Comparison of Continuous and Discontinuous Charging Models for the Electric Bus Scheduling Problem

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Abstract: Electric buses offer an alternative to conventional vehicles in the public transport system due to their low operational costs and low emissions. Therefore, the standard problems must be resolved with respect to the nature of the electric buses, which is mainly the reduced driving range and charging time. In this paper, we deal with the electric bus scheduling problem. We propose changes to the previously presented model, which allowed only the continuous charging. These changes will allow the model to describe also discontinuous charging when the electric bus can be unplugged during the charging, then let another electric bus to charge and after that plug-in to charger again. These two formulations are tested by IP solver and the solutions and performance of both discontinuous charging and continuous charging formulations are compared on the datasets generated from the data provided by public transport system provider DPMŽ in the city of Žilina.

1 INTRODUCTION

In recent years the importance of electric vehicles is increasing. The countries are trying to improve the ecology and therefore reduce the emissions. The electric vehicles are a way that can reduce CO_2 emissions. From the point of public transport system providers, the use of electric vehicles can reduce operational costs.

With the application of the electric vehicles in the public transport system, different problems must be addressed. These problems are for example line planning, vehicle, and crew scheduling. The electric vehicles have reduced driving range, based on the capacity of the battery and also the charging time must be considered, because it is much longer than the time needed to refuel a conventional vehicle. Due to these facts, the problems must be remodeled and new methods to solve these problems need to be proposed.

In our paper, we address the electric bus scheduling problem (EBSP) which is a special case of vehicle scheduling problem with the constraints of energy and driving range. In this problem, we are assigning electric buses to service trips, that need to be served. In this paper, we are proposing changes to our previously presented mathematical model (Janovec and Kohání, 2019b; Janovec and Kohání, 2019a) which allowed only continuous charging, that would enable discontinuous charging. That means during the charging the unplugging is possible and then after an interval of waiting the bus can be plugged in again and continue charging. The discontinuous charging has the potential to improve some types of objectives like minimizing the charged energy but can worsen solutions of different objectives like time spent waiting. In this paper, we are researching the objective of minimizing the number of used electric buses. Due to the complexity of the EBSP, we do not consider the schedule of the drivers, therefore we assume that the bus can be driven by different drivers and it is possible to change drivers during the duty of the electric bus.

The proposed model is tested with an IP solver on several datasets to find if discontinuous charging changes the results obtained by the continuous charging model and the changes in the computational time.

In section 2 the state-of-the-art in the field is mentioned. Then in section 3 the problem of scheduling electric buses is described and a linear mathematical model for continuous charging is mentioned. Also, the changes to the model that would enable discontinuous charging are proposed in this section. In section 4 the numerical experiments are described and the results we obtained are discussed. The last section concludes the research and suggests possible future possibilities.

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2 RELATED WORK

The vehicle scheduling problem is a well-researched problem and a lot of variants are known. Some basic ideas on how the vehicle scheduling problem is modeled and solved were mentioned in (Bunte and Kliewer, 2009). There the authors present an overview of the models for a single depot vehicle scheduling problem as well as models for a multiple depot scheduling problem. The basic ideas, how the problem is solved, are mentioned for each model. The authors also present different extensions of the vehicle scheduling problem, like heterogeneous fleet, time windows and route constraints. All these extensions make the scheduling an NP-hard problem, but for some specific cases, polynomial algorithms exist. One example is a two depot vehicle scheduling problem, for which a specific solution method based on the graph theory was proposed in (Czimmermann, 2006).

The scheduling of electric vehicles has two directions based on the used technology, specifically the battery exchange system and the electric bus charging system. The scheduling with the battery exchange system was addressed in (Kim et al., 2015) where the authors proved the usability of the system by simulation from the data gathered during an experimental application of the electric buses in Soul. Scheduling of electric buses with the battery exchange system was researched by (Chao and Xiaohong, 2013), where the authors present a mathematical model for the single depot vehicle scheduling problem with two objective functions. This problem was solved by the Non-Dominated Sorting Genetic Algorithm.

The bus charging system is specific by charging the buses during their operation or during the night in the depot. The scheduling problem for this technology was addressed in (Sassi and Oulamara, 2017), where a linear mathematical model for the scheduling problem was presented. Besides the standard constraints of the battery, the model includes constraints of maximal charging power that can be obtained from the power grid. The authors also proposed two heuristic algorithms to solve the problem and proved the NP-hardness of the electric bus scheduling problem.

Next, two mathematical models were proposed by (van Kooten Niekerk et al., 2017). The first model assumed that charging is a linear process and the second model was able to describe also a non-linear charging process by discretization of the battery energy state. The proposed models were solved by exact methods and by the column generation method.

Non-linear model for the electric bus scheduling problem was presented by (Rogge et al., 2018) and was solved by grouping genetic algorithm. This model considered only charging at one location, specifically the depot. Also, the electric bus always charges to the maximum capacity.

All of the above-mentioned authors who researched the electric bus scheduling problem with bus charging technology assumed the bus is charging continuously. Therefore, we focus on the discontinuous charging, where the bus can be unplugged during the charging and then again plugged-in after some time of not charging. The possible advantages of this approach to charging electric vehicles were mentioned in (Yanjin et al., 2016).

3 PROBLEM DESCRIPTION

In the problem of the electric bus scheduling problem (EBSP) we assign available electric buses to the tasks that need to be served, which create a schedule for each electric bus. The schedule is composed of two different tasks (Fig. 1). The first type of task is serving a service trip, which is a required type of task. The second type of task is charging, when the electric bus is plugged into a charger and it charges the battery. This type of task is voluntary and is performed only when needed.

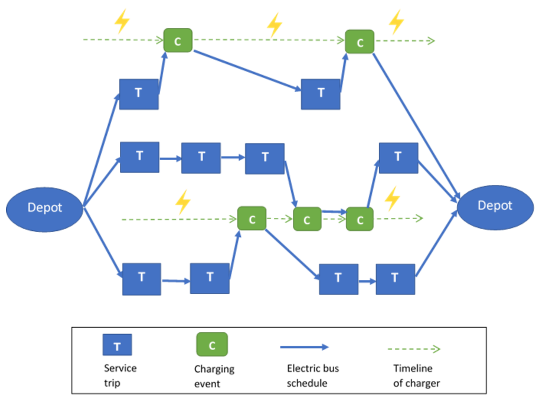

Figure 1: Illustration of electric bus schedules with arcs connecting tasks of one electric bus schedule and arcs connecting charging events on one charger.

Each schedule of electric buses must met certain conditions to be called feasible. The most important condition is that each service trip is assigned to one electric bus. Also, the electric bus can not be assigned to more trips simultaneously. Furthermore, due to the nature of electric buses, we add conditions that the bus must have enough energy during the whole schedule and that the battery capacity can not be exceeded during the charging.

3.1 Formal Formulation of the Problem

In the models, we use the set N of all the service trips. Next, we add the depot, where the morning depot is represented by the node D_0 and the evening depot is represented by the nodes D_n, where we add one possible depot node for each service trip. The set R is a set of all chargers. At each charger $r \in R$ we have T^r charging events, which are time intervals. Lastly, the set of all electric bus types is denoted as K.

Each service trip $i \in N$ is defined by the start time s_i and its duration t_i. The energy consumption during the trip is denoted as constant c_i. Next, the constant t_{ij} represents the time of transfer between the end terminal of trip i and the start terminal of the trip j. The energy consumed during this transfer trip is represented by constant c_{ij}.

A charger $r \in R$ has its charging speed q_r defining how much energy is charged during one unit of time. The next needed information about the charger is its location. This is represented by the travel time between the end terminal of service trip i and the charger r denoted as t_{ir} with the energy consumption of c_{ir}. Location is also defined by the travel time t_{rj} between the charger r and the starting terminal of service trip j with the energy consumption of c_{rj}.

At each charger r we defined a set of charging events T^r. Each charging event is derived from the service trip. In other words, we create a charging event on each charger for every service trip. The charging event t at charger r is characterized by its starting time s_{rt}, which is connected to the corresponding service trip and is defined as $s_{rt} = s_i + t_i + t_{ir}$, where s_i is starting time of corresponding service trip, t_i is its duration and t_{ir} is the transfer time between the trip and the charger. Also, the charging events in the set T^r at charger $r \in R$ are ordered by ascending starting time, which divides the whole available charging time into intervals with different length

An available electric bus type $k \in K$ is characterized by its battery. The battery has maximal capacity represented by constant SoC^k_{max} for each bus type k. We also define a minimal battery capacitySoC^k_{min} for each bus type k, which can be used as the minimum reserve of energy.

3.2 Mathematical Models

In this section, we list both of the linear mathematical models for the problem of scheduling of electric buses. The first model is specific by continuous charging and the second represents also the alternative of the discontinuous charging.

3.2.1 Continuous Charging Model

This model was presented in our previous work (Janovec and Koháni, 2019a; Janovec and Koháni, 2019b), and there only continuous charging is enabled. That means the electric bus after plugging-in to charger can start charging, but after unplugging, the bus must continue to serve a service trip.

In this model we use sets F_i, B_i, Fc_{ri}, Bc_{ri} for each charger $i \in N$, which are used to reduce the number of the decision variables. Set F_i represent all possible following service trips, to which the electric bus can transfer after the end of the service trip i. In other words for trip j to be in set F_i of trip i the condition $s_j \geq s_i + t_i$ must hold, where s_j is starting time of service trip j, s_i is starting time of trip i and t_i is duration of service trip i. Similarly the set B_i is a set of all possible previous service trips to trip i.

The set Fc_{ri} is connected to charging events and it contains the charging events at charger r which can be visited after finishing the service trip i. For each charging event $t \in T^r$ at charger $r \in R$ in the set Fc_{ri} the condition $s_{rt} \geq s_i + t_i + t_{ir}$ must hold, where s_{rt} is starting time of charging event t at charger r, s_i is starting time of trip i, t_i is duration of trip i and t_{ir} is transfer time between ending point of trip i and charger r. Similarly the set Bc_{ri} is a set of all possible previous charging events at charger r of service trip i, for which the condition $s_{rt} + t_{ri} \leq s_i$ is true, where t_{ri} is transfer time from the charger r to the starting point of trip i.

For each charging event t at charger r the set $Fi(r,t)$ define the following service trips and set $Bi(r,t)$ define previous charging events. For trip i to become a part of set $Fi(r,t)$ the condition $s_{rt} + t_{ri} \leq s_i$ must be satisfied. Similarly the condition $s_{rt} \geq s_i + t_i + t_{ir}$ must hold for trip i to be in set $Bi(r,t)$ of charging event t at charger r.

Next, we define the decision variable x^k_{ij} which describes the decision to serve the service trip j just after serving service trip i with the vehicle k. The next variables y^k_{irt} and z^k_{rtj} are connected to the transfer to and from charger. Variable y^k_{irt} represents the transfer from the end terminal of service trip i to the charger r to charge during the charging event t with the vehicle k. The transfer from the charger r after charging during charging event t to the starting terminal of service trip j by the vehicle k is represented by decision variable z^k_{rtj}.

The variable w^k_{rt} represents the principle of continuous charging. The variable represents the decision to continue charging during the following charging event $t+1$ which begins just after the end of charging event t at charger r with the vehicle k.

To keep track of the energy state of the battery in each used electric bus the last two variables e_i^k and ε_{rt}^k are introduced. The variable e_i^k is energy state of bus k just before the service trip i. The variable ε_{rt}^k is similar, but it represent the energy state of bus k just before the start of charging event t at charger r.

Objective

$$minimize \sum_{k\in K}\sum_{j\in F_{D_0}} x_{D_0 j}^k + \sum_{k\in K}\sum_{r\in R}\sum_{t\in Fc_{rD_0}} y_{D_0 rt}^k \qquad (1)$$

The objective function (1) minimizes the number of used electric buses, where the first sum is the number of electric buses that depart from depot to the service trip and the second sum is the number of electric buses that depart from depot to the charger.

Vehicle Scheduling Constraints

$$\sum_{k\in K}\sum_{i\in B_j} x_{ij}^k + \sum_{k\in K}\sum_{r\in R}\sum_{t\in Bc_{rj}} z_{rtj}^k = 1 \quad \forall\, j\in N \qquad (2)$$

$$\sum_{k\in K}\sum_{j\in Bi_{rt}} y_{jrt}^k + \sum_{k\in K} w_{rt-1}^k \leq 1 \quad \forall\, r\in R, t\in T^r \qquad (3)$$

$$\sum_{i\in B_j} x_{ij}^k + \sum_{r\in R}\sum_{t\in Bc_{rj}} z_{rtj}^k = \sum_{l