# Comparison of Binary Images based on Jaccard Measure using Symmetry Information

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Abstract: Method of comparing binary raster images using information about the axes of symmetry of the shapes is proposed, which will allow to take into account the translation, rotation and scaling of a pair of images. The symmetry axis of the figure is searched by one of the previously developed methods: based on the skeleton representation of the figure (Kushnir et al., 2016), the adjustment of the skeleton axis or exhaustive search (Kushnir et al., 2019). Jaccard measure is used as a measure of similarity. Three comparison algorithms were developed. The paper demonstrates that using information about the symmetry of the shapes with simple principle of comparison as the Jaccard measure allows to obtain significant results. The possibility of using this approach for image classification is also investigated. The algorithms were experimentally studied on the "Flavia" and "Butterflies" datasets.

## 1 INTRODUCTION

The task of comparing binary raster images arises in many applications, such as classifying objects (for example, when determining the type of leaves by their shape) or detecting tumors in medical images.

When comparing images, it is necessary to consider that objects can be of different sizes, shifted relative to each other and rotated. To solve this problem, it is proposed to use the axes of approximate symmetry as the basis for matching the compared figures. Obviously, it is quite easy to bring two images to the same size, location and angle of inclination using information about the location and size of the axes of symmetry.

The task of symmetry detection and symmetry measure evaluation for 2D shapes is well-known. There are both effective and efficient methods for its solution. In this work we use an exact algorithm for finding the axis of reflection symmetry based on the Jaccard similarity of two image parts (for binary sets it is also known as Tanimoto (Lesot et al., 2009)):

$$\mu(B) = \frac{|S(B) \cap S(B_r)|}{|S(B) \cup S(B_r)|}, \tag{1}$$

where $B$ – binary image, the brightness of the black pixels denote 1, white – 0; $B_r$ – reflection of the binary image $B$ with respect to a line, $S(B)$ – set of pixels belonging to the image $B$, the brightness of which is equal to 1. The measure possesses the basic "good" measure properties: $0 \leq \mu(B) \leq 1$, and $\mu(B) = 1$, if $B$ – absolutely symmetric, and $\mu(B) = 0$, if $B$ and $B_r$ not overlapped.

The exact algorithm for determining reflection symmetry routine iterates over all possible lines crossing the figure and finds the one for which the symmetry measure (Jaccard similarity) is maximum. This line will be the axis of reflection symmetry of the image.

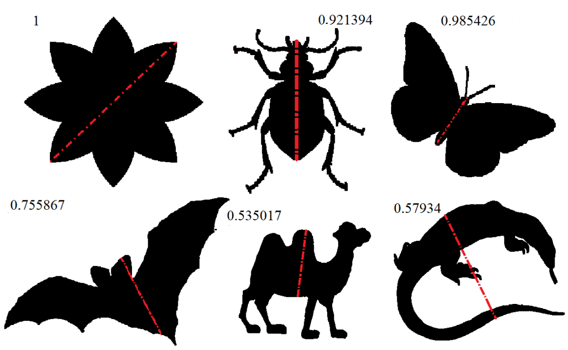


Figure 1: Examples of images with higher (top) or lower (bottom) value of symmetry measure evaluated on the basis of Jaccard similarity.

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The purpose of the work is to study the possibility of image comparison using information about the symmetry. Figure 1 shows examples of axes for both perfectly symmetrical images (a figure with a Jaccard measure equal to 1) and asymmetric ones. In identical images, the axes of symmetry will be located on the figure in approximately the same place. For almost similar images (for example, images of butterflies with opened wings), the axis of symmetry will always be located along the body. Thus, on the found axis of symmetry, two images of butterflies will be correctly combined, and it will be necessary to calculate the measure of similarity. In this work we use the Jaccard similarity as a measure of the similarity of two images. We will not use the comparison of parts of one image, but two different shapes – $A$ and $B$:

$$\mu(A,B) = \frac{|S(A) \cap S(B)|}{|S(A) \cup S(B)|}. \tag{2}$$

The quality of the obtained measure and its application for image recognition are also investigated.

