Solving the Multi-activity Shift Scheduling Problem using Variable Neighbourhood Search

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Abstract: This paper presents a set of benchmarks instances for the multi-activity shift scheduling problem and the results produced using a variable neighbourhood search method. The data set is intended as a resource to generate and verify novel research on an important and practical but challenging problem. The variable neighbourhood search uses four different neighbourhood operators and can produce feasible solutions within short computation times.

1 INTRODUCTION

The multi-activity shift scheduling problem is a widely occurring, difficult optimisation problem. It is commonly found in retail environments for example. At shops and stores staff are required to work different activities at different times throughout a day. Each shop could have a different number of activities. Staff may be required to operate tills, or assist customers on shop floors, or manage stock in a warehouse, or supervise other staff etc. The demand for each activity may also fluctuate throughout the day as busy periods arise for that activity. For example, there could be times when more staff are required in the warehouse when deliveries arrive. The demand will also vary per day as some days are busier than others. There are also constraints on the employee's individual schedule. They will have a maximum number of contracted hours. They may also have a minimum number of hours work they must be assigned. There are constraints on their shift lengths and when they can start. There are often working directives on how much rest they must have between shifts and so on. The objective of the scheduler is often to minimise costs from overstaffing whilst satisfying all other constraints.

One of the earliest attempts to solve the problem is (Loucks & Jacobs, 1991). Until recently it was a relatively under-studied problem. Possibly due to its size and complexity most papers have focussed on solving a single stage of the decomposed problem, such as day off scheduling, shift scheduling or tour scheduling. More recently mathematical programming-based approaches have been used on different variations of the problem (Gérard, Clautiaux, & Sadykov, 2016; Restrepo, Gendron, & Rousseau, 2016, 2018; Restrepo, Lozano, & Medaglia, 2012; Salvagnin & Walsh, 2012) and also neighbourhood search methods (Dahmen & Rekik, 2015; S Pan, Akplogan, Létocart, Touati, & Calvo, 2016; Stefania Pan et al., 2018; Quimper & Rousseau, 2010).

The benchmark data set captures the core features of the problem. Instances of varying planning length, numbers of staff and numbers of activities have been produced. There are instances of length 7, 14 and 28 days. The number of staff varies from 10 to 150 and the numbers of different activities varies from 1 up to 19 in the largest instances. There are 225 instances in total. The instances and their characteristics are listed in Table 1. Due to the way that the solutions were created it is known that every instance does have a feasible solution. When creating the instances, it was also observed that at least three of the instances have zero cost solutions (that means their total penalty score is zero). The instances are available for download from www.schedulingbenchmarks.org.

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2 PROBLEM DESCRIPTION

The problem requires the assignment of shifts to employees and the assignment of activities within the shifts.

The planning horizon is divided into 15 minute intervals and shifts must start and finish at the beginning of a 15 minute interval. For example, 09:00-17:15 would be a valid shift but 09:08-17:15 would not be, nor would 09:00-17:04.

Shifts consist of one or more activities. For example, Figure 1. shows a representation of a shift which starts at 09:00 and finishes at 17:00. Within this shift an employee starts on Activity 1 (green) for 3.5 hours and then transfers to Activity 2 (grey) for the remaining 4.5 hours of the shift.

Figure 1: Example shift with one transfer.

Figure 2 shows another 8 hour shift but in this example there are two activity changes. The employee starts with Activity 1 and then switches to Activity 2 after 3.5 hours and then transfers back to Activity 1 for the final hour of the shift.

Figure 2: Example shift with two transfers.

There are no limits on the number of different activities that a shift can contain or how many activity changes can occur. However, every activity duration must be at least one hour before the employee changes to a different activity or the shift ends. Activity changes must also occur at the beginning of a 15 minute interval. For example, an activity change could occur at 12:00 or 12:15 but not at any time between 12:00-12:15.

Staff demand requirements (also called cover) are provided for each activity for every 15-minute time interval in the planning horizon e.g. 00:00-00:15, 00:15-00:30 … 23:30-23:45, 23:45-00:00.

In all the instances the planning horizon starts at 06:00 on the first day and finishes at 06:00 on the last day. Therefore if the planning horizon is 7 days then it runs from 06:00 on day 1 to 06:00 on day 8.

Shifts must be designed and then assigned such that a minimum demand for each activity is satisfied as well as the employees' constraints and the organisation's work regulations. The other constraints are as follows:

2.1 Constraints

C1: Maximum one shift per day - Employees cannot start more than one shift on a day (where a day is considered as starting at midnight and finishing 24 hours later).

