Influence of Sampling Point Setting on Fitting Error of Ideal Gaussian Beam

Yan Baozhu, Liu Wenguang, Zhou Qiong, Sun Quan and Yang Yi
College of Advanced Interdisciplinary Studies, National University of Defense Technology, Changsha, Hunan, P.R. China
azhu711@163.com

Keywords: Gaussian Beam, Least Square Fitting, Fitting Error, Sampling Point.

Abstract: The setting of sampling points on the detector array will affect the fitting error of Gaussian beam. Based on MATLAB and least square method, the fitting of ideal Gaussian beam in one dimension and two dimensions was simulated, and the influence of sampling point interval on the fitting error of characteristic parameters, such as facula center, facula radius and center power density, were studied. The results show that, the number of sampling points in the two-dimensional simulation is greater, so the fitting accuracy is better than that in the one-dimensional simulation under the same condition of sampling point interval. In the range of initial conditions of simulation calculation, the interval of sampling points shall be d⩽50mm, then the fitting error would be controlled within the range of admissible one.

1 INTRODUCTION

Generally, the detector array is used to measure the power density distribution of the facula(C. Higgs, P.C. Grey, J.G. Mooney. 1999; J. Thomas Knudtson, Kenneth L. Ratzlaff. 1983). The characteristic parameters of the facula are acquried by the least square fitting. The setting of sampling points on detector array will affect the fitting error of facula. It can be predicted that, the smaller the interval between sampling points, and the larger the sampling range, the smaller the fitting error of facula. In the design of detector array, due to the limitation of single detector size and data processing capacity, the sampling point interval cannot be small infinitely, and the sampling range cannot be large infinitely, so it is necessary to make a balance between sampling point setting and fitting error. It is great to use as few sampling points as possible to obtain the fitting error that meets the requirements. In this paper, based on MATLAB and least square method, one-dimensional and two-dimensional simulation are carried out for the sampling point setting on the Gaussian facula, and the influence of sampling point interval on the fitting error is studied.

2 ONE-DIMENSIONAL SIMULATION

2.1 Calculation Method of One-dimensional Simulation

The general expression of power density distribution function of one-dimensional Gaussian beam is (Bingkun Zhou, Yizhi Gao, Tirong Chen, et al.. 2000):

$$I(x) = I_0 \cdot \exp\left(-\frac{2(x-x_0)^2}{w_0^2}\right) \tag{1}$$

Where x_0 is the facula center, w_0 is the facula radius, and I_0 is the center power density.

First of all, equation (1) is transformed by logarithm operation on both sides of the equal sign(Bing Kong, Zhao Wang, Yusan Tan, 2002), then:

$$z = \ln I(x) = ax^2 + bx + c, \quad \begin{cases} a = -\dfrac{2}{w_0^2}, \\ b = \dfrac{4x_0}{w_0^2} \\ c = \ln I_0 - \dfrac{2x_0^2}{w_0^2} \end{cases} \tag{2}$$

The coefficients of a, b and c can be obtained by the second order polynomial fitting of the transformed function. According to the relationship between the transformed function and the original Gaussian function, the corresponding parameters of the Gaussian function can be obtained as follow:

$$I_0 = \exp\left(c - \frac{b^2}{4a}\right), x_0 = -\frac{b}{2a}, w_0 = \sqrt{-\frac{2}{a}} \qquad (3)$$

The position of the sampling point on the actual detector array is given, and the facula wobbles within a certain range. For the convenience of calculation, it is assumed that the facula is fixed, and sampling points are set uniformly. In this way, three parameters are involved in the distribution of sampling points, as shown in Figure 1. They are d, wr, and h, which stand for the interval of sampling points, the ratio of sampling point range to facula diameter, and the distance between the facula center and the sampling point on the left side of it. Parameters of d and wr are related to the design of detector array. Parameter of h changes randomly in the actual measurement, with the range of 0~d/2.

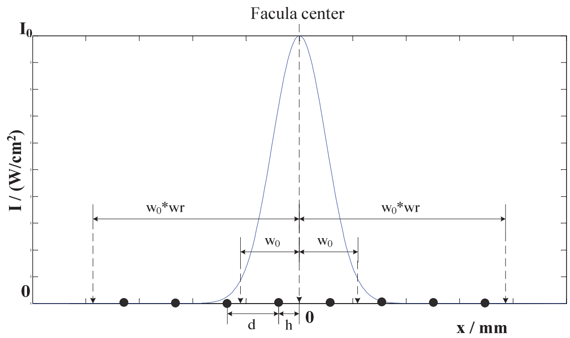

Figure 1: Physical meaning of parameters in simulation calculation.

