Matching-aware Shape Simplification

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Keywords: Polygon Simplification, Spatio-temporal Data, Shape Matching.

Abstract: Current research has shown significant interest in spatio-temporal data. The acquisition of spatio-temporal data usually begins with the segmentation of the objects of interest from raw data, which are then simplified and represented as polygons (contours). However, the simplification is usually performed individually, i.e., one polygon at a time, without considering additional information that can be inferred by looking at the correspondences between the polygons obtained from consecutive snapshots. This can reduce the quality of polygon matching, as the simplification algorithm may choose to remove vertices that would be relevant for the matching and maintain other less relevant ones. This causes undesired situations like unmatched vertices and multiple matched vertices. This paper presents a new methodology for polygon simplification that operates on pairs of shapes. The aim is to reduce the occurrence of unmatched and multiple matched vertices, while maintaining relevant vertices for image representation. We evaluated our method on synthetic and real world data and performed an extensive comparative study with two well-known simplification algorithms. The results show that our method outperforms current simplification algorithms, as it reduces the amount of unmatched vertexes and of vertexes with multiple matches.

1 INTRODUCTION

Over the last few decades, the amount of geographical data being generated is increasing greatly. Developments in sensor technology have expanded the availability of geographical data being used for several purposes. With the growing body of spatial data, there is also an increasing demand for tools to deal with spatio-temporal data. The applications can be as diverse as monitoring Glacier-Ocean boundaries, forest cover and iceberg tracking.

Spatio-temporal data is often acquired at discrete times and represented as an ordered sequence of snapshots. Each snapshot consists of a timestamp and a geometry. The acquisition of each snapshot can be summarized in the following steps: 1) obtain the raw data, 2) use image segmentation to extract the contour of the objects of interest and 3) apply simplification of the contour to obtain the polygon to be stored in a database. These steps are depicted in Figure 1. The extracted polygons are then interpolated as slices between two shapes (McKenney and Webb, 2010). Thus, spatio-temporal data can be represented by a geometry and a function used to represent its evolution over time (Forlizzi et al., 2000). Such high-level abstraction model treats real world entities as *moving points*, *moving lines* or *moving regions*. One key aspect of such representation is the selection of the function used to represent the data evolution, which is often called in the context of *moving regions* as *Region Interpolation Problem* (Tøssebro and Güting, 2001; Heinz and Güting, 2016). The interpolation methods derive the function of transition for a pair of shapes, and build the whole evolution by interpolating each pair of shapes sequentially. Any simplification algorithm that can be applied to a pair shapes can be used on the snapshots being interpolated. It can then be applied to a whole time series, two snapshots at a time, on the same way that interpolation methods are executed.

Region interpolation methods often require the definition of a matching between the vertices of two shapes representing an object of interest in two consecutive observations. *Shape Matching* or *Vertex Correspondence Problem* is the process of finding good matches (Liu et al., 2004; Van Kaick et al., 2011).

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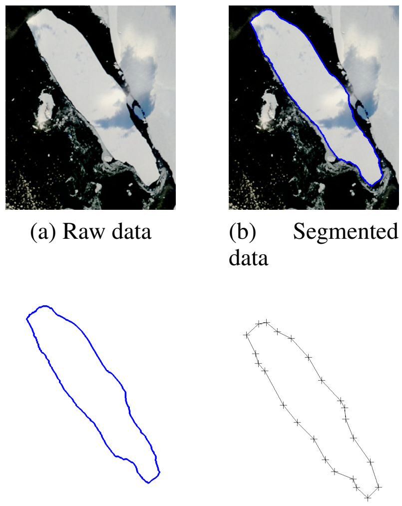
Miranda, E., Costa, R., Dias, P. and Moreira, J.
Matching-aware Shape Simplification.
DOI: 10.5220/0008969402790286
In *Proceedings of the 15th International Joint Conference on Computer Vision, Imaging and Computer Graphics Theory and Applications (VISIGRAPP 2020) - Volume 1: GRAPP*, pages 279-286
ISBN: 978-989-758-402-2; ISSN: 2184-4321



(a) Raw data (b) Segmented data

(c) Contour Extraction (d) Simplified Polygon

Figure 1: Data acquisition process.