## 2 RELATED WORKS

The task of symmetry detection and symmetry measure evaluation for 2D shapes is well-known, and there are many effective methods for its solution based on: 1) Fourier series expansion of parametric contour representation (Van Otterloo, 1988), 2) contour representation by turning function (Sheynin et al., 1999), 3) contour representation by critical points and computation of similarity measure for two sub-contours via vectors of geodesic distances (Yang et al., 2008), 4) model of Electrical Charge Distribution on the Shape (ECDS) (Li et al., 2014), 5) Boundary-Skeleton Function (BSF) (Niu et al., 2015), 6) pair-wise comparison of sub-sequences of skeleton primitives (Kushnir et al., 2016), 7) Fourier descriptor of the image contour (Mestetskiy and Zhuravskaya, 2019), 8) image gradient (Sun and Si, 1999).

However, there are a few works devoted to the use of the symmetry information in other applications, such as image comparison. In particular, (Hauagge and Snavely, 2012) describes the use of local symmetries of architectural structures to compare images of buildings.

## 3 COMPARISON OF IMAGES

In this paper, we propose to use information about the symmetry of shapes to compare images. We will rely on the procedures for finding the axis of symmetry proposed in previous works. To achieve reliability in the verification of the proposed procedure, an exact algorithm for determining the reflection symmetry of binary raster images (Kushnir et al., 2016) and its parallel version (Fedotova et al., 2017) will be used, requiring a complete search of all potential axes of symmetry. Calculations were carried out on a supercomputer, however, in the works (Kushnir et al., 2016; Kushnir et al., 2019) methods of significant acceleration of the computational procedure were developed.

This algorithm searches for the axis of symmetry by iterating over all possible lines passing through a pair of points on the contour of the figure. The value of symmetry is calculated with respect to each line. The line with respect to which the symmetry measure is maximal is considered to be the symmetry axis. To estimate the value of symmetry, the Jaccard measure is used, which shows the degree of similarity of two sets. The sets are the pixels of the binary image. When searching for a symmetry measure, the image is mirrored relative to the selected line and overlays the original one. Consequently, the areas of intersection and union of two sets are formed, and the Jaccard measure is calculated as their ratio (1). It is worth noting that the axes of symmetry found with the help of the Jaccard measure do not always coincide with the visual assessment by a person, a discussion is given in Section 7 of the paper (Kushnir et al., 2016).

After the symmetry axes are found for the pair of images being compared, it is proposed to align the axes in the images with each other and calculate the Jaccard measure (2), which will show the value of similarity, as shown in figure 2.

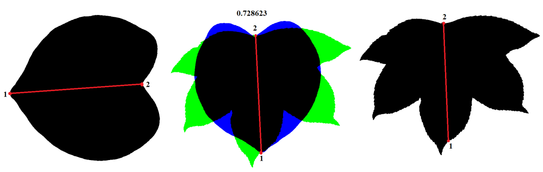


Figure 2: Example of intersection of two images (black color in the central shape indicates the intersection zone, blue and green-areas that do not match).

It is obvious that two different shapes can be different sizes, rotated through a certain angle and biased. Just using information about the location of the axes of symmetry we will achieve invariance to the shift, rotation and scale.

In similar images, the axes of symmetry will be in the shape approximately the same. Knowing the location of the axis of symmetry, we can calculate the affine transformation, which will allow to overlay (match) one image on another, aligning their axes of symmetry.

Three points are required to calculate the affine transformation. The axis of symmetry is defined by two points of intersection of the line with the contour. The third point is defined on the median perpendicular to the axis of symmetry at a distance equal to half of the symmetry segment. The coordinates of the third point are calculated as:

$$x = \frac{a}{\sqrt{k_n^2+1}} + x_0, y = \frac{k_n a}{\sqrt{k_n^2+1}} + y_0, \quad (3)$$

where $a$ is half the length of the axis of symmetry $k_n$ is the tangent of the slope of the line orthogonal to the axis of symmetry; $x_0$, $y_0$ – the coordinates of the center of the segment lying between the points of symmetry axis.