For each set of d, wr and h, the coordinates x_i of each sampling point can be determined. Then the true value $I(x_i)$ of each sampling point can be obtained by substituting x_i to formula (1). There is a certain measurement error for each sampling points, which follows the normal distribution with the mean value of 0 and the standard deviation of δ_0. A group of random error values δ_i (i=1, 2, ⋯, n, n is the number of sampling points) that meet the above normal distribution are selected, so the measured value of each sampling point is $I'(x_i) = I(x_i)*(1+\delta_i)$. Substituting it into formula (2), $z'(x_i)=\ln(I'(x_i))$ is obtained. In MATLAB, the least square fitting of (x_i, $z'(x_i)$) is carried out using the polyfit function (Shenyong Ruan, Yongli Wang, Qunfang Sang, 2004), and three coefficients of a_1, b_1 and c_1 are obtained. Then, the fitting coefficients x_{01}, w_{01} and I_{01} are calculated according to formula (3). Here, the subscript "1" represents the fitting result.

Therefore, the power density distribution function obtained by fitting is:

$$I_1(x) = I_{01} \cdot \exp\left(-\frac{2(x - x_{01})^2}{w_{01}^2}\right) \qquad (4)$$

Compared with the true value in formula (1), the fitting error values of facula center ($x_{01}-x_0$), facula radius ($w_{01}-w_0$), and center power density ($I_{01}-I_0$) are obtained.

A number of m groups of random error are selected, and a group of fitting error, such as $(x_{01}-x_0)_j$, $(w_{01}-w_0)_j$ and $(I_{01}-I_0)_j$, j = 1, 2, ... , m, is obtained for each group of random error according to the above process. Then the fitting error under the conditions of d, wr and h is acquired by the standard deviation of m groups of fitting error values is calculated.

2.2 Initial Conditions of One-dimensional Simulation

The initial conditions used in the calculation are:
1) The facula center x_0=0. The facula radius w_0=50mm. The center power density I_0=100mW/cm^2.
2) The ratio of the sampling point range to the facula diameter wr=2. The sampling point interval d is changed from 30mm to 60mm, and the step is 1mm. The distance between the facula center and the sampling point on the left side of it h is changed from 0 to d/2, and the step is d/8.
3) the standard deviation of sampling point error is δ_0=15%.
4) The number of groups of random error m = 10000.

2.3 Results of One-dimensional Simulation

When d=30mm, wr=2, h=0mm, a group of random error of sampling points is selected, and the fitting result for the measured values of sampling points is as Figure 2:

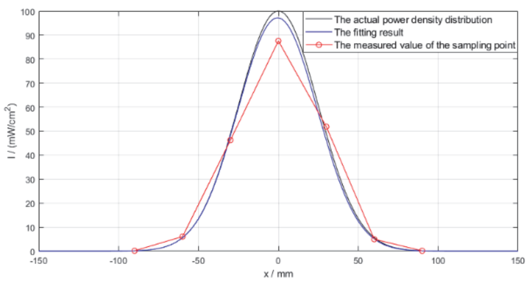

Figure 2: Result of one-dimensional fitting.

In the figure, the black line represents the actual power density distribution, the red circle represents the measured value of the sampling point, and the blue line represents the power density distribution obtained by fitting the measured value of the sampling points, where x_{01}=0.46mm, w_{01}=49.80mm and I_{01}=97.18mW/cm^2, and the fitting errors are (x_{01}-x_0)=0.46mm, (w_{01}-w_0)=-0.20mm, and (I_{01}-I_0)=-2.82mW/cm^2.

A number of m=10000 groups of random error values are selected, and a number of m=10000 groups of the fitting error is obtained. Then the fitting error under the conditions of d=30mm, wr=2 and h=0mm is obtained: the fitting error of the facula center is δ_{x0}=0.61mm, the fitting error of the facula radius is δ_{w0}=0.59mm, and the fitting error of the center power density is δ_{I0}=8.73mW/cm^2.

When d and h change in the calculation range, the simulation results of the fitting errors are as shown in Figure 3 ~ Figure 5.