There are two main issues that can arise during shape matching. First, some vertices in a shape may have no corresponding vertices in the other shape. Second, two or more vertices in a shape may correspond to a single vertex in the other shape. Such problems may be influenced by the simplification algorithm. Since some interpolation methods require an one-to-one correspondence between the vertices of the two shapes (Moreira et al., 2016), a post-processing step is usually need to fix these issues.

This paper presents a novel technique for simplification with a focus on matching that we call Matching Aware Simplification (MAS). Our technique is meant to be applied to two shapes of the same object at different times or in shapes to be matched. The focus is on preserving vertices that are important to define a correspondence with vertices of the other shape, which would be discarded by other simplifications algorithms (e.g. Ramer-Douglar-Peucker algorithms (Douglas and Peucker, 1973) and Visvalingam-Whyatt algorithm (Visvalingam and Whyatt, 1993)). Our technique is able to deliver polygons that lead to better automatic vertex matching. We evaluate the results on real world and artificial data, and we perform a wide numeric evaluation over a publicly available dataset.

This paper is organized as follows. The next section presents an overview on related work. Section 3 details the proposed method. Section 4 presents the experimental results. Finally, Section 5 presents the conclusion and guidelines for future work.

2 BACKGROUND AND RELATED WORK

2.1 Polygon Simplification

Image segmentation algorithms return all pixels (vertices) defining the boundaries of an object of interest. That means the representation of the objects' geometry is noisy and may have hundreds or thousands of vertices. However, a large number of vertices can be detrimental to the performance of algorithms, e.g., to find a correspondence between the vertices of two shapes (Shape Matching), or to estimate the transformation of an object between two snapshots (Region Interpolation). In addition, noisy boundaries can create unwanted behavior during the interpolations.

Boundaries and shapes can be simplified using different families of algorithms. The first algorithm for simplification was the Ramer-Douglas-Peucker (RDP) algorithm, published in 1972 (Ramer, 1972) and 1973 (Douglas and Peucker, 1973). In the RDP algorithm, a line segment is defined between the first and last vertexes of a polyline to be simplified. Then, the most distant vertex from this line segment is included in the simplified line (a polyline) - as long as the vertex distance is over an accepted threshold value (ξ). This process is now applied recursively on all sub lines on the polyline until there are no vertices over ξ distance from the line. In order to apply this process to a polygon, we need to pick two vertices as reference vertices for the polylines.

Another common approach to polygon simplification is using Visvalingam-Whyatt (VW) algorithm (Visvalingam and Whyatt, 1993). In the VW algorithm, for every vertex in a line or polygon, we build a triangle between this vertex and the previous and next vertices. Then, the area of each triangle is calculated, and all vertices of triangles with an area below a threshold ξ are removed. This process is applied iteratively until no triagle has an area below ξ.

Accounting that the morphing or interpolation happens between two different shapes, one can use this additional information to create different simplifications. Baxter et al. (Baxter III et al., 2009) proposed a method for compatible embedding of two shapes, and later generalized the approach for multiple shapes (Baxter et al., 2009). This method reduces the number of vertices on a polygon, however to keep the textures inside the polygon during the morphing process, the reduction of vertices aims at embedding and not at simplification of the geometry. While suited for morphing animations, embedding creates deformations on the polygon when used on real world data.

Take iceberg tracking for instance, where one would rather be close to the original shape and area than ensure embedding on a bigger polygon that loses information.

These techniques are characteristic in that they are suited for simplification of a single polygon or polyline, with minimal information loss (RDP and VW), or they are suited for animation (embedding). But they have no regard to the temporal behaviors of the shape. Our method, on the other hand, uses the information of two consecutive representations of the same shape in order to simplify each object. We maintain vertices that are more relevant to represent the temporal evolution of the object.

