One Step Ahead Optimal Control of a Single Echelon Supply Chain using Mathematical Programming

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Keywords: Supply Chain, Mathematical Programming, Optimal Control, Discrete-time Dynamical System.

Abstract: A single echelon supply chain model problem, consisting of a store with known inventory and shipping capacities, a known delivery delay or lead time and a random demand for a product at the store is formulated as an optimal control problem. In the practical case when only current and past demands are known, using the concept of one step ahead optimal control, the problem is reformulated as the mathematical programming problem of maximizing economic value added (EVA), subject to the dynamics and constraints, such as inventory size. Illustrative examples are given and performance indices are proposed to evaluate the performance of the proposed controller, which exhibits good efficiency and no bullwhip effect.

1 INTRODUCTION

(McGarvey and Hannon, 2004) describe a simple supply chain problem in the following terms: "*One of our most difficult issues involves understanding the complex way in which the world of the customer connects with the world of retail business. The vagaries of pleasing customers on the one hand while dealing with suppliers on the other can sometimes seem overwhelming. Charge too much or not have the article in stock, and you may never see the customer again. Order too much or too often, and you may run up the inventory cost, causing profits to vanish. This is a balancing act if ever there was one. How can you walk that tightrope and survive? Reliance on mental models of a process fraught with randomness, feedbacks, and delays makes the most astute of mental management models unreliable.*"

In this paper we will formulate the single echelon supply chain control problem discussed in (McGarvey and Hannon, 2004, p.179-187) as an optimal control problem, when the demand is known over the entire planning horizon and then propose feedback controllers for unknown demands, based on this approach. There is a large and ever-growing literature on control of supply chains and a brief review is given in the following section, focusing only on the papers most relevant to the approach proposed here.

1.1 Literature Review

(Ivanov et al., 2018) in their recent survey write "*Modern production and logistics systems, supply chains, and Industry 4.0 networks are challenged by increased uncertainty and risks, multiple feedback cycles, and dynamics. Control theory is an interesting research avenue which contributes to further insights concerning the management of the given challenges in operations and supply chain management.*" They also identify one of the main contributions of control theory as being the application of dynamic feedback control to production-inventory systems and point out that a wide range of control-theoretic tools have been used in this context, ranging from the classical PID control to model predictive control. We also point the reader to the recent survey (Lin et al., 2017) on control-theoretic approaches to the inventory control problem.

In the specific area of feedback control of a supply chain, based on a state model of its dynamics, most of the literature has been devoted to models that apply standard control ideas such as set point control, which involves choosing set points or targets for system variables, such as inventory level. These approaches can be viewed as parametrized versions of several standard algorithms from the SCM literature, such as order-up-to-inventory control. A large class of

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such controllers is known by the acronym IOBPCS, which stands for inventory and order based production control system. Various prefixes, such as AP (for automatic pipeline) and MP (for matched parameter) identify variants of the basic IOBPCS scheme. Papers in this area also assume linear models and study the effect of the controller on the bullwhip effect and on stability with regard to, for example, step changes in the demand: see, for example, (Lin et al., 2019; Lin and Naim, 2019; Hoberg et al., 2007).

The model predictive control (MPC) approach uses a model of the real plant, as well as past and current system data to successively predict on some horizon; optimize some suitable performance index, on a (possibly different) horizon, using the predictions; and, finally, apply one or more of the computed optimal controls to the plant, before beginning a new predict-optimize-control sequence. For examples of this approach, see (Wang et al., 2007) and references therein. If the performance index takes economic factors into account, the MPC approach is referred to as economic MPC and a recent example in the supply chain context is (Subramanian et al., 2014).

This paper uses an approach that is related to the MPC approach; however, it uses only past values of the demand and system state, does not use a predictor, and uses a stage cost that is economic: it is the so called Economic Value Added (EVA) performance index, commonly used in the business and financial world (McGarvey and Hannon, 2004). The main idea, which can be described by the term greedy control (Lavretsky, 2000), is to optimize EVA using past demands, for just one step ahead of the current time, apply this optimal control to the system, update its state and repeat the cycle of one step ahead optimization of EVA, until the end of the horizon. Note that the control scheme just described is set up to compute an ordering decision (or control) in real time, although it can also be used in a planning mode. The main differences between the approach of this paper and existing results are as follows: (i) an economic stage cost function is used, similar to the one in (Subramanian et al., 2014), but also including discounting of the profit to its present value and **not** including any target or tracking costs, (ii) using only past demands and the current demand to compute the present value of the control, **not** involving any prediction, in contrast with (Wang et al., 2007). The result is a real-time control that is computationally cheap and, as will be shown, efficient when compared with the globally optimal omniscient control, which will be defined in the sequel.