The two images are overlaid on three points which are combined on corresponding numbers by affine transformation (the first point with the first, the second with the second, the third with the third). The similarity measure (Jaccard measure) is calculated.

However, it is noted that the image orientation is not known in advance (who knows where "top" and where "the bottom"), and therefore, the imposition of the axes must be performed two times. In addition, we use a reflection copy of one of the compared images. Of the four measures obtained, the best one will be chosen, which will be considered a measure of the similarity of the two images.

Figure 3 shows an example of an overlay of two images: black area corresponds to the intersection of two sets, blue and green areas correspond to the mismatched parts of the two images.

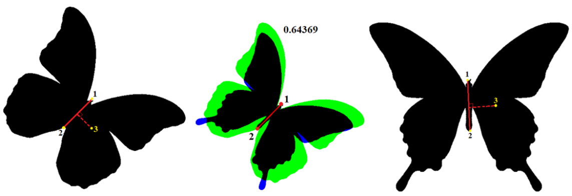


Figure 3: An example of matching two images by three points and corresponding measure of similarity.

Two more variants of algorithm were also developed. In the second algorithm it is proposed to use the intersection points of the symmetry axis with the minimum bounding circle around the binary image. Figure 4 shows an example of combining two images at the intersection of the axis of symmetry with the circumscribed circle.

As in the first version of the overlay, the orientation of the image is not known in advance, so the axis overlay must be performed twice. Of the two measures obtained, the best one will be chosen, which will be considered a measure of the similarity of the two images.

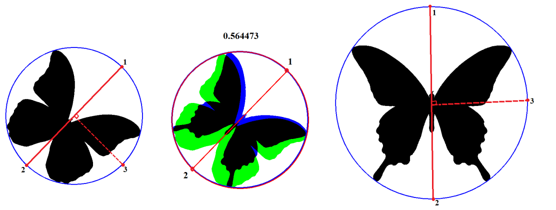


Figure 4: An example of combining two images on three points using the intersection points of the axis of symmetry with the circle described near the figure.

The third algorithm also uses the intersection points of the symmetry axis with the circumscribed circle, but the overlay operation is performed in two stages. First, it is required to determine the centers of the segments of the axes of symmetry, bounded by the contour of the figure. Images are aligned so that these centers coincide. The scaling factor is calculated as the ratio of the radiuses of the circumscribed circles. In order for the axes of two figures to lie on the same straight line, the rotation matrix is calculated. For an image that has a longer axis of symmetry, the operations of transfer, scaling, and rotation are applied sequentially. For the obtained images, the Jaccard measure is calculated. Figure 5 shows an example of combining pair of images using three points by the third algorithm with every process stage explanation. It can be noted that the three proposed algorithms give different measures of similarity on a pair of images.

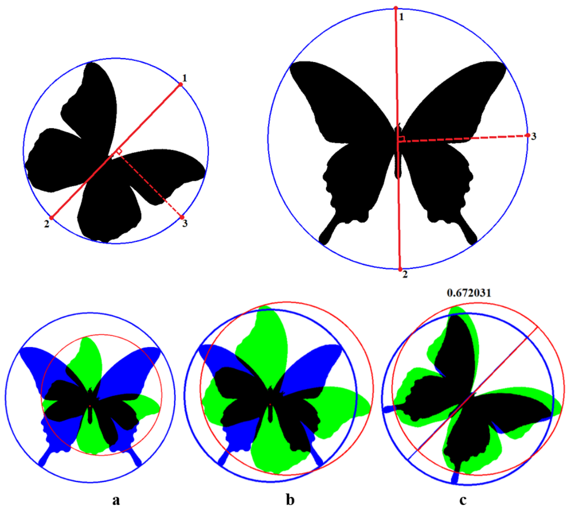


Figure 5: Example of step-by-step alignment of two images on three points using the third algorithm: a) alignment of image centers, b) scaling, c) rotation.