C2: Maximum Five Consecutive Working Days - The maximum number of consecutive working days that can be assigned to an employee is 5. A day is considered as a working day if a shift is started on that day (where the day is considered as starting at midnight and finishing 24 hours later). This constraint always assumes that the last day of the previous planning period was a day off and the first day of the next planning period is a day off.

C3: Minimum Total Minutes - Each employee has a minimum total time in minutes that must be assigned over the whole planning horizon. This is specified in the data file for each employee. The duration of each shift is from the start time to the end time.

C4: Maximum Total Minutes – Each employee has a maximum total time in minutes that can be assigned over the whole planning horizon. This is specified in the data file for each employee. The duration of each shift is from the start time to the end time.

C5: Minimum Rest Time between Shifts - There must be a minimum of 14 hours rest after every shift. This means that after a shift finishes the employee cannot start another shift until at least 14 hours later.

C6: Minimum Activity Duration - A shift can contain any number of different activities, but each activity duration must be at least 1 hour. This means that a shift cannot contain an activity duration that is less than one hour. For example, a shift which starts at 09:00 and has the first 45 minutes assigned to Activity 1 and then switches to Activity 2 at 09:45 is not valid, but if the shift switches to Activity 2 at 10:00 then it would be valid.

C7: Minimum Shift Length - The minimum shift duration is 6 hours.

C8: Maximum Shift Length - The maximum shift duration is 10 hours.

C9: Valid Shift Start Times - Shifts can only start between the following times each day: 06:00-10:00, 14:00-18:00 and 20:00-00:00. A shift cannot start outside one of these intervals. For example, a shift could start at 10:00 but not 10:15.

C10: Minimum Cover Requirements - The minimum number of required staff for every time interval and every activity must be satisfied. These requirements are specified in the instance data files.

2.2 Objective Function

The objective is to minimise assigning more staff that is required for each activity for every interval. It is modelled as a quadratic function to ensure that over-assignment is spread out over the planning horizon rather than occurring in a small number of intervals, which could happen with a linear function.

A maximum cover requirement is given for every activity and every interval. If more than the maximum required number of staff at the specified time interval for the specified activity is assigned then a penalty cost is added to the objective function value: If the number assigned (x) is more than the maximum required number then the penalty for that activity and time interval is:

(x - max) * (x - max) * weight

The weight for all instances is 1.

The solution's total penalty is the sum of all the penalties for every time interval and activity requirement.

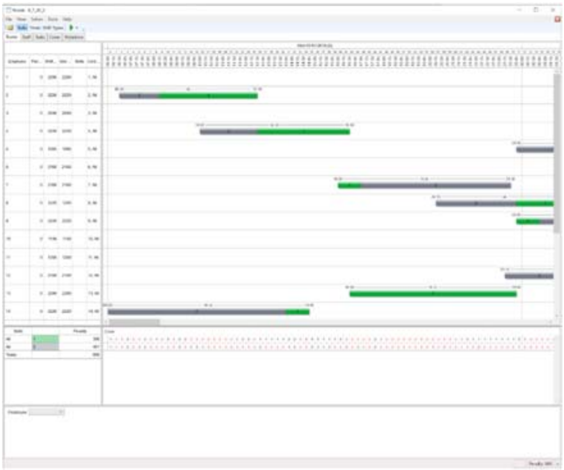

Figure 3: Screenshot of verifier.

2.3 Solution Verification

To ensure the accuracy of new computational results a verifier has been made available (screenshot in Figure 3). The solutions can be saved in a defined XML format. These XML files can be read by and opened using the verifier. The verifier will display the objective function as well as displaying any errors or constraint violations that may have been accidentally introduced. This will help researchers to verify their solutions and identify any errors if their solution does not match the verifier's calculated objective function. The solution visualisation may also be useful in designing and testing new algorithms. The verifier is available at www.schedulingbenchmarks.org.

3 VARIABLE NEIGHBOURHOOD SEARCH

The variable neighbourhood search (VNS) uses four different neighbourhood operators. Each operator is applied to the solution in an iterative process until the solution is a local optimum with respect to all four operators. Once the local optimum is reached, all shifts are un-assigned and the process is repeated. When the time limit is reached the solution with the best local optimum is returned.

To make the search space more connected and to assist the neighbourhood search to reach better local optima, all constraints except C1, C5 and C9 are relaxed and made soft constraints but with very high weights. The search neighbourhood operators are as follows:

N1: For each shift, try replacing it with a new shift. When creating the new shift, every possible start time and shift length is tried and each combination of three different activities within the shift is tried.