2 DISCRETE-TIME SUPPLY CHAIN DYNAMICS

The supply chain considered in this paper is that of a store, the example being taken from (McGarvey and Hannon, 2004). It has two main components: a delay between ordering and receiving, corresponding to a block named conveyor or pipeline and an accumulator, which receives the ordered items after the stipulated delay, corresponding to the store. Selling of items in the store occurs in accordance with the demand and it is assumed that all but w items (the ones in the display windows) in the store can be sold. The store manager's objective is to choose an ordering sequence, in response to the demand, that maximizes cumulative profit over a given time horizon, assuming that there are costs associated to handling, shipping, storage and shortage (being out-of-stock affects sales negatively, since customers are turned away). Finally, profits (=sales revenues minus costs) are discounted using a fixed interest rate and then added over the given horizon to obtain the cumulative profit, referred to as economic value added (EVA). The following notation will be used to formulate the mathematical model.

– k: discrete time counter.
– $x_i(k)$: quantity of items in the ith stage of shipping, $i = 1,\ldots,D$, where $i = 1$ corresponds to receiving and $i = D+1$ to ordering.
– D: delay between receiving and ordering.
– $s_t(k)$: stock level in store at discrete time instant k.
– $d(k)$: customer demand at store at time k.
– $s_e(k)$: sales (selling) at time k.
– p: price per unit of material sold.
– w: quantity of items to be displayed in the store window (and not sold), also called display stock.
– $c_{os}(k)$: out-of-stock cost at time k.
– $\alpha_{os}(k)$: out-of-stock cost per unit of demand not met at time k.
– $c_h(k)$: handling cost at time k.
– α_h: handling cost per unit of material handled.
– $c_{st}(k)$: cost of maintaining stock level in store at time k.
– α_{st}: storage cost per unit of material stored.
– $c_{sh}(k)$: shipping cost of material in transit.
– α_{sh}: shipping cost per unit of material being shipped.
– $c_t(k)$: total cost at time k.
– $d_p(k)$: discounted profit at time k.
– ρ: discount rate.
– $L(k)$: cumulative profit at time k.
– K_f: horizon over which cumulative profit is to be maximized.

The delay between ordering and receiving is written by introducing a string of D one unit delays as follows:

$$x_i(k+1) = x_{i+1}(k), \;\; i = 1, \ldots, D \qquad (1)$$

$$x_{D+1}(k) = o(k) \qquad (2)$$

Thus the quantity of material in transit (i.e., being shipped) at time k, denoted by $s_h(k)$, can be written as:

$$s_h(k) = \sum_{i=1}^{D} x_i(k), \qquad (3)$$

while the quantity of material being received at the store is $x_1(k)$ and corresponds to an order placed D time units in the past, so that the variable ordering, denoted $o(k)$ is just $x_{D+1}(k)$.

Evolution of the stock level in the store at time k is determined by the quantity of material being received ($x_1(k)$) plus the quantity currently in the store $s_t(k)$ (setting aside a quantity w to be put in the display window and not sold) less the quantity sold ($s_e(k)$) in response to the demand. The corresponding equation is as follows.

$$s_t(k+1) = s_t(k) - w + x_1(k) - s_e(k) \qquad (4)$$

In order to describe the sales $s_e(k)$ at time k, observe that sales can only occur if the store level $s_t(k)$ plus the received material $x_1(k)$ less the quantity in the display window w is greater than the demand $d(k)$. Thus sales can be written as follows:

$$s_e(k) = d(k) + \min\{0, s_t(k) - w + x_1(k) - d(k)\}. \quad (5)$$

Thus the quantity of sales matches the demand $d(k)$ if it is less than the effective available store level $s_t(k) - w + x_1(k)$, otherwise it is just equal to the latter.