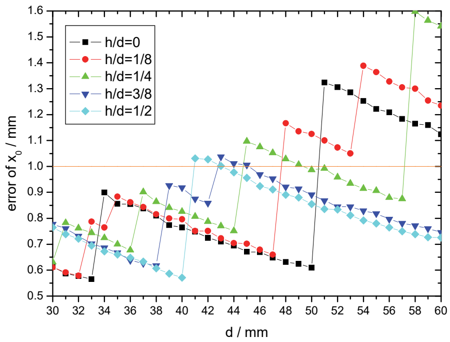

Figure 3: Fitting error of the facula center for one-dimensional simulation.

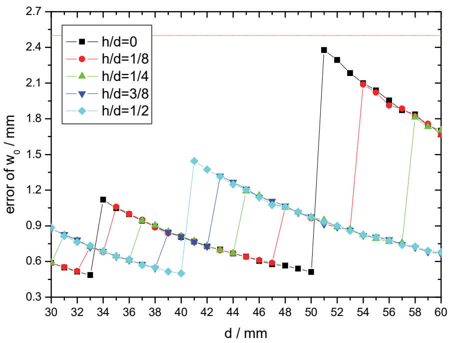

Figure 4: Fitting error of the facula radius for one-dimensional simulation.

It can be seen from the figure:

1) The relationship between fitting error and sampling interval is not monotonous increasing or decreasing, but segmented. With the increase of d, the overall errors of the next section is higher than that of the

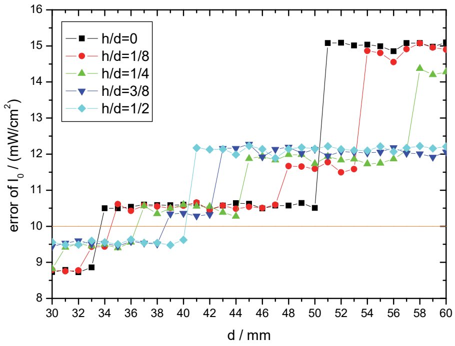

Figure 5: Fitting error of the center power density for one-dimensional simulation.

previous section. While in a certain section, it is basically monotonic decreasing. This is because in a certain range, the number of sampling points is constant. The larger d is, the wider the distribution of sampling points is, the more information is detected, and the smaller the fitting error is. When d increases to a certain value, because wr is limited to 2, the number of sampling points decreases, so the detection information decreases, and the fitting error suddenly increases. Taking h=0 as an example, the range of sampling points is limited to w_0*wr=50*2=100mm. When d=30-33mm, there are three sampling points on both sides of the facula center. When d=34-50mm, there are two sampling points on both sides of the facula center. When d=51-60mm, there is one sampling point on both sides of the facula center. Therefore, the boundary between segments are between d=33mm and d=34mm, d=50mm and d=51mm.

2) It is assumed that, the admissible errors of the facula center, facula radius and center power density are 1mm, 2.5mm (5%) and 10mW/cm^2 (10%), respectively, which are represented by solid red lines in the figures. When selecting the interval of sampling points d, the fitting errors should not exceed the admissible ones under all h conditions. It can be seen from the figures that, within the initial condition range of simulation calculation, the fitting error of facula radius is smaller than the admissible one, and the fitting errors of facula center and center power density do not exceed the admissible ones when d$\leqslant$40mm and d$\leqslant$33mm respectively. Therefore, d$\leqslant$33mm should be selected to ensure the fitting accuracy.

3 TWO-DIMENSIONAL SIMULATION

3.1 Calculation Method of Two-dimensional Simulation

The general expression of the power density distribution function of the two-dimensional Gaussian beam cross-section facula is as follows:

$$I(x,y)=I_0\cdot\exp\left(-\frac{2(x-x_0)^2}{w_{0x}^2}-\frac{2(y-y_0)^2}{w_{0y}^2}\right)\tag{5}$$

Where (x_0, y_0) is the facula center, (w_{0x}, w_{0y}) is the facula radius in the X and Y directions, and I_0 is the center power density.

