,j}$. Also, an integer variable $\delta_i^{v,t}$ is the arrival time of the vehicle $v \in V$ at location $i \in C$ on day $t \in T$. y_i^t is a binary decision variable where its value is 1 when no vehicle make visit at location $i \in C$ on day $t \in T$, and $y_i^t = 0$ indicates otherwise. A variable $p_j^{v,t}$ is a variable between 0 to 1 for determining delivery proportion. In this problem, it is possible to delivery partial demands as the supplies are very limited and rationing must be enforce to distribute the commodity equally for all demand points. Therefore, we define β_i as the proportion of delivered supplies and the total demands at location i for the whole time horizon. This proportion β_i can be written as

$$\beta_i = \frac{\sum_{t \in T} \sum_{v \in V} p_i^{v,t} \cdot s_i^t}{\sum_{t \in T} s_i^t}.$$

In order to measure equal distribution, we then minimize the variance of β_i. Note that we use $\bar{\beta}$ as a notation of an average of all β_i.

From the above notation, we present the mathematical model for the multi-day distribution problem with shortage supplies below.

$$\begin{aligned} \text{Min.} \quad & \omega_1 \sum_{t \in T} \sum_{i,j \in C} \sum_{v \in V} d_{i,j} \cdot x_{i,j}^{v,t} \\ & + \omega_2 \sum_{t \in T} \sum_{i \in C} \sum_{v \in V} (s_i^t - s_i^t \cdot p_i^{v,t}) \\ & + \omega_3 \cdot \sum_{j \in C} \frac{(\beta_j - \bar{\beta})^2}{|C|} \end{aligned} \tag{1}$$

Table 1: Notations for the multi-day distribution problem with shortage supplies.

Sets	
C	A set of locations.
V	A set of vehicles.
T	A set of time period. ($T = \{1, 2, ..., 7\}$)
Parameters	
$d_{i,j}$	Distance between node i and j.
M	Penalty cost of not assigning visit a node.
$\alpha_{i,j}$	Travel time from node i to j.
γ_i^t	Weight for each location i at time t.
q_v	Vehicle capacity of vehicle v
s_i^t	Demand of location i on day t
ω_1	A weight associated to total transportation cost.
ω_2	A weight associated to unsatisfied cost.
ω_3	A weight associated to variance of proportion of delivered supply.
Variables	
$x_{i,j}^{v,t}$	Binary decision variable with value 1 indicating that vehicle v is assigned to visit i and then move to visit j on day t, and 0 otherwise.
y_i^t	Binary decision variable with value of 1 indicating that visit i on day t is unassigned, and 0 otherwise.
$\delta_i^{v,t}$	Arrival time at node i assigned to vehicle v on day t
$p_i^{v,t}$	Delivery proportion delivered to customer i by vehicle v on day t ($0 \le p_i^{v,t} \le 1$)

Subject to

$$\sum_{v\in V}\sum_{i\in C\cup\{c_0\}} x_{i,l}^{v,t} + y_l^t = 1 \quad \forall l \in C, t \in T \quad (2)$$

$$\sum_{i\in C\cup\{c_0'\}} x_{c_0,i}^{v,t} = 1 \quad \forall v \in V, t \in T \quad (3)$$

$$\sum_{i\in C\cup\{c_0\}} x_{i,c_0'}^{v,t} = 1 \quad \forall v \in V, t \in T \quad (4)$$

$$\sum_{i\in C\cup\{c_0\}} x_{i,r}^{v,t} = \sum_{j\in C\cup\{c_0'\}} x_{r,j}^{v,t} \quad \forall v \in V, r \in C, t \in T \quad (5)$$

$$\sum_{i\in C} s_i^t \cdot p_i^{v,t} \leq q_v \quad \forall v \in V, t \in T \quad (6)$$

$$\sum_{i\in C\cup\{c_0\}} x_{i,j}^{v,t} \geq p_j^{v,t} \quad \forall v \in V, j \in C, t \in T \quad (7)$$

$$\sum_{i\in C\cup\{c_0\}} x_{i,j}^{v,t} \leq M \cdot p_j^{v,t} \quad \forall v \in V, j \in C, t \in T \quad (8)$$