The costs are expressed as follows:

$$c_o(k) = \alpha_{os}(d(k) - s_e(k)) \quad \text{(out of stock cost)} \quad (6)$$

$$c_{st}(k) = \alpha_{st} s_t(k) \quad \text{(storage cost)} \quad (7)$$

$$c_h(k) = \alpha_h(x_1(k) + o(k)) \quad \text{(handling cost)} \quad (8)$$

$$c_{sh}(k) = \alpha_{sh} \sum_{i=1}^{D} x_i(k) \quad \text{(shipping cost)} \quad (9)$$

$$c_t(k) = c_o(k) + c_{st}(k) + c_h(k) + c_{sh}(k) \quad \text{(total cost)} \quad (10)$$

Discounted profit is the revenue from sales less the total cost (all at time k), appropriately discounted:

$$d_p(k) = (p s_e(k) - c_t(k)) e^{-\rho k}. \qquad (11)$$

The cumulative profit $L(k)$ accumulates the discounted profit:

$$L(k+1) = L(k) + d_p(k). \qquad (12)$$

The objective function is simply the cumulative profit at the end of the time horizon and thus, **given the demand** $d(k), k = 1, \ldots, K_f - 1$ **over the whole planning horizon**, the optimization problem to be solved is an optimal control problem, which will be referred to as the **omniscient** optimal control problem (since the demand over the entire planning horizon is assumed to be known), and can be written as follows:

$$\max_{\{o(k)\}_{k=1}^{K_f-1}} L(K_f) \qquad (13)$$

In optimal control terminology, this is a problem of choosing an optimal ordering sequence that maximizes the final cumulative profit $L(K_f)$. More realistically, the following constraints are also needed:

$$o(k) \geq 0, \quad k = 1, \ldots, K_f - 1 \qquad (14)$$

$$s_h(k) \leq C_{max\_ship}, \quad k = 1, \ldots, K_f - 1 \qquad (15)$$

$$s_t(k) \leq C_{max\_store}, \quad k = 1, \ldots, K_f - 1 \qquad (16)$$

For future reference, the **state vector** of the supply chain at instant k is denoted as $\mathbf{z}(k)$ and defined as:

$$\mathbf{z}(k) = [x_1(k), \ldots, x_D(k), s_t(k)] \qquad (17)$$

3 ONE STEP AHEAD OPTIMAL CONTROL

In practice, of course, only current and past demands are known to the supply chain manager, who has to make the ordering decision based on this information and observation of the current variables, such as inventory or store level, amount of goods in the shipping pipeline, sales levels, etc. This section formulates the so called **one step ahead optimal control (OSAOC) problem** that respects this practical information constraint.

Given the delay D between ordering and receiving, it is clear that, at time instant k, the value of the objective function $L(k+1)$ (the cumulative profit) at the next time instant $k+1$ is dictated by the ordering $o(k-D)$. This means that, since the past demands and the current demand are assumed known, one can optimize over the backward horizon $\{k-D, k-D+1, \ldots, , k\}$ (the past), to obtain the optimal sequence $\{o^*(i)\}_{i=k-D}^{k}$, assuming the state $\mathbf{z}(k) = [x_1(k), \ldots, x_D(k), s_t(k)]$ to be given. Note that, from (1)-(2), the state $\mathbf{z}(k)$ can also be expressed as $\mathbf{z}(k) = [o(k-D), \ldots, o(k-1), s_t(k)]$. Thus, after the optimization step is carried out, the state is updated to $\mathbf{z}(k+1) = [o(k-D+1), \ldots, o^*(k), s_t(k+1)]$, introducing only the current input and discarding the older ones, which are given and cannot be changed. Now a new optimization step can be carried out, from this new initial condition. In other words, at each step k, although the entire vector $\{o^*(i)\}_{i=k-D}^{k}$ is determined, only the first element of the sequence $o^*(k)$ is used: it is injected into the sytem state and its effect on the objective function is only observed D instants later. The OSAOC problem just described is denoted $OS(\mathbf{z}(k))$ and written formally as follows:

$$\begin{aligned}&\max_{\{o(i)\}_{i=k-D}^{k}} L(k+1)\\ &\text{subject to (1)-(12)}\\ &\text{and } \mathbf{z}(k) = [x_1(k),\dots,x_D(k),s_t(k)] \text{ (specified i.c.)}\end{aligned} \tag{18}$$

Let the optimal control computed as the solution to (18) be denoted as follows:

$$[o^*(k-D),\dots,o^*(k)] := \operatorname{argmax}_{\{o(i)\}_{i=k-D}^{k}} L(k+1) \tag{19}$$

With this notation in place, the iterative one step ahead control (OSAOC) scheme can be written as shown (Algorithm 1):

Algorithm 1: One step Ahead Optimal Control (OSAOC).

Initial state $\mathbf{z}_0 := \mathbf{z}(k_0)$, $k_0 = D$ given
while $D \le k \le K_f - 1$ **do**
  $\mathbf{z}(k) := \mathbf{z}_0 = [o(k-D),\dots,o(k-1),s_t(k)]$
  solve $OS(\mathbf{z}(k))$ defined in (18)
  to get $[o^*(k-D);\dots;o^*(k)]$
  $\mathbf{z}_0 \leftarrow [o(k-D+1),\dots,o^*(k),s_t(k+1)]$
  $k \leftarrow k+1$
end while

Remark: Algorithm 1 bears a resemblance to a model predictive control (MPC) scheme. Note, however, that it relies only on past values of the demand and uses no prediction.