## 4 EXPERIMENTAL STUDY

The developed methods were experimentally studied on two image datasets: "Butterflies" dataset (accessible at http://lda.tsu.tula.ru/papers/ Butterflies.zip) and the well-known FLAVIA binary image dataset (Yang et al., 2008).

The ”Butterfly” dataset contains 30 images with resolution of 400 by 600 pixels which were found on public Internet resources and binarized.

Symmetry axes and Jaccard measures for images from the ”Butterfly” and FLAVIA datasets were obtained by an exact algorithm implemented on supercomputer ”Lomonosov” (Kushnir et al., 2019; Voevodin et al., 2019).

Class ”Butterflies” and 4, 8 classes from FLAVIA dataset were taken for experiment. Class 4 of leaves from the FLAVIA dataset contains 72 images, class 8 –52 images. Figure 6 shows some sample images from each class.

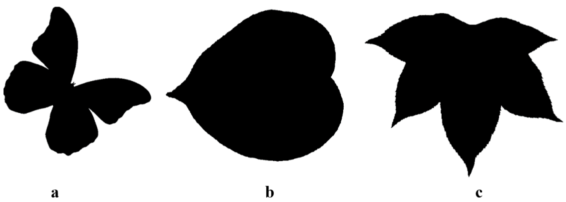


Figure 6: Examples of images from class a) ”Butterflies”, b) class 4 from FLAVIA leaf base, c) class 8 from FLAVIA.

To assess the applicability of the proposed image comparison approach in the classification problem, a similarity matrix among all objects of the three classes was calculated. The result is shown in Figure 7.

In Figure 7 different colors indicate the values of the similarity measure (Jaccard measure) for 154 images of three classes of objects. Dark blue color corresponds to the minimum of Jaccard measure (equal to zero), yellow – the maximum (equal to one). Comparing the object with itself gives a measure of similarity equal to 1 - the bright yellow diagonal of the matrix.

The figure highlights the first class – the Butterfly class for all variants of the comparison algorithm. The class 4 of leaves is well separated by algorithms 2 and 3. However, class 8 is not so well separated.

Figure 8 shows the mapping of the distance matrix to the three-dimensional feature space. Distances are obtained as a addition of the Jaccard measure to unit $\mu(A,B) = 1 - \mu(A,B)$. FastMap algorithm (Faloutsos and Lin, 1995) was used to visualize the distance matrix.

To determine the possibility of solving classification problems on the obtained similarity matrices, the so-called Bulls-eye test was performed (Bai et al., 2014). It is traditionally used to assess the quality of object comparison procedures in recognition problems. The significance of the study is as follows. For a particular object a given number $M$ of its nearest