$$s_j^t \cdot p_j^{v,t} \geq 0.05 \cdot q_v \cdot x_{i,j}^{v,t} \quad \forall v \in V, i, j \in C\cup\{c_0\}, t \in T \quad (9)$$

$$x_{i,j}^{v,t} + x_{j,i}^{v,t} \leq 1 \quad \forall v \in V, i, j \in C, i \neq j, t \in T \quad (10)$$

$$\delta_i^{v,t} + \alpha_{i,j} - (1 - x_{i,j}^{v,t}) \cdot M \leq \delta_j^{v,t} \quad \forall v \in V, i \in C\cup\{c_0\}, j \in C\cup\{c_0'\}, i \neq j, t \in T \quad (11)$$

The objective function of the proposed multi-day distribution with shortage supplies is to minimize a summation of three weighted costs (1). The first cost is the total travelling distances for the whole time horizon. The second cost is the unsatisfied cost, which is measured from the number of accumulated demands that have not been served at the end of the time horizon. The third cost is to have equal supply distribution, which is measured by the percentage of delivered supply variance. The percentage delivered supply variance is to measure differences between delivery proportion, in which the ideal solution should be the case where all locations get the same proportion of supply or the delivery proportion variance should equal to 0. Therefore, the proposed model is to minimising the percentage supply variance should provide a fair distribution plan. There are corresponding weights to the above objective function costs which are ω_1, ω_2, and ω_3, respectively.

We set the variance as the highest priority to minimize. Hence, the weight ω_3 is the highest in this objective function. The second priority is to minimize the cost of unsatisfied demands where the corresponding weight is ω_2. The lowest priority is to minimize the cost of travelling distances, where the weight is ω_1. We followed weight parameter setting from (Rasmussen et al., 2012). These weights are set by $\omega_1 = 1, \omega_2 = \sum_{i\in B}\sum_{j\in B} d_{i,j}$ and $\omega_3 = \omega_2 |C| \max_{i\in C, t\in T} \gamma_i^t$.

The proposed mathematical model has the following constraints to complete the multi-day distribution problem with shortage supplies. First, constraint (2) ensures that a visit $l \in C$ is either assigned to a vehicle $v \in V$ by having one of $x_{i,l}^{v,t} = 1$ or left unassigned by having $y_l^t = 1$ for every time period $t \in T$. For each day, a vehicle must leave a depot from constraint (3) and it must return to depot at the end of the day (4). During the day, a vehicle must travel to visit a list of locations, where the flow conservation constraint guarantees the condition such that a vehicle v arrives at a location l, it must leave that location (5). Constraint (6) defines maximum vehicle capacity for which the total delivery must not exceed. Constraints (7) and (8) control that the $p_j^{v,t}$ can be more than 0 only if the sum of $x_{i,j}^{v,t}$ is 1. Furthermore, a visit to be made must delivery supplies at least 5 % of the vehicle capacity. Constraint (10) prevents vehicle v to return to a location $i \in C$ if it has been left the location. Constraint (11) is a sub-tour elimination constraint indicating that the arrival time at location $j \in C$ must be more than the arrival time at location $i \in C$, given that the vehicle $v \in V$ must visit location j after location i ($x_{i,j}^{v,t} = 1$) (Moshref-Javadi and Lee, 2016).

The mathematical programming model for the multi-day distribution problem with shortage supplies above is a quadratic programming model as it applies variance as one of the objective function. The problem is a np-hard so that applying exact method to solve this problem may not be the best approach. Therefore, we propose a method to sequentially solve daily sub-problems using a mixed integer programming model and mathematical solver to tackle this problem.

4 SOLUTION METHOD

This section explains our proposed method to solve the multi-day distribution problem with shortage supplies. We formulate a MIP model for a daily problem and use it to solve the multi-day problem. The multi-day problem is then split into multiple daily problems. Each daily problem is then solved by the exact solver to get the best solution with a limited computation time. A next day problem is then updated with demands and historical supplies to adjust the next day delivery plan.