2.2 Shape Matching

In order to use the region interpolation functions on the continuous spatio-temporal data representation, simplified polygons generated for consecutive snapshots need to be matched. The matching can be based on segments or concavities (Tøssebro and Güting, 2001) (McKenney and Webb, 2010) or based on vertices (Moreira et al., 2016) (Baxter III et al., 2009). The Shape Matching algorithms must provide a correspondence between the vertices of two shapes, as depicted in Figure 2. It can also be seen that some vertices might have no match, or match more than one vertex. There are surveys on Shape Matching or Vertex Correspondence Problem (Van Kaick et al., 2011).

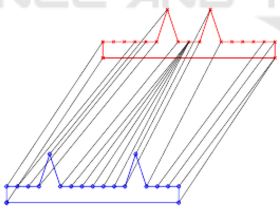


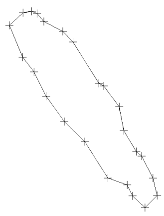
Figure 2: Vertex matching (Van Kaick et al., 2007).

One algorithm with good results for fully automatic 2D vertex matching is presented on (Van Kaick et al., 2007). This algorithm performs vertex matching so that the vertices matched are both in similar places in space, while also aiming at keeping the geodesic distance (distances within the contour) between every pair of vertices similar. The additional requirement that a pair of vertices in one shape maintain similar distances to the matched pair of vertices on the other shape leads to a matching that is relatively robust.

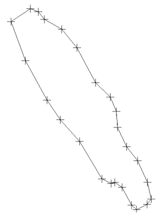
Since the existing simplification algorithms are focused exclusively on the simplification of shapes one-by-one, two implications arise: 1) the simplification may select vertices in a shape that are distant from the

(a) Raw data from ice01

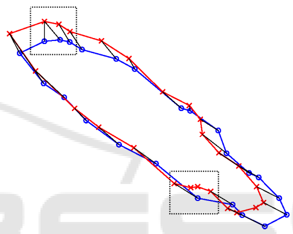
(b) Raw data from ice02



(c) Polygon from ice01



(d) Polygon from ice02



(e) Vertex matching for B-15a

Figure 3: B-15a extraction and matching.

possible corresponding vertices in the other shape; or 2) the corresponding parts of two shapes may have a different number of vertices. These cases are depicted in Figure 3, where the two polygons represent the shape of iceberg B-15a at different times (from now on called *ice01* and *ice02*). The shapes were simplified using RDP and the correspondences between the vertices were performed using (Van Kaick et al., 2007).

3 MULTIPLE-SHAPE AWARE SIMPLIFICATION

The workflow to prepare spatio-temporal data for interpolation can be summarized in 3 main steps (Duarte et al., 2018). First, we extract the region-of-interest from each raster image (Segmentation). Then, the amount of vertices on the shape is reduced (Simplification). Finally, a one-to-one correspondence between the vertices must be found. In this workflow, only the third step takes into account more than one shape. We aim to improve this process by extending the use of information from a pair of shapes into the simplification step, reducing matching errors.

3.1 Compatible Simplification

Our method relies on implicit information, as we know that we are simplifying a pair of related shapes instead of an individual shape. We call this simplification of two shapes simultaneously as compatible simplification. The aim is to be able to consider the importance of the vertices to represent a shape at a given time and also their importance to establish correspondence with vertices in the other shape. For instance, we want to keep vertices that represent distinct features in a shape. We also want vertices that should represent that distinct feature in the matched shape. Our method operates before the vertex correspondence problem, avoiding the need of optimization and user interaction. This method can also be paired with any correspondence algorithm.

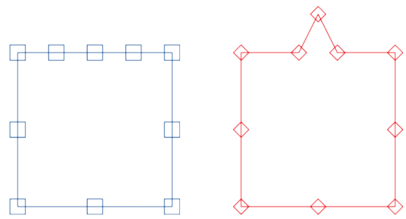
Our method has two main objectives and one minor objective. The first is to simplify a polygon in a way that will reduce the need for adjustments during the matching step, like adding extra vertices to solve the vertex correspondence problem. Notice that the added vertices were part of the full set of vertices before simplification. Re-adding a vertex into a contour that was removed during simplification means that local information was lost. Since our method deals with simplification using knowledge that is relevant to perform vertex correspondences in a later step, we are call it Matching Aware Simplification (*MAS*). Our proposed method strives to keep vertices that will be representative on future or past shapes by design.