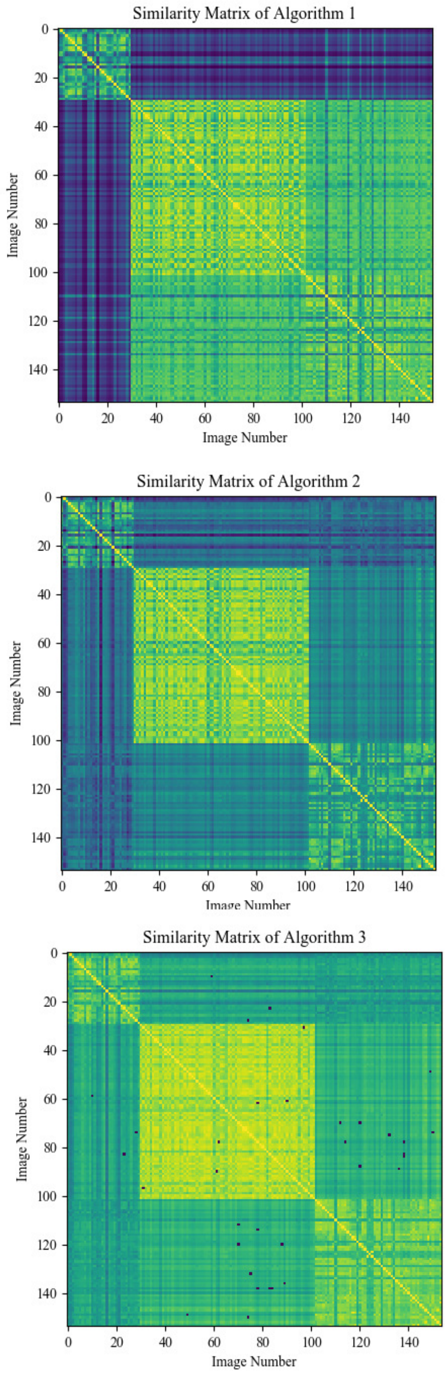


Figure 7: Similarity matrix among objects of classes Butterflies, 4 and 8 classes from FLAVIA.

neighbors is determined, among them the proportion $m$ of objects of the same class as the original object is calculated. The ratio $m$ to $M$ averaged for all objects determines the ability of the developed algorithms to produce compact groups for different classes. Figure 9 shows graphs of the dependence of recognition quality on the number of nearest neighbors for the

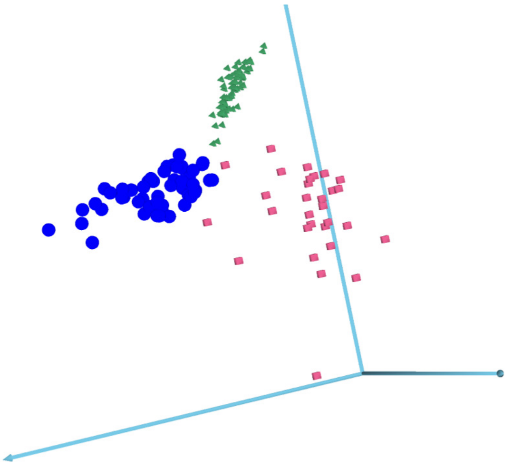

Figure 8: Visualization of distance matrix between objects of classes “Butterflies” (square markers), 4 (triangular markers) and 8 (round markers) classes from FLAVIA.

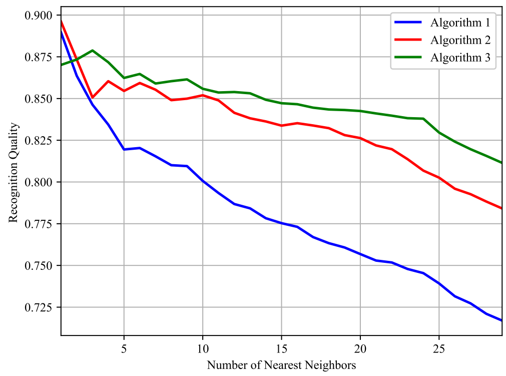


Figure 9: Graphs of recognition quality dependence on the number of nearest neighbors for three variants of image overlay algorithms.

three developed variants of the image overlay algorithm.

In addition, the study of the obtained similarity measures for a simple method of recognition-the algorithm of the nearest centers. In particular, in each of the classes were found ”centers” - the significance of the objects with the maximum total score of similarity for objects within the class. Recognition is performed as a calculation of similarity with class centers.

Classification accuracy above 0.9 was obtained for all three proposed algorithms. The results show that this measure can be apply for image comparing. In particular, the proposed measure can be used in classification problems as an additional characteristic (modality) in featureless pattern recognition based only on pairwise dis(similarity) function.

Table 1: The confusion matrix of nearest centers method.