Similar to the one-dimensional simulation calculation, equation (5) is transformed by logarithm operation on both sides of the equal sign.

$$z=\ln I(x,y)=ax^2+bx+cy^2+dy+e,\quad \begin{cases} a=-\dfrac{2}{w_{0x}^2}, \\ b=\dfrac{4x_0}{w_{0x}^2} \\ c=-\dfrac{2}{w_{0y}^2} \\ d=\dfrac{4y_0}{w_{0y}^2} \\ e=\ln I_0-\dfrac{2x_0^2}{w_{0x}^2}-\dfrac{2y_0^2}{w_{0y}^2} \end{cases}\tag{6}$$

The coefficients a, b, c, d and e can be obtained by the second order polynomial fitting of the transformed function. According to the relationship between the transformed function and the original Gaussian function, the corresponding parameters of the Gaussian function can be obtained as follow:

$$\begin{aligned} &I_0=\exp\left(e-\frac{b^2}{4a}-\frac{d^2}{4c}\right), x_0=-\frac{b}{2a}, \\ &w_{0x}=\sqrt{-\frac{2}{a}}, y_0=-\frac{d}{2c}, w_{0x}=\sqrt{-\frac{2}{c}} \end{aligned}\tag{7}$$

Similar to the one-dimensional simulation calculation shown in Figure 1, four parameters are involved in the two-dimensional distribution of sampling points.They are the interval of sampling points d, the ratio of sampling point range to facula diameter wr, the distance between the facula center and the sampling point on the left side of it hx, and the distance between the facula center and the sampling point under it hy.Parameters of d and wr are related to the design of detector array. Parameters of hx and hy change randomly in the actual measurement, with the range of 0~d/2.

For each set of d, wr, hx and hy, the coordinates (x_i, y_j) of each sampling point can be determined, where i=1, 2,... , nx, j=1, 2,... , ny, nx and ny represent the number of columns and rows of sampling points respectively. The true value I(x_i, y_j) of each sampling point is obtained by substituting (x_i, y_j) to formula (5). A group of random error values δ_{ij} are selected, and the measured value of each sampling point is I'(x_i, y_j)= I(x_i, y_j)*(1+δ_{ij}). Substituting it into formula (6), z'(x_i, y_j)=ln(I'(x_i, y_j)) is obtained. Then the least square fitting of ((x_i , y_j) , z'(x_i, y_j)) is carried out, and five coefficients of a_1, b_1, c_1, d_1 and e_1 are obtained. So, the fitting coefficients (x_{01}, y_{01}), (w_{0x1}, w_{0y1}) and I_{01} are calculated according to formula (7).

So far, the power density distribution function of the two-dimensional Gaussian beam cross-section facula is obtained by fitting:

$$I_1(x,y)=I_{01}\cdot\exp\left[-\frac{2(x-x_{01})^2}{w_{0x1}^2}-\frac{2(y-y_{01})^2}{w_{0y1}^2}\right]\tag{8}$$

Compared with the true value in formula (5), the fitting errors of facula center (x_{01}-x_0, y_{01}-y_0), facula radius (w_{0x1}-w_{0x}, w_{0y1}-w_{0y}), and center power density (I_{01}-I_0) are obtained.

A number of m groups of random error are selected, and a group of fitting error, such as (x_{01}-x_0, y_{01}-y_0)$_j$, (w_{0x1}-w_{0x}, w_{0y1}-w_{0y})$_j$ and (I_{01}-I_0)$_j$, j = 1, 2, ... , m, is obtained for each group of random error according to the above process. Then The fitting error under the conditions of d, wr hx and hy is acquired by the standard deviation of m groups of fitting error values is calculated.

3.2 Initial Conditions of Two-dimensional Simulation

The initial conditions used in the calculation are:

1) The facula center (x_0,y_0)=(0,0). The facula radius w_{0x}=w_{0y}=50mm. The center power density I_0=100mW/cm^2.

2) The ratio of the sampling point range to the facula diameter wr=2. The sampling point interval d is changed from 30mm to 60mm, and the step is 1mm. The distance between the facula center and the sampling point on the left side of it hx is changed from 0 to d/2, and the step is d/4. The distance between the facula center and the sampling point under it hy is changed from 0 to hx, and the step is d/4.

3) the standard deviation of sampling point error is δ_0=15%.

4) The number of groups of random error m = 10000.

3.3 Results of Two-dimensional Simulation

The actual distribution of the power density of the two-dimensional Gaussian facula is shown in Figure 6. When d=30mm, wr=2, hx=hy=0mm, a group of random error of sampling points is selected, and the measured value of sampling points is obtain as shown in the red dot in Figure 6. The power density distribution obtained by fitting the measured value of the sampling points is shown in Figure 7, where x_{01}=0.30mm, w_{0x1}=49.82mm, y_{01}=0.32mm, w_{0y1}=49.98mm and I_{01}=101.00mW/cm^2, and the fitting errors are $(x_{01}-x_0)$=0.30mm, $(w_{0x1}-w_{0x})$=-0.18mm, $(y_{01}-y_0)$=0.32mm, $(w_{0y1}-w_{0y})$=-0.02mm and $(I_{01}-I_0)$=1.00mW/cm^2, respectively.