Algorithm 1 illustrates the steps of our proposed method. A multi-period vehicle routing problem can be denoted by $P = (V, B)$, where V is the set of vehicles and B is the set of locations. The goal is to assign vehicles to make visits at multiple locations within the time horizon. Our proposed method has three steps, which are decomposing problem, solving sub-problems and updating parameters. At the updating parameter step, demands and delivery priority of a problem at period t depend on the solution of the problem at period $t-1$. This step allows us to solve the problem as a day-by-day in sequential basis. The updating parameter step is the key to balance the supply delivery.

Algorithm 1: Algorithm for solution method.

```
Data: Problem P=(V,B), V is the set of
      vehicles and B is the set of all
      locations
Result: {SolutionPath} sub_sol
begin
    {Problem} T=ProblemDecomposition
     (V,B);
    SortSubProblem(t);
    for day t ∈ T do
        sub_sol (t)=cplex.solve (t, s_i^t, γ_i^t);
        Update_data (d_i^t, γ_i^t);
    end
end
```

4.1 Problem Decomposition

Our proposed method tackles this problem in a day-by-day basis. Therefore, we split this one week problem into seven daily sub-problems. These sub-problems will be solved sequentially, such that the sub-problem at period t will be processed before the sub-problem at period $t+1$. Structure of each sub-problem is identical with the full problem such that it has a set of vehicles and a set of locations with all constraints applied.

The daily sub-problems will be formulated as a mixed integer programming model and to be solved by a mathematical solver.

4.2 Daily Sub-problem

We implement a mixed integer programming model for the daily sub-problems. Originally, our multi-day problem is a quadratic programming problem as it is required to have balance supply distribution. In this daily problem, we re-implement the quadratic term into linear factor. Hence, we replace the term $\sum_{j\in C}\frac{(\beta_j-\bar{\beta})^2}{|C|}$ with a weighted delivered demands $\gamma_j^k \cdot s_j^{t_k} \cdot p_j^{v,t_k}$. To balance delivery priority, we use weight adjustment step which will be explained in the next subsection.

Therefore, mixed integer programming model for the daily problem is presented below. Note that at time period $t_k \in T$, all variables are considered at time t_k.

$$\text{Min.} \sum_{i,j\in C\cup\{c_0,c_0'\}} \sum_{v\in V} d_{i,j}\cdot x_{i,j}^{v,t_k} + \sum_{i\in C} M\cdot y_i^{t_k} - \sum_{v\in V}\sum_{j\in C} \gamma_j^k \cdot s_j^{t_k}\cdot p_j^{v,t_k} \tag{12}$$

Subject to

$$\sum_{v\in V}\sum_{i\in C\cup\{c_0\}} x_{i,j}^{v,t_k} + y_j^{t_k} = 1 \quad \forall j\in C \tag{13}$$

$$\sum_{j\in C\cup\{c_0'\}} x_{c_0,j}^{v,t_k} = 1 \quad \forall v\in V \tag{14}$$

$$\sum_{i\in C\cup\{c_0\}} x_{i,c_0'}^{v,t_k} = 1 \quad \forall v\in V \tag{15}$$

$$\sum_{i\in C\cup\{c_0\}} x_{i,r}^{v,t_k} = \sum_{j\in C\cup\{c_0'\}} x_{r,j}^{v,t_k} \quad \forall v\in V, r\in C \tag{16}$$

$$\sum_{j\in C} s_j^{t_k}\times p_j^{v,t_k} \le q_v \quad \forall v\in V \tag{17}$$

$$\sum_{i\in C\cup\{c_0\}} x_{i,j}^{v,t_k} \ge p_j^{v,t_k} \quad \forall v\in V, j\in C \tag{18}$$

$$\sum_{i\in C\cup\{c_0\}} x_{i,j}^{v,t_k} \le M\cdot p_j^{v,t_k} \quad \forall v\in V, j\in C \tag{19}$$

$$s_j^{t_k}\cdot p_j^{v,t_k} \ge 0.05\cdot v_v\cdot x_{i,j}^{v,t_k} \quad \forall v\in V, i\in C\cup\{c_0\}, j\in C \tag{20}$$

$$x_{i,j}^{v,t_k} + x_{j,i}^{v,t_k} \le 1 \quad \forall v\in V, i,j\in C, i\ne j \tag{21}$$

$$\delta_i^{v,t_k} + \alpha_{i,j} - (1-x_{i,j}^{v,t_k})\cdot M \le \delta_i^{v,t_k} \quad \forall v\in V, i\in C\cup\{c_0\}, j\in C\cup\{c_0'\}, i\ne j \tag{22}$$