4 IMPLEMENTATION OF THE ALGORITHM

The MATLAB implementation uses the Optimization Toolbox, which is set up to carry out black box optimization of a function, using interior-point, sequential quadratic programming (SQP) and active set methods, amongst others. The black box in question is the system model (1)-(5) of the supply chain dynamics. In the examples presented in Sec. 5, the MATLAB Optimization Toolbox `fmincon` command was used to optimize, with the algorithm being chosen as `interior point`.

5 SIMULATION EXAMPLES

In this section, an example of a simple demand curve is first given to highlight the differences between omniscient and one step ahead optimal control. This is followed by a more realistic example of a demand generated from a Poisson distribution. In all cases, the comparison is between the final value of the cumulative profit attained by the OSAOC (L_{OSA}) and the omniscient control (L_{omni}) for the same demand sequence. The ratio between L_{OSA}/L_{omni} is referred to as the efficiency η.

5.1 Demand with Step Change

The simple demand used to illustrate the differences in behavior between the omniscient and OSAO control is constant, with one step change. The parameters used in the simulation are as follows: $\alpha_{os} = 20$, $\alpha_{st} = 5$, $\alpha_h = 10$, $\alpha_{sh} = 5$, $p = 100$, $\rho = 0.005$, $C_{max\_store} = 30$, $C_{max\_ship} = 50$, $s_t(0) = 10$, $D = 7$, $K_f = 37$, $s_h(0) = 0$. Finally the demand is as follows: constant at value 4 from $k = 0$ to $k = 20$, after which a step change to the value 6 occurs. The cumulative profits L_{OSA} and L_{omni} attained by the OSAOC and omniscient algorithms at the end of a 30 day period are, respectively, 7288.7 and 8605.9, so that the OSAOC achieves an efficiency $\eta = 84.6\%$, with respect to the globally optimum (omniscient) result. Figures 1 to 3 show the main results for the demand with step change and their captions provide additional details and observations.

5.2 Demand with Poisson Distribution

Experiments in this subsection are carried out with a demand d that has a Poisson distribution with $\lambda = 1$ shifted by 1 (to eliminate demands equal to zero), generated by the MATLAB command `d = poissrnd(1,37,1)+1`. The parameters used in the simulation are as follows: $\alpha_{os} = 20$, $\alpha_{st} = 5$, $\alpha_h = 10$, $\alpha_{sh} = 5$, $p = 100$, $\rho = 0.005$, $C_{max\_store} = 30$, $C_{max\_ship} = 50$, $s_t(0) = 10$, $D = 7$, $K_f = 37$, $s_h(0) = 0$. The cumulative profits L_{OSA} and L_{omni} attained by the OSAOC and omniscient algorithms at the end of a 30 day period are, respectively, 2107.6 and 2598.6, so that the OSAOC achieves an efficiency $\eta = 81.1\%$ with respect to the globally optimum (omniscient) result. Figures 4 to 9 show the main results for the Poisson demand and their captions provide additional details and observations.

5.3 Comparison of Average Performance of OSAOC versus Omniscient Control

Ten samples each of Poisson and uniformly distributed demands were generated and the average efficiency (resp. standard deviation) were found to be 78.25%(3.28%) (Poisson) and 73.52%(5.77%) (uniform).

The coefficient of variation is a measure of relative variability, defined as the ratio of the standard deviation to the mean of the variable under consideration. Thus, a measure of amplification of demand variability, also known as bullwhip, is the ratio of the coefficients of variation of the ordering and the demand, and is denoted B^x_{od}, where the subscript *od* refers to the ratio being calculated and the superscript x can be *os* (resp. *om*) for one step (resp.omniscient). A related measure is the ratio of the coefficients of variation of the inventory (store) level and the demand, denoted B^x_{std}. For the Poisson demand of sec. 5.2, the calculations, carried out over the interval $[D, K_f - D]$ so as to exclude both initial transient due to pipeline initialization, and final transient due to the turnpike effect for the omniscient case, yield: $B^{os}_{od} = 0.43, B^{os}_{std} = 0.32, B^{om}_{od} = 0.70, B^{om}_{std} = 0.28$. Since all these measures are less than unity, the conclusion is that the proposed one step ahead controller does not cause amplification of the demand uncertainty. This should be contrasted with the behavior of controllers of the IOBPCS family (Lin and Naim, 2019) and the MPC family with targets (Subramanian et al., 2014).