The second main objective is to allow simpler matching by providing matching algorithms with locally-aware vertices in order to reduce vertex matching complexity. Since algorithms can work with locality of data in order to help finding the vertex correspondences (Van Kaick et al., 2011), providing the algorithms with better locality should help the matching process.

In order to keep vertices that allow better matching, similar sub regions should be represented with similar resolution or density of vertices. This minor objective follows from the two main objectives.

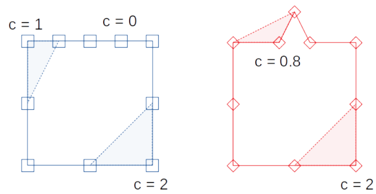
3.2 The MAS Algorithm

In the following, we consider the raw contours to be simplified as P, the source polygon, and the matched polygon as Q, the target polygon. We assume that P and Q were previously aligned to account for both rotation and translation. We also define $p \in P$ and $q \in Q$



(a) Source (P) (b) Target (Q)

Figure 4: Artificial Polygons for illustration.



(a) Source (P) (b) Target (Q)

Figure 5: Local cost for vertices (triangle area).

as vertices. Figure 4 presents two artificial polygons that will be used to illustrate cost calculations.

We start by defining a cost function for each vertex, which represents the loss of information for removing this vertex (Equation 1). It introduces a parameter, t_factor, representing the preference between keeping temporal information or single shape information.

$$cost_p = max(cost\_single_p, cost\_matched_p * t\_factor) \tag{1}$$

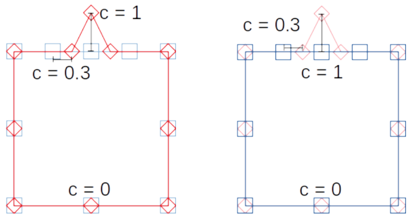
In order to consider the loss of information on of P and Q, we assume the cost to remove the vertex p as the maximum of the cost to remove that vertex considering a single shape (P) and of the cost for loss of feature representation on the matched shape (Q).

We define the cost to remove a vertex for a single shape as the area of the triangle given by p, $p-1$ and $p+1$, similar to VW algorithm (Equation 2). It is also possible to use other measures, like the distance between p and the line connecting $p-1$ and $p+1$, which would lead to a MAS simplification based on the RDP algorithm. Any future cost metric for single-shape simplification could also be applied at this step. Figure 5 shows a calculation for some vertices.

$$cost\_single_p = area\_triangle(p, p+1, p-1) \tag{2}$$

We represent the loss of matching information on two different steps. A significant vertex represents either a feature present in Q and not in P, or a vertex on P needed to morph into the feature of Q. The first case where P has a distinct and unique feature (like a curvature or a distinctive deformation), the vertex has a $cost\_unique\_feature$ as seen on Equation 3. This cost measures how distant the vertex is from the other shape - a vertex far from the other shape represents a distinct feature. Figure 6a shows this metric for three different vertices of Q.

$$cost\_unique\_feature_p = min(d_{pq}) \forall q \in Q) \tag{3}$$



(a) Distinct features cost on Q (P on background) (b) Matching vertices on P (Q on background)

Figure 6: Matching costs.

We also define a cost for a matched feature, which is complementary to the distinct feature. This cost is defined in Equation 4. This cost prioritizes vertices of P that are the closest vertices for a distinct feature of Q, and could be good candidates for future matching. In the example in Figure 6b, the vertex at the top (with $c = 1$) is important to morph into the topmost vertex of Q.

$$\begin{aligned} cost\_matched\_feature_i &= max(d_{pq}) \forall q \in Q| \\ d_{pq} &= min(d_{kq}) \forall k \in P \end{aligned} \tag{4}$$

Finally, we define the loss of information of vertex p in the matching with Q as the maximum of the two complementary costs, as seen on Equation 5.

$$\begin{aligned} cost\_time_i = max(&cost\_unique\_feature_p, \\ &cost\_matched\_feature_i) \end{aligned} \tag{5}$$