N2: For each shift, test making it up to 30 minutes longer or shorter and up to 30 minutes earlier or later and simultaneously making another shift for the same employee on a different day up to 30 minutes longer or shorter and up to 30 minutes earlier or later. Similarly to neighbourhood 1, it will also test each combination of three different activities within the shift

N3: For each day, for each pair of employees, try swapping the shifts assigned between the two employees on that day. This is repeated for that day but also simultaneously swapping over the next x consecutive days where x is 1..4.

N4: For each shift, try moving it up to one hour earlier or later and up to 15 minutes shorter or longer and simultaneously moving another shift for another employee on the same day up to one hour earlier or later and up to 15 minutes shorter or longer. When moving the first shift, it will also test different combinations of activities within the shift. For the second shift it will keep the activities within them the same and the absolute times of the activity changes.

4 RESULTS

The algorithm was run for 10 minutes on each instance. The best solution found after 5 minutes and 10 minutes on each instance is listed in Table 1. If no feasible solution was found in the time allowed, then the row contains '-'. The experiments were conducted on an Intel Core i5-4690K CPU 3.50GHz.

The VNS method can find feasible solutions for most of the smaller instances within 5 minutes. An extra five minutes further improves the solutions on some instances and results in solutions for some other instances that could not be solved within 5 minutes. The larger instances with more staff, more activities or longer planning horizons appear to be more difficult to solve and often a feasible solution could not be found within the time provided.

5 CONCLUSIONS

New benchmark instances have been introduced for the multi-activity shift scheduling problem. They are publicly available for download from www.schedulingbenchmarks.org. A verifier with a graphical user interface is also available to validate new results and assist with development. A Variable Neighbourhood Search that uses four different neighbourhoods has also been presented. It can find feasible solutions to the smaller and medium sized instances in relatively short computation times. Future research should focus on methods for solving the larger instances. Data sets which also consider break scheduling and task scheduling within the shifts may also be introduced in the future.

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APPENDIX

Table 1: Results.

Instance	Days	Staff	Tasks	5 mins	10 mins
1	7	10	1	387	387
2	7	10	1	192	176
3	7	10	1	317	317
4	7	10	1	330	328
5	7	10	2	115	115
6	7	20	1	900	900
7	7	20	1	879	818
8	7	20	2	884	884
9	7	20	2	513	500
10	7	20	3	274	268
11	7	30	1	909	844
12	7	30	2	1541	1541
13	7	30	2	1440	1440
14	7	30	3	1476	1469
15	7	30	4	553	553
16	7	40	2	1946	1883
17	7	40	2	1831	1831
18	7	40	3	1737	1737
19	7	40	4	1437	1437
20	7	40	5	990	955
21	7	50	2	1740	1740
22	7	50	3	2646	2646

Table 1: Results (continue).

23	7	50	4	2446	2446
24	7	50	5	1795	1795
25	7	50	7	-	1344
26	7	60	2	1734	1734
27	7	60	3	2904	2904
28	7	60	4	3276	3248
29	7	60	6	2463	2463
30	7	60	8	-	-
31	7	70	3	2574	2574
32	7	70	4	3288	3288
33	7	70	5	3170	3170
34	7	70	7	-	-
35	7	70	9	-	-
36	7	80	3	2858	2709
37	7	80	4	3335	3335
38	7	80	6	3894	3894
39	7	80	8	-	-
40	7	80	10	-	-
41	7	90	3	2575	2575
42	7	90	5	4317	4317
43	7	90	6	4971	4877
44	7	90	9	-	-
45	7	90	12	-	-
46	7	100	4	3471	3471
47	7	100	5	5139	4837
48	7	100	7	5492	5302
49	7	100	10	-	-
50	7	100	13	-	-
51	7	110	4	3338	3338
52	7	110	6	5084	5084
53	7	110	8	6291	6237
54	7	110	11	-	-
55	7	110	14	-	-
56	7	120	4	3486	3486
57	7	120	6	5991	5991
58	7	120	8	-	6749
59	7	120	12	-	-
60	7	120	15	-	-
61	7	130	5	4932	4932
62	7	130	7	-	6720
63	7	130	9	-	7086
64	7	130	13	-	-
65	7	130	17	-	-
66	7	140	5	4057	4057
67	7	140	7	6009	6009
68	7	140	10	-	-
69	7	140	14	-	-
70	7	140	18	-	-
71	7	150	5	4063	4063
72	7	150	8	-	7590
73	7	150	10	-	-
74	7	150	15	-	-
75	7	150	19	-	-
76	14	10	1	604	598
77	14	10	1	814	814
78	14	10	1	634	634
79	14	10	1	607	607
80	14	10	2	292	292
81	14	20	1	1659	1659
82	14	20	1	1643	1643
83	14	20	2	1387	1387
84	14	20	2	1279	1168
85	14	20	3	520	520
86	14	30	1	1738	1738
87	14	30	2	2672	2672
88	14	30	2	2780	2780
89	14	30	3	2551	2551
90	14	30	4	-	-
91	14	40	2	3514	3514
92	14	40	2	3767	3767
93	14	40	3	3942	3820
94	14	40	4	-	3980
95	14	40	5	-	-
96	14	50	2	3666	3666
97	14	50	3	4921	4921
98	14	50	4	4802	4802
99	14	50	5	-	-
100	14	50	7	-	-
101	14	60	2	3419	3419
102	14	60	3	5473	5473
103	14	60	4	5942	5942
104	14	60	6	-	5620
105	14	60	8	-	-
106	14	70	3	5137	5137
107	14	70	4	7208	6892
108	14	70	5	-	-
109	14	70	7	-	-
110	14	70	9	-	-
111	14	80	3	-	5510
112	14	80	4	6748	6748
113	14	80	6	-	8124
114	14	80	8	-	-
115	14	80	10	-	-
116	14	90	3	5715	5598
117	14	90	5	-	8818
118	14	90	6	-	-
119	14	90	9	-	-
120	14	90	12	-	-
121	14	100	4	-	-
122	14	100	5	-	-
123	14	100	7	-	-
124	14	100	10	-	-
125	14	100	13	-	-