4.3 Updating Parameters

Updating parameter step is to balance delivery distribution. The problem assumption does not require that all locations have to be visited in one day, but visits may distribute for the whole horizon. For one day plan, it is possible that some locations left unassigned or some locations may be visited but they are received partial demands.

Table 2 presents settings for γ_j^t parameter. The parameter sets delivery priority for which the assignments should prioritise a visit j over the other visits at day t when the visit has the highest γ_j^t. In this paper, we propose three parameter settings. The first setting is the case where the γ_j^t remains the same for every time period. The second setting is to double the priority value to locations that have not been visited in the time period $t-1$. The third setting is to double the

Table 2: Three setups for changing weight of locations.

Setup	Description
1	$\gamma_j^t = \gamma_j^{t-1}$ for all locations.
2	$\gamma_j^t = 2\gamma_j^{t-1}$ for all locations that have not been visited on day $t-1$.
3	$\gamma_j^t = 2\gamma_j^{t-1}$ for some locations that have not been visited on day $t-1$.

priority value for some locations that have not been visited on day $t-1$.

For all these cases, we assume that their demands are accumulated if the needs have not been delivered. Therefore, the demand parameter is to be changed by $d_i^t = d_i^{t-1} + d_i^{t-1} \cdot (1 - p_i^{v,t-1})$. For example, location a has ordered 10 units of product every day. On the first day, this location may receive the commodity for 6 units. Thus, on day 2, the location a may request this product for another 10 units. Hence, an updated demand for location a on day 2 is equal to 10+(10-6)=14.

The method was implemented in JAVA with a well-known commercial mathematical solver, IBM ILOG CPLEX Optimization Studio (CPLEX) in a process to solve the daily MIP model.

5 EXPERIMENTS AND COMPUTATIONAL RESULTS

In this section, we explain our modified datasets, experimental setups and results of the experiment.

5.1 Datasets

We use modified Solomon's datasets in our experiment. The datasets have three instance types which are randomly generated type (R), clustered type (C), and mixed type (RC). Originally, Solomon's datasets are created for a single period vehicle routing problem with time windows. The objective function of the original datasets are to find a routing plan which has the lowest number of assigned vehicles and the shortest total travel distances. The constraints of the original problem are vehicle capacity, delivery time windows and the demands for each location which every location must be served.

In our modified Solomon's dataset, a single period VRPTW is expanded to be a seven-day scheduling problem. There are four instances for each original dataset type. Each new instance has 25 visit locations and the original demands are requested repeatedly for seven days. The modified problems do not require delivery time window, in which the delivery can be made anytime in a day. Therefore, we created 12 modified seven-day instances for our experiment. We also set initial location visiting priority randomly, which had range between 1 to 3.

5.2 Results

In this section, we present results from our experiments. This experiment compared solutions of the modified Solomon's datasets which obtained from the proposed algorithm. We set the computational time to return the best feasible solution if the daily computational time exceed five minutes. Thus, the maximum computation times for a weekly problem is 35 minutes.

Table 3 presents objective values of solutions from three visiting priority setups. Each setup in the table presents four objective values, which are total travel distance (Distance), unsatisfied cost (Unsatisfied), supply distribution variance (Variance), and the total weekly cost (Weekly cost). There are 12 modified Solomon's instances presented in the table. Bold texts in the table present the best objective value among the three setups.

Consider the number of the cheapest weekly cost solutions, there are 3, 2, and 7 best solutions from Setup 1, Setup 2, and Setup 3, respectively. The weighted summation of objective values shows that the unsatisfied cost is the most influential value, followed by the percentage of delivered supply variance, and the total travel distance.