6 CONCLUDING REMARKS

This paper proposed a one step ahead optimal control scheme for a single echelon supply chain that does not require prediction but only the solution of a low-dimensional (in terms of the number of decision variables) optimization problem at each time step, thus being perfectly adequate for real time supply chain control. This one step ahead or greedy control is shown to be quite efficient, attaining, for Poisson or uniformly distributed demands, at least 70% of the (unattainable) omniscient global optimum. In addition, experiments show that the proposed OSAO control scheme does not cause amplification of the demand uncertainty or bullwhip, which is attributable to the fact that no target levels for inventory or pipeline are used. Another important observation is that the model (1)-(12), with the exception of (5), consists of linear equations. It is easy to write (5) in piecewise linear form, which means that both the omniscient (13) and OSAOC optimization problems (18) can be rewritten as linear programming problems, which, from a computational viewpoint, implies that the simple examples in Sec. 5, can easily be scaled to much larger dimension. It should also be noted that a large number of supply chain problems, including the multi-echelon case, are described by linear or piecewise linear dynamics. Thus, the proposed scheme is scalable and can: (i) be generalized to large multi-echelon supply chains, which will be the subject of future work, and (ii) used in a planning mode, to choose inventory size, shipping capacity requirements and prices that ensure profit margins, since these choices can be recast as feasibility problems. This is also the subject of ongoing work, to be reported in a future version of this paper.

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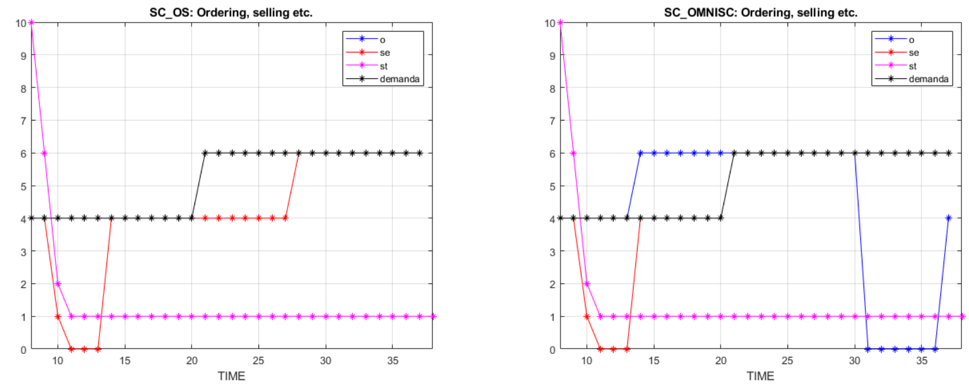

Figure 1: Showing ordering and selling over the horizon $k = 0$ to $k = K_f = 37$, with a step increase in the constant demand at $k = 20$: OSAOC (left), omniscient control (right). Note the turnpike effect for omniscient control over the horizon $k_f = 37$, with the ordering dropping to zero D instants before the end of the horizon is reached, because the demand will be met with the orders in the pipeline. For this reason, all simulation examples are run till $K_f = 37$, but the efficiency η is calculated at $k = 30$ (in order to exclude the turnpike effect). Also note that, the omniscient control, with complete knowledge of the demand, anticipates ordering in order to meet the demand perfectly, given a sufficiently large initial inventory, and maintains the inventory at the minimum, which is the specified display stock of 1, thus operating in just-in-time mode.

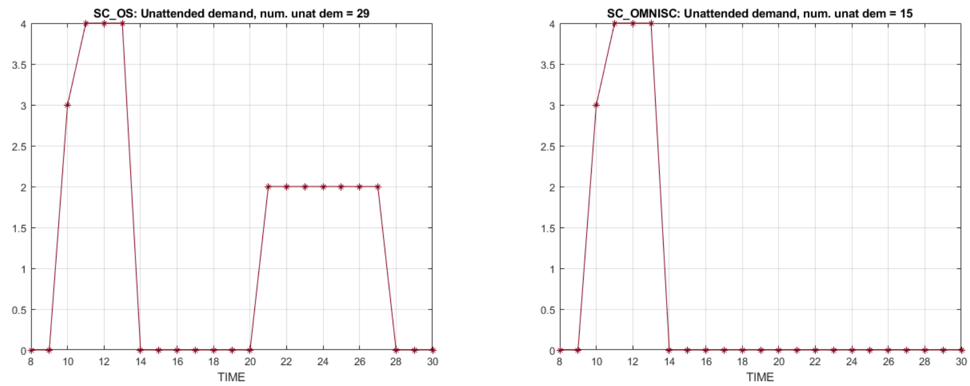

Figure 2: Showing lost sales: OSAOC (left) and omniscient control (right), until $k = 30$. Observe that both controllers lead to lost sales in the initial segment, between $k = 9$ and $k = 14$, because the pipeline is initially empty. However, the OSAO controller also results in lost sales when the step increase in demand occurs, since it is causal (non-anticipative) and this, of course, does not occur with the omniscient noncausal controller, which is able to anticipate the future demand.