Given this definition of cost, our $SIMPLIFY$ function starts by removing the vertex in P with lowest cost, then removing the vertex in Q with lowest cost, and iterating as many times as necessary to achieve the desired number or vertices in the simplified polygon.

function SIMPLIFY(P, Q, size)
  while $\|P\| > size \vee \|Q\| > size$ **do**
    if $\|P\| > size$ **then**
      $r \leftarrow p \in P | cost(p) = min(cost(k) \forall k \in P)$
      $P \leftarrow P - r$
    end if
    if $\|Q\| > size$ **then**
      $r \leftarrow q \in Q | cost(q) = min(cost(k) \forall k \in Q)$
      $Q \leftarrow Q - r$
    end if
  end while
end function

4 EXPERIMENTAL RESULTS

In order to evaluate our proposals, we made several experiments on publicly available datasets of artificial and real world data, and compared the use of state-of-the art simplification algorithms. In all experiments, the tolerance for each algorithm until 95% of the vertices of the original contour were removed. This way, the algorithms can be compared because they all keep the same amount of information.

In Sections 4.1 and 4.2, we discuss the simplification quality presenting a visual analysis of selected pairs of images. Then, in Section 4.3, we present a detailed numeric comparison of MAS matching quality (using several metrics) for every pair of image in the 216 Binary Shape Database from Brown University (Sebastian et al., 2004).

4.1 Visual Qualitative Analysis - Artificial Data

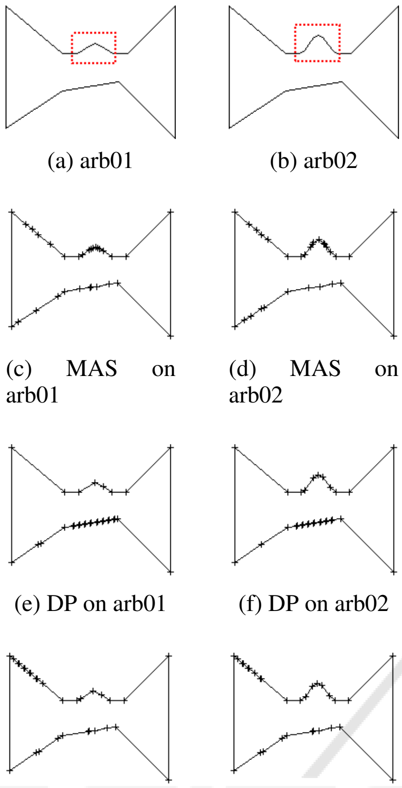
For the artificial data, we used two shapes from the Brown University Binary Image dataset (Sebastian et al., 2004), named arb01 and arb02. We can see that for both arb01 and arb02 (Figure 7) the simplified shapes cannot be visually distinguished from the original image, leading to similar results of the contour features.

Considering arb01, the feature highlighted in Figure 8 can be represented using few vertices, since its shape is triangular. However, in arb02, more vertices are needed to represent the same feature, because the shape is a round curve. It is important to keep a similar number of vertices in arb01 and arb02 for the vertex matching. This is depicted in Figure 8, where the simplification algorithms are compared side-by-side, showing how MAS keeps more points on the details than both RDP and VW.

Our method chooses the vertices to be kept considering the context of the target shape. We can see on Figure 7 that RDP and VW keep vertices on lines that could be considered not-important (because they do not play an important role on the definition of the source and target shapes or on the definition of the correspondences between them), like the lower inclined line in RDP or the left upper line in VW.

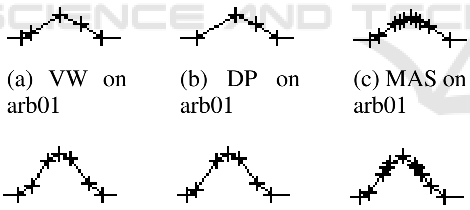
4.2 Visual Qualitative Analysis - Real-world Data

For the real world data, we used two images of Iceberg B-15a taken at different times (Figure 3) (RossSea subsets, 2016). Each simplification method produces a slightly different shape, as we can see in



(a) arb01 (b) arb02 (c) MAS on arb01 (d) MAS on arb02 (e) DP on arb01 (f) DP on arb02 (g) VW on arb01 (h) VW on arb02

Figure 7: Comparison of arb01 and arb02.