| | | | | |
|---|---|---|---|---|
| **Algorithm 1** | | | | |
| Overall accuracy: 0.9156 | | Ground-Truth | | |
| | | Butterfly | FLAVIA Class 4 | FLAVIA Class 8 |
| Decision | Butterfly | 30 | 0 | 1 |
| | FLAVIA Class 4 | 0 | 72 | 12 |
| | FLAVIA Class 8 | 0 | 0 | 39 |
| **Algorithm 2** | | | | |
| Overall accuracy: 0.9545 | | Ground-Truth | | |
| | | Butterfly | FLAVIA Class 4 | FLAVIA Class 8 |
| Decision | Butterfly | 27 | 0 | 0 |
| | FLAVIA Class 4 | 0 | 72 | 4 |
| | FLAVIA Class 8 | 3 | 0 | 48 |
| **Algorithm 3** | | | | |
| Overall accuracy: 0.9740 | | Ground-Truth | | |
| | | Butterfly | FLAVIA Class 4 | FLAVIA Class 8 |
| Decision | Butterfly | 30 | 0 | 0 |
| | FLAVIA Class 4 | 0 | 72 | 4 |
| | FLAVIA Class 8 | 0 | 0 | 48 |

## 5 IMAGES WITH ”HOLES”

There are such morphologically complex objects (figures) that not only have an external contour, but also contain holes, that is, have internal contours, as shown in figure 10. It is obvious that holes shift the axis of symmetry in such images and image comparing procedure will give different result.

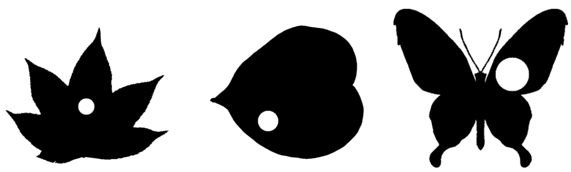

Figure 10: Examples of images with randomly added ”holes”.

Let us analyze how stable the proposed measure of similarity of images will be in case of changing the axis of mirror symmetry in the presence of holes in the image. In our experiment, we find the axis of symmetry in the image without holes (preliminary filling of internal contours is carried out), then the measure of similarity with respect to this line is calculated, but already in the image with a hole. As an alternative, you can consider the option when the axis of symmetry is determined directly from the image with holes without filling. Apparently, this approach can be used in cases where a comparison with an accurate reference image is performed, for example, the silhouette of a part on a conveyor is analyzed for the purpose of its rejection.

In this experiment, holes were artificially added to the images. In one case the hole was set a random center location and a radius in the range $[0.01R; 0.1R]$, where the radius described around the figure of the circle, the other radius is specified as a range $[0.01R; 0.2R]$.

Figure 11 shows graphs of the dependence of recognition quality on the number of nearest neighbors for images without holes, images with holes with a radius not exceeding 0.1 and 0.2 radius of the circumscribed circle. Algorithm 2 was used to compare the images in this experiment. We were puzzled by the fact that the results with small holes turned out to be better than for the original images, in future studies we will try to answer this question.

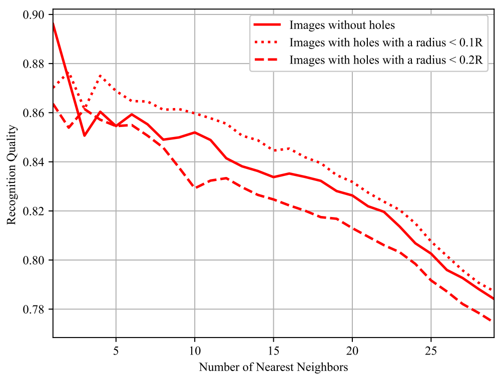


Figure 11: Graphs of the dependence of recognition quality for Algorithm 2 on the number of nearest neighbors for images with holes.

From the analysis of the curves in figures 9-11 and table 1 as a whole, it can be concluded that the proposed simple procedure can not only give quite acceptable quality in the problems of recognizing shapes, but also solve the problem of holes in the compared shapes.

# 6 CONCLUSIONS

The paper shows that even such a simple principle of image comparison as the Jaccard measure, but using information about the symmetry of the compared figures, allows to obtain a significant results. In particular, after the symmetry axes are found for a pair of compared images, it is proposed to combine these segments with each other and calculate the Jaccard measure. Three comparison algorithms have been developed and tested. The idea of method lies in the sphere of relational discriminant analysis - there is no need to hand craft some features for particular tasks. Good pairwise (dis)similarity function translate ptoblem of classification into the area of featureless pattern recognition. Proposed similarity measure can be used while solving featureless classification task with combination of another relational modalities of images (Mottl et al., 2005).