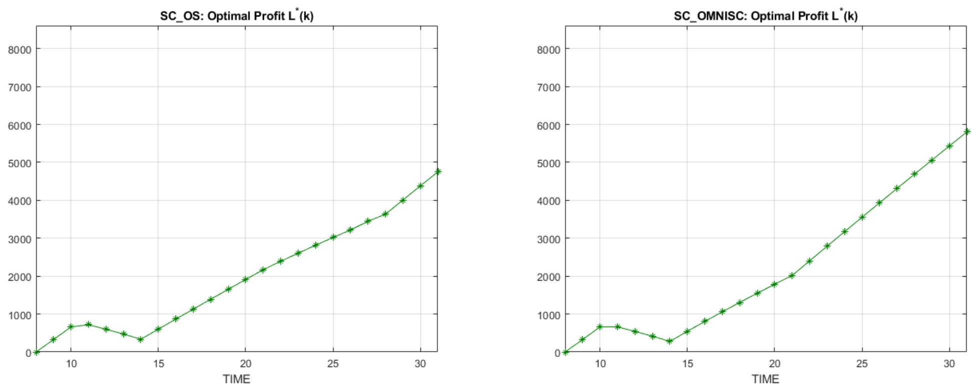

Figure 3: Showing evolution of cumulative profit: OSAOC (left), omniscient control (right), with the OSAO controller attaining an efficiency $\eta = L_{OSA}/L_{omni} = 84.6\%$. Note that, after the end of the transient caused by the zero initial pipeline ($s_h(0) = 0$), the cumulative profit curve is monotonically increasing.

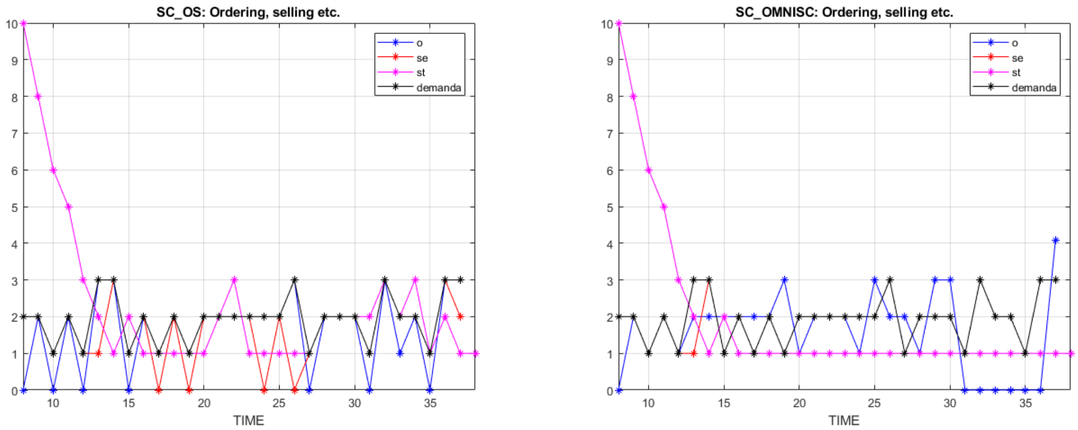

Figure 4: Showing the evolution of ordering, selling, store for a Poisson demand: OSAOC (left), omniscient (right) for the horizon $k = 0$ to $k = K_f = 37$, with the turnpike effect for the omniscient control starting from $k = 30$.