We analyze the result based on the original Solomon's dataset types which are randomly generated dataset (R), clustered generated dataset (C), and mixed generated dataset (RC).

For dataset type R, Setup 3 provided the best overall solutions, in which the Setup 3 found the lowest weekly cost solutions for three instances. The main contribution was that the Setup 3 had the lowest overall unsatisfied delivery plan. Setup 1 provided one solution with the lowest weekly cost. Also, Setup 1 had the lowest travel distances for every solution for this dataset type. On the other hand, Setup 2 did not find the best solution and had the highest weekly cost.

For data type C, Setup 2 had the best solution for two instances when comparing the weekly cost while Setup 1 and Setup 3 had the best solution for one instance for each setup. Setup 3 also had solution with the shortest distance for three instances.

For data type RC, Setup 3 provided the minimum unsatisfied cost, variance and weekly cost of three datasets. Setup 1 and Setup 2 had two solutions providing the shortest distances. From the result, Setup 2

Table 3: Solution weighted costs for the modified Solomon's Dataset.

Datasets	Setup 1				Setup 2				Setup 3			
	Distance	Unsatisfied	Variance	Weekly cost	Distance	Unsatisfied	Variance	Weekly cost	Distance	Unsatisfied	Variance	Weekly cost
Dataset 1-Type R	**3,717**	5,257,411	39,755	5,300,883	3,789	6,519,331	53,007	6,576,127	3,999	**4,849,787**	**26,504**	**4,880,290**
Dataset 2-Type R	**3,751**	**11,555,526**	**92,762**	**11,652,039**	4,709	12,321,124	92,762	12,418,595	4,036	11,780,452	92,762	11,877,251
Dataset 3-Type R	**3,465**	8,486,067	66,259	8,555,791	4,058	8,170,676	79,511	8,254,244	4,091	**7,094,634**	**53,007**	**7,151,732**
Dataset 4-Type R	**2,670**	9,910,719	**66,259**	9,979,648	3,467	11,361,697	106,014	11,471,178	3,042	**9,741,450**	119,266	**9,863,757**
Dataset 1-Type C	2,510	**19,648,281**	159,021	19,809,813	2,460	19,660,473	**145,769**	**19,808,702**	**2,320**	19,682,029	172,273	19,856,622
Dataset 2-Type C	**2,276**	**12,176,945**	119,266	**12,298,487**	2,760	13,080,714	106,014	13,189,488	2,627	12,211,753	**92,762**	12,307,142
Dataset 3-Type C	3,482	24,555,669	172,273	24,731,424	3,445	**24,554,433**	**172,273**	**24,730,150**	**3,393**	24,610,267	172,273	24,785,932
Dataset 4-Type C	3,791	18,510,574	145,769	18,660,135	3,696	19,305,326	145,769	19,454,791	**3,595**	**18,402,617**	**145,769**	**18,551,981**
Dataset 1-Type RC	**3,603**	**29,517,301**	185,525	**29,706,428**	3,636	29,551,403	198,776	29,753,815	4,789	29,610,417	**172,273**	29,787,479
Dataset 2-Type RC	**4,550**	18,110,902	159,021	18,274,473	4,554	17,572,174	159,021	17,735,749	4,899	**15,971,893**	**145,769**	**16,122,561**
Dataset 3-Type RC	3,779	6,944,624	66,259	7,014,661	**3,560**	6,980,138	66,259	7,049,957	3,625	**6,907,342**	**66,259**	**6,977,226**
Dataset 4-Type RC	3,504	12,491,983	**92,762**	12,588,249	**3,468**	13,007,388	106,014	13,116,870	3,551	**12,118,460**	145,769	**12,267,781**

where $\omega_1 = 1$, $\omega_2 = 17669$ and $\omega_3 = 1325175$

Remark: Bold texts present the best objective value among the three setups.