To determine the possibility of solving classification problems on the obtained similarity matrices, Bull's-eye test and recognition by the method of nearest centers were carried out. From the analysis of the results, it can be concluded that the proposed simple procedure can not only give quite acceptable quality (more than 0.9 classification accuracy rate) in the problems of recognizing shapes, but also possible to applied for the problem of holes in the compared shapes.

In our opinion, the use of the axis of symmetry for image comparison should give a more stable result in classification problems compared to the method based on the main axis (PCA) of the figure. We will try to test this hypothesis in future works.

# ACKNOWLEDGEMENTS

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# REFERENCES

Bai, X., Rao, C., and Wang, X. (2014). Shape vocabulary: A robust and efficient shape representation for shape matching. *IEEE Transactions on Image Processing*, 23(9):3935–3949.

Faloutsos, C. and Lin, K.-I. (1995). *FastMap: A fast algorithm for indexing, data-mining and visualization of traditional and multimedia datasets*, volume 24. ACM.

Fedotova, S., Seredin, O., and Kushnir, O. (2017). The parallel implementation of algorithms for finding the reflection symmetry of the binary images. *The International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences*, 42:179.

Hauagge, D. C. and Snavely, N. (2012). Image matching using local symmetry features. In *2012 IEEE Conference on Computer Vision and Pattern Recognition*, pages 206–213. IEEE.

Kushnir, O., Fedotova, S., Seredin, O., and Karkishchenko, A. (2016). Reflection symmetry of shapes based on skeleton primitive chains. In *International Conference on Analysis of Images, Social Networks and Texts*. Springer.

Kushnir, O., Seredin, O., and Fedotova, S. (2019). Algorithms for adjustment of symmetry axis found for 2d shapes by the skeleton comparison method. *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, 42(2/W12).

Lesot, M.-J., Rifqi, M., and Benhadda, H. (2009). Similarity measures for binary and numerical data: a survey. *International Journal of Knowledge Engineering and Soft Data Paradigms*, 1(1):63.

Li, Z., Xu, Y., Qu, W., Qi, H., and Cao, J. (2014). Robust symmetry detection for 2d shapes based on electrical charge distribution. *Journal of Information and Computational Science*, 11(9):2887–2894.

Mestetskiy, L. and Zhuravskaya, A. (2019). Method for assessing the symmetry of objects on digital binary images based on fourier descriptor. *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, 42(2/W12).

Mottl, V., Krasotkina, O., Seredin, O., and Muchnik, I. (2005). Principles of multi-kernel data mining. In *International Workshop on Machine Learning and Data Mining in Pattern Recognition*, pages 52–61. Springer.

Niu, D., Zhang, C., Li, W., and Zhou, Y. (2015). A novel approach for detecting symmetries in two-dimensional shapes. *Journal of Information and Computational Science*, 12(10):3915–3925.

Sheynin, S., Tuzikov, A., and Volgin, D. (1999). Computation of symmetry measures for polygonal shapes. In *International Conference on Computer Analysis of Images and Patterns*, pages 183–190. Springer.

Sun, C. and Si, D. (1999). Fast reflectional symmetry detection using orientation histograms. *Real-Time Imaging*, 5(1):63–74.

Van Otterloo, P. J. (1988). A contour-oriented approach to digital shape analysis.

Voevodin, V. V., Antonov, A. S., Nikitenko, D. A., Shvets, P. A., Sobolev, S. I., Sidorov, I. Y., Stefanov, K. S., Voevodin, V. V., and Zhumatiy, S. A. (2019). Supercomputer lomonosov-2: large scale, deep monitoring and fine analytics for the user community. *Supercomputing Frontiers and Innovations*, 6(2):4–11.

Yang, X., Adluru, N., Latecki, L. J., Bai, X., and Pizlo, Z. (2008). Symmetry of shapes via self-similarity. In *International Symposium on Visual Computing*, pages 561–570. Springer.