Table 1: Results (continue).

126	14	110	4	-	7573
127	14	110	6	-	-
128	14	110	8	-	-
129	14	110	11	-	-
130	14	110	14	-	-
131	14	120	4	-	7475
132	14	120	6	-	-
133	14	120	8	-	-
134	14	120	12	-	-
135	14	120	15	-	-
136	14	130	5	-	-
137	14	130	7	-	-
138	14	130	9	-	-
139	14	130	13	-	-
140	14	130	17	-	-
141	14	140	5	-	8013
142	14	140	7	-	-
143	14	140	10	-	-
144	14	140	14	-	-
145	14	140	18	-	-
146	14	150	5	-	-
147	14	150	8	-	-
148	14	150	10	-	-
149	14	150	15	-	-
150	14	150	19	-	-
151	28	10	1	1677	1677
152	28	10	1	1509	1509
153	28	10	1	1729	1729
154	28	10	1	1535	1535
155	28	10	2	-	-
156	28	20	1	3766	3766
157	28	20	1	3541	3523
158	28	20	2	3327	3327
159	28	20	2	2989	2989
160	28	20	3	-	1803
161	28	30	1	3513	3505
162	28	30	2	-	6551
163	28	30	2	6209	6209
164	28	30	3	-	-
165	28	30	4	-	-
166	28	40	2	-	7613
167	28	40	2	7317	7317
168	28	40	3	-	8270
169	28	40	4	-	-
170	28	40	5	-	-
171	28	50	2	7000	6843
172	28	50	3	-	-
173	28	50	4	-	-
174	28	50	5	-	-
175	28	50	7	-	-
176	28	60	2	7179	7179
177	28	60	3	-	-
178	28	60	4	-	-
179	28	60	6	-	-
180	28	60	8	-	-
181	28	70	3	-	-
182	28	70	4	-	-
183	28	70	5	-	-
184	28	70	7	-	-
185	28	70	9	-	-
186	28	80	3	-	11181
187	28	80	4	-	-
188	28	80	6	-	-
189	28	80	8	-	-
190	28	80	10	-	-
191	28	90	3	-	-
192	28	90	5	-	-
193	28	90	6	-	-
194	28	90	9	-	-
195	28	90	12	-	-
196	28	100	4	-	-
197	28	100	5	-	-
198	28	100	7	-	-
199	28	100	10	-	-
200	28	100	13	-	-
201	28	110	4	-	-
202	28	110	6	-	-
203	28	110	8	-	-
204	28	110	11	-	-
205	28	110	14	-	-
206	28	120	4	-	-
207	28	120	6	-	-
208	28	120	8	-	-
209	28	120	12	-	-
210	28	120	15	-	-
211	28	130	5	-	-
212	28	130	7	-	-
213	28	130	9	-	-
214	28	130	13	-	-
215	28	130	17	-	-
216	28	140	5	-	-
217	28	140	7	-	-
218	28	140	10	-	-
219	28	140	14	-	-
220	28	140	18	-	-
221	28	150	5	-	-
222	28	150	8	-	-
223	28	150	10	-	-
224	28	150	15	-	-
225	28	150	19	-	-