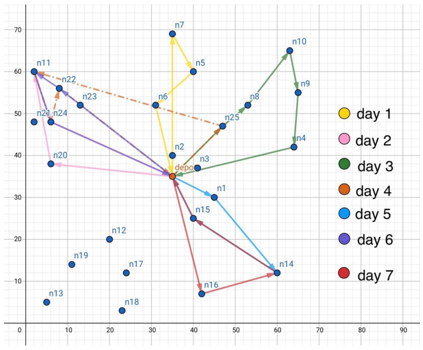

Figure 1: A routing plan of vehicle 1 for dataset 2 type R.

did not perform well when considered weekly costs, unsatisfied demands and supply distribution variance.

Overall, Setup 3 found solutions with the lowest weekly cost, which contributed from low unsatisfied demands and delivered supply variance. Setup 1 had the shortest total distance.

As we mention that we preferred a solution with equal supplies distribution. From this view, we can conclude that Setup 3 is the best setup because it provided the best solution for seven instances. From Table 3, we recommended that the dataset type R should be solved by applying Setup 1 because it provides the best overall costs. Next, we present a solution from using Setup 1 to solve Dataset 2-Type R.

Figure 1 illustrates a 7-day vehicle scheduling solution for vehicle 1 applying setup 1 to solve Dataset 2-Type R. The instance has 25 visiting locations and one service depot. There are six vehicles, each has capacity of 50 units. From the figure, vehicle 1 is deployed every day. There are locations where the vehicle has visited multiple days such as a path from location n_{14} to location n_{15} and return to depot on day 5 and day 7. The solution also shows the case where the visiting order may not so efficient in term of travelling distances, such as a route on day 1 (yellow path). This is because the sub-problem solution is not the optimal solution because the mathematical solver reaches the

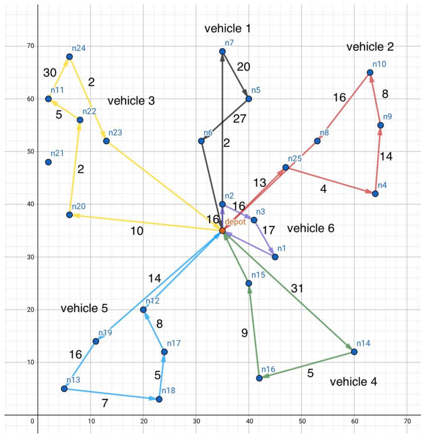

Figure 2: A routing plan of all vehicles on the first period using Setup 1.

computation time limit.

Figure 2 displays a one-day plan of all vehicles. Each line represents a vehicle path to delivery commodities. Numbers on each line represent the number of commodity to delivery at destination location. For example, the plan for vehicle 5 is to delivery 14 units at location n_{19}, 16 units at location n_{13}, 7 units at location n_{18}, 5 units at location n_{17}, 8 units at location n_{12} and return to depot. From this one-day plan, six vehicles make visits to 24 locations and only one location, n_{21}, has not been visited on day 1. The plan to visit location n_{21} is on day 3 and day 6.

6 CONCLUSIONS AND FUTURE WORK

This paper explains a multi-day distribution problem while supplies are shortage or there are transportation difficulties limiting supply delivery. The aim is to build visiting plans having fair supply distribution over planning horizon. We assume a constant demand from every location every day, which then accumulated with unsatisfactory demands from previous days.

We propose a hybrid heuristic method to tackle this multi-day problem as multiple daily sub-problems. Each sub-problem was solved by a mixed-integer programming model. We applied heuristic delivery priority distribution, so that the locations that had not been visited on previous days had higher visiting priority. There were three priority distribution setups applied in our experiment, which were double the priority every locations that had not been visited, double the priority for some locations that had not been visited, and no priority changes as a control group. We tested our proposed method with 12 modified Solomon's instances.

Our future works includes expanding problem cases, improving the current algorithm and exploring other solution approaches. To expand problem cases, we may added due date and time window constraints can be included as our future works. It is necessary to improve the current algorithm to obtain an optimal solution. Our proposed algorithm can also be improved to tackle larger problems. Also, our future works will apply other heuristic methods to this problem, such as neighbourhood search, genetic algorithm, and constructive heuristic methods to compare results with this hybrid heuristic approach.

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