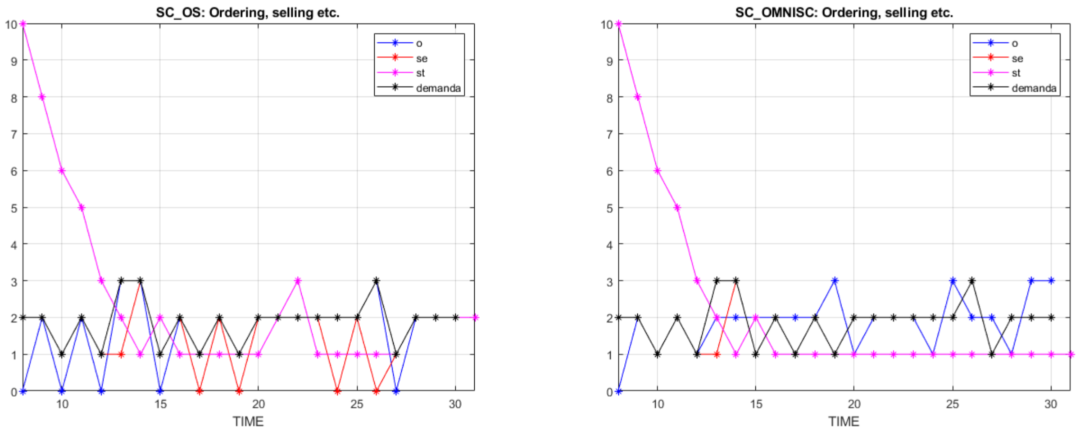

Figure 5: Showing the evolution of ordering, selling, store variables for a Poisson demand, with the graphs plotted from instant $k = 8$ to $k = 30$, in order to start from the first computed ordering, assuming that demands from $k = 1$ to $k = 7$ are used as the initial past demands, and also, for purposes of comparison, truncating the horizon at $k = 30$ to avoid the turnpike effect, which occurs at the end of the horizon, for the omniscient case: OSAOC (left), omniscient (right).

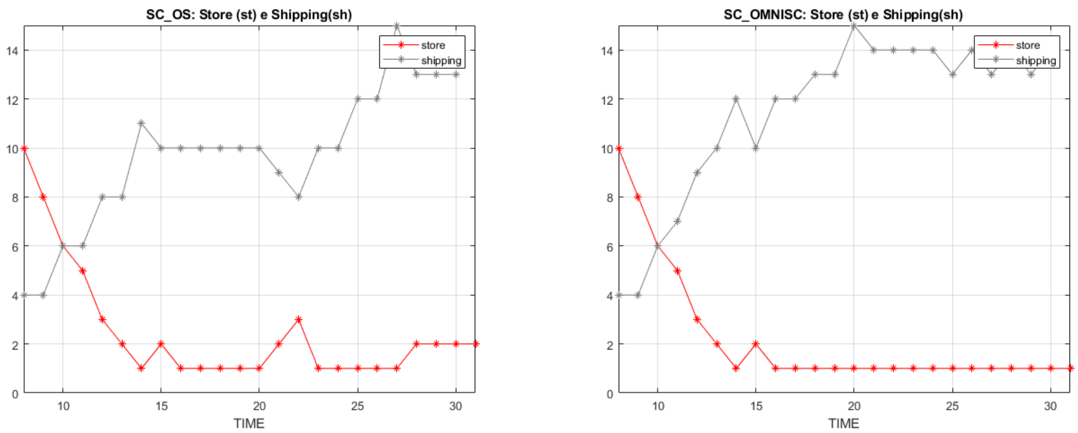

Figure 6: Showing the evolution of the store and shipping variables, for a Poisson demand, for a 30 day period: OSAOC (left), omniscient (right). Note that, after the initial transient (i.e., $k \geq 16$), the OSAOC is not able keep the inventory level at the minimum, which is the specified display stock of 1, unlike the anticipative omniscient controller.