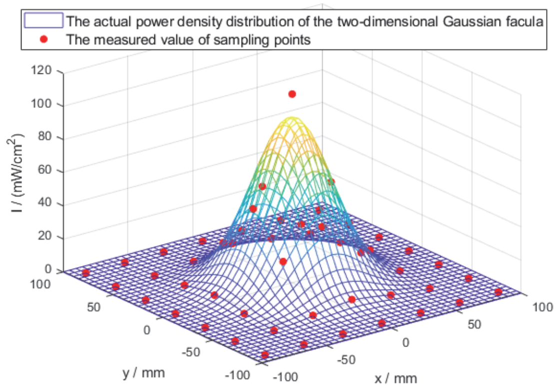

Figure 6: Gaussian facula and measured value of sampling points.

A number of m=10000 groups of random error values are selected, and a number of m=10000 groups of the fitting errors $(x_{01}-x_0)_j$, $(w_{0x1}-w_{0x})_j$, $(y_{01}-y_0)_j$, $(w_{0y1}-w_{0y})_j$ and $(I_{01}-I_0)_j$ are obtained. Then the fitting errors under the conditions of d=30mm, wr=2 and hx=hy=0mm are obtained: the fitting error of the facula center is $(\delta_{xo}, \delta_{y0})$=(0.23mm, 0.23mm), the fitting error of the facula radius is $(\delta_{w0x}, \delta_{w0y})$=(0.22mm, 0.22mm), and the fitting error of the center power density is δ_{I0}=4.16mW/cm^2.

When d, hx and hy change in the calculation range, the simulation results of the fitting errors are as shown in Figure 8 ~ Figure 12.

It can be seen from the above data:

1) Within the range of initial conditions of simulation calculation, the fitting error of two-dimensional simulation is less than that of one-dimensional simulation. The reason is: there are more sampling points distributed in two-dimensional, and more

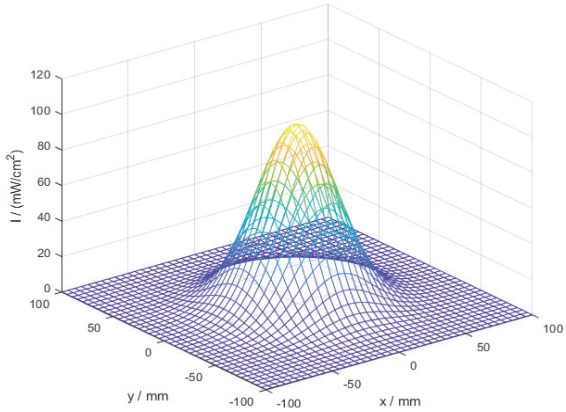

Figure 7: Result of two-dimensional fitting.

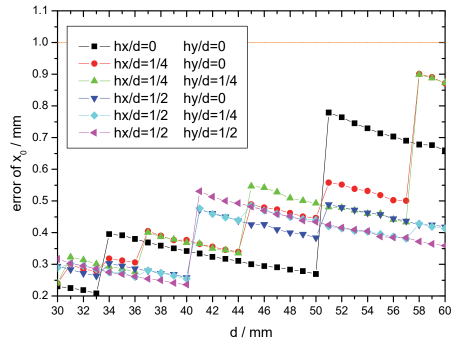

Figure 8: Fitting error of the facula center x_0 for two-dimensional simulation.

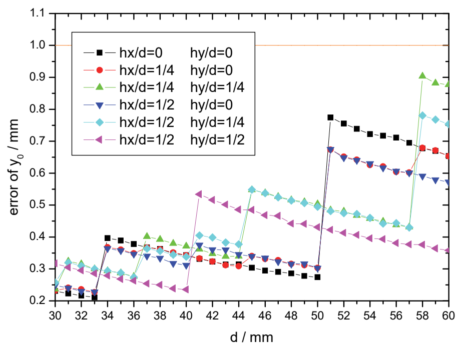

Figure 9: Fitting error of the facula center y_0 for two-dimensional simulation.

information is detected, then the fitting error is smaller.

2) The relationship between the fitting error and the interval of sampling points is not monotonous increasing or decreasing, but segmented. Because the number of sensors has changed. With the increase of d, the overall error of the next section is higher than that of the previous section. While in a certain section, it is basically monotonic decreasing. This is consistent with one-dimensional simulation.