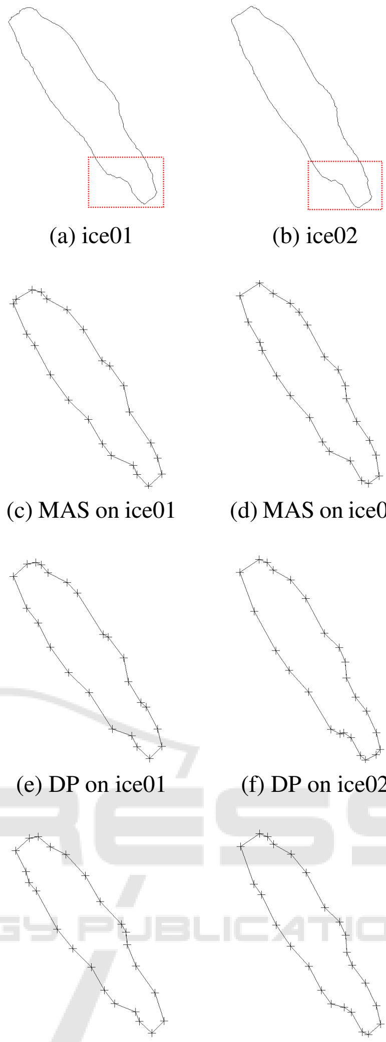


(a) VW on arb01 (b) DP on arb01 (c) MAS on arb01 (d) VW on arb02 (e) DP on arb02 (f) MAS on arb02

Figure 8: Highlight on the feature-area representation to all simplification algorithms.

Figure 9. However, the results of the methods are visually very similar.

It is visually perceptible in the highlighted area of Figure 9 that the distribution of the vertices in the simplified shapes is more similar in our method (considering the number of vertices and the spacing between them). This better distribution of number of vertices and spacing can also be seen on the left side of the shape. Thus, our method performed as expected, providing more similar distributions of vertices on both shapes with noisy borders.



(a) ice01 (b) ice02 (c) MAS on ice01 (d) MAS on ice02 (e) DP on ice01 (f) DP on ice02 (g) VW on ice01 (h) VW on ice02

Figure 9: Comparison of ice01 and ice02.

When dealing with real world data the borders are noisy, and the noise can influence RDP or VW on selecting the vertices to keep. Our goal was to remove these influences by using information from the other polygon and we can see in Figure 9 that our method provides a more similar density of vertices along the boundaries.

4.3 Simplification and Matching Analysis - Binary Shape Database

In order to provide a complete evaluation, we applied our method on the 216 Binary Shape Brown dataset (Sebastian et al., 2004). It is composed of 216 images divided in 18 classes. This dataset is widely used in polygon matching and image retrieval benchmarks.

For the testing, we first aligned the shapes using the Iterative Closest Point method (Tihonkih et al., 2016). For the hammer class the ICP method failed to provide several correct rotations of the objects, and so, they were removed from the test dataset.

We evaluated the performance of the simplification in the Vertex Correspondence Problem. For this test, we used the VCP algorithm developed by (Van Kaick et al., 2007) and recorded the number of vertices without a correspondence and the number of vertices with multiple matches, and considered both cases as anomalous vertices.

Since the VCP algorithm is an heuristic, it can generate distinct results on consecutive executions. To account for this, we ran 35 replications for each pair of images. Our method was able to be the better than RDP and VW on 35% of the images, and being the best tied with either RDP or VW on 21%. Thus, our method was the best choice on more than half of the images. The results can be seen on Figure 10.