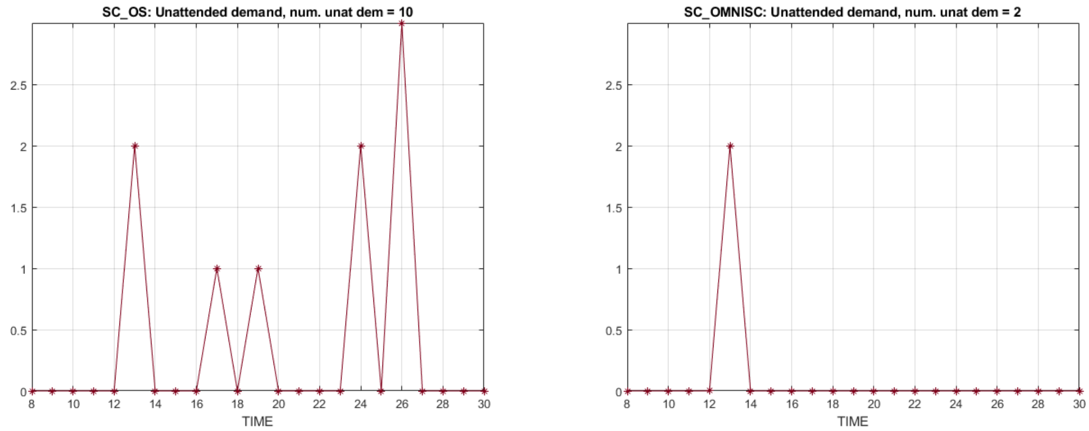

Figure 7: Showing the evolution of lost sales, for a Poisson demand: OSAOC (left), omniscient (right). Note that, lost sales occur at $k = 17, 19, 24, 26$ only for OSAOC after the initial transient, but not for the omniscient controller. This means that, in Figure 6, the OSAOC is at minimum store level at these instants, since it is unable to meet the demand. In contrast, the omniscient controller, also at the minimum store level, is meeting demand with the orders being received, in the just-in-time mode.

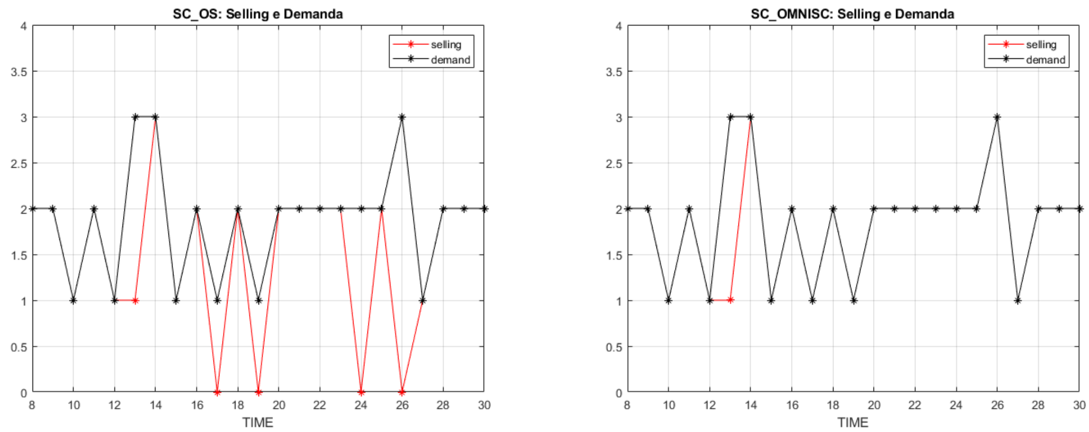

Figure 8: Showing the evolution of selling for a Poisson demand: OSAOC (left), omniscient (right). Note that, after the initial transient, the omniscient selling matches the demand perfectly, operating, as explained in the caption to Fig. 7.

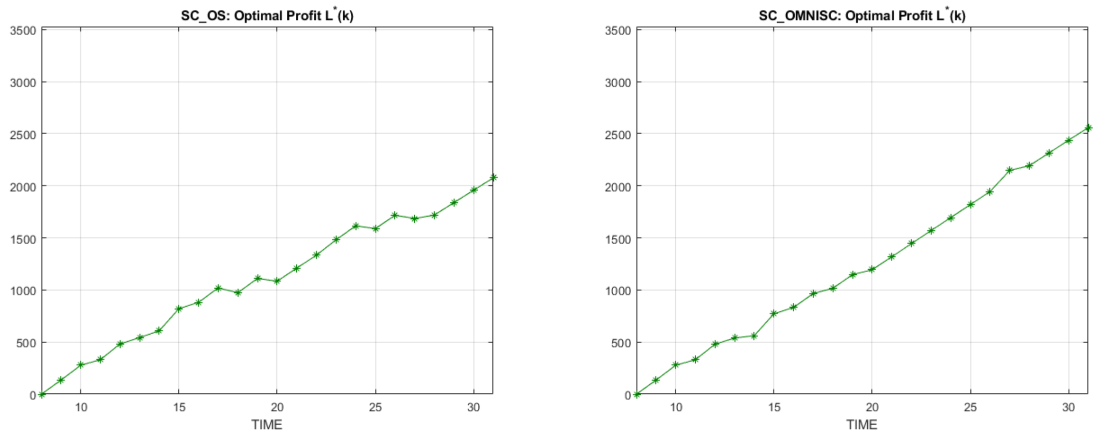

Figure 9: Showing the evolution of the cumulative discounted profit (EVA): OSAOC (left), omniscient (right), with the OSAO controller attaining an efficiency $\eta = L_{OSA}/L_{omni} = 81.1\%$. Despite the zero initial pipeline, the initial store level is able to meet the initial demands during the lead time of $D = 7$.