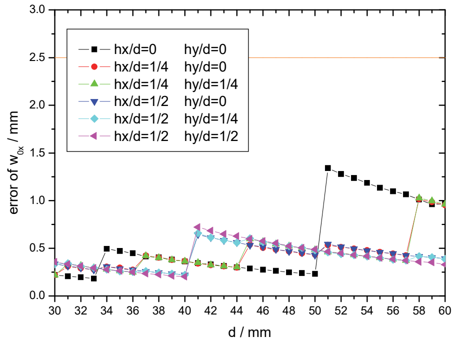

Figure 10: Fitting error of the facula radius w_{0x} for two-dimensional simulation.

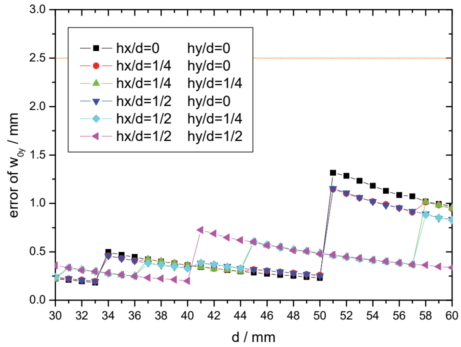

Figure 11: Fitting error of the facula radius w_{0y} for two-dimensional simulation.

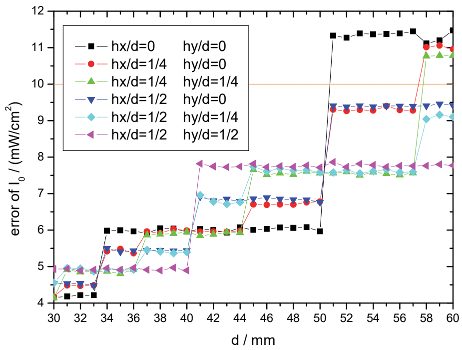

Figure 12: Fitting error of the center power density for two-dimensional simulation.

3) It is assumed that, the admissible errors of the facula center, facula radius and center power density are 1mm, 2.5mm (5%) and 10mW/cm^2 (10%), respectively, which are represented by solid red lines in the figure. When selecting the interval of sampling points d, the fitting errors should not exceed the admissible ones under all hx and hy conditions. It can be seen from the figure that, in the initial condition range of the simulation calculation, the fitting errors of the facula center and the facula radius are smaller than the admissible ones, and the fitting error of the center power density exceeds the admissible one when d>50mm. Therefore, in the design, d≤50mm should be selected to ensure that all fitting errors do not exceed the admissible ones. The interval range of sampling points is larger than that allowed by one-dimensional simulation.

4 CONCLUSIONS

Based on MATLAB and least square method, the one-dimensional and two-dimensional fitting of Gaussian distribution facula are carried out. The influence of sampling point layout on fitting error is studied, and the relationship between the fitting error and sampling point interval is analyzed. The results show that, the number of sampling points in the two-dimensional simulation is larger, and the fitting accuracy is better than that in the one-dimensional simulation under the same sampling point interval. Because the actual facula is a two-dimensional Gaussian distribution, the two-dimensional simulation results shall be prioritized.

In the range of initial conditions of simulation calculation, the interval of sampling points should be d ≤50mm, so that the fitting errors of facula center, facula radius and center power density can be controlled within the admissible ones.

REFERENCES

Bingkun Zhou, Yizhi Gao, Tirong Chen, et al.. 2000. Laser principle(4th edition), National Denfense Industry Press, Beijing.

Shenyong Ruan, Yongli Wang, Qunfang Sang, 2004. MATLAB program design, Publishing house of electronics industry, Beijing.

Lili Wang, Zhongwen Hu, Hangxin Ji, 2012. Laser facula center location algorithm based on Gaussian fitting, Journal of Applied Optics, 33(5), 985.

Bing Kong, Zhao Wang, Yusan Tan, 2002. Gaussian fitting technique of laser facula, Laser Technology, 26(4),277.

C. Higgs, P.C. Grey, J.G. Mooney. 1999. Dynamic target board for ABL ACT performance characterization, Proceeding of SPIE, 3706, 216

J. Thomas Knudtson, Kenneth L. Ratzlaff. 1983. Laser beam spatial profile analysis using a two-dimensional photodiode array, Rev.Sci.Instrum, 54(7),856