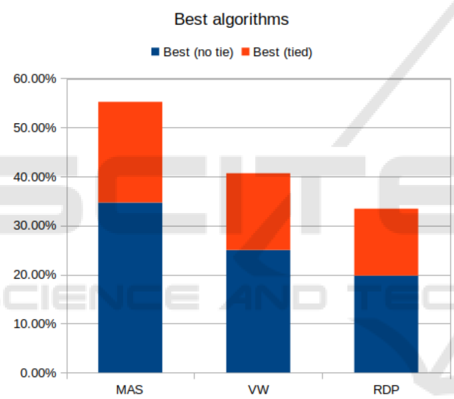


Figure 10: Performance comparison between algorithms.

It is also important to evaluate the gap of performance instead of just which algorithm is best. For this analysis, we compared the number of anomalous points generated by each method. Figure 11 presents these results. Our method is able to improve by a wide margin on some categories. However, when our method is not the best the gap is much smaller. Overall, our method can perform significantly better on the majority of the cases, or slightly worse on a few cases.

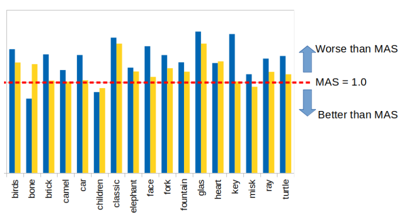


Figure 11: Anomalous vertices generated.

Finally, we selected a pair of shapes for visual assessing of our method (Figure 12). It can be seen that MAS represents all fork prongs with 2 points in both shapes. Both VW and RDP chose to simplify matching prongs with a different number of points on each shape. Also, on the fork handle,the number of pointed obtained using VW and RDP greatly differs, and MAS performs with closer density on both shapes. These findings are highlighted in the images.

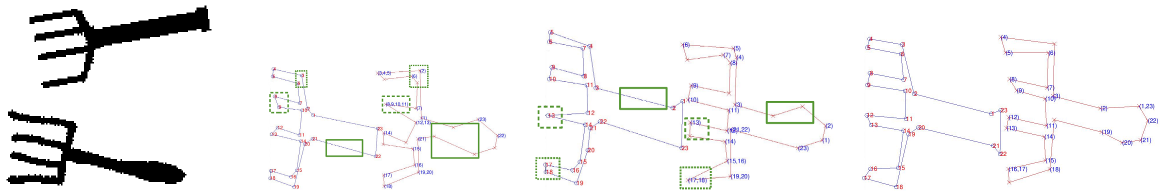
5 CONCLUSION

Current technology has enabled us to gather a rich set of spatial data, leading to a growing body of historical data. This has led to an increased interest in spatio-temporal data processing and analysis.

Using the continuous spatio-temporal model to represent real world data involves several steps, including image acquisition and segmentation, object simplification and shape matching. Although data acquisition can have a significant impact on the quality of the data, few works exist on transforming raw data into spatio-temporal data representations. Also, after image acquisition and segmentation the set of points of the contour has to be simplified in order to obtain a polygon. Current simplification algorithms account only for a single shape at a time. This can lead to a loss of information about the evolution of the shapes over time.

In this work, we deal with shape simplification for polygon matching. We presented an algorithm for simplification of 2D polygons. Our method makes use of implicit information that arises from the knowledge that we have more than one shape to be matched. Our Matching Aware Simplification method improves simplification quality, leading to less anomalous points during matching than other simplification methods. Our method can also be combined with any matching algorithm on the next stage leading to improved matching results. The proposed simplification technique should allow for easier automatic correspondence of vertices on real world phenomena shapes.

In the future, we are interested in expanding research on the next step of the spatio-temporal acquisition workflow: the vertex correspondence problem. Since it is expected that avoiding to remove vertices from a shape (source) that may have a correspondence with vertices of another shape (target) in a sequence of observations will lead to more natural interpolations, we also aim at developing workflows to evaluate the impact of each step of the process on the quality of the



(a) Fork19.pgm and Fork03.pgm (b) VW matching (c) RDP matching (d) MAS matching

Figure 12: Matchings between Fork03 and Fork19.

interpolations. Finally, we would also want to work on all steps of the interpolation process of multiple snapshots, instead of two - including simplification, matching and interpolation functions.

ACKNOWLEDGEMENTS

This work is partially funded by National Funds through the FCT in the context of the projects UID/CEC/00127/2013 and POCI-01-0145-FEDER-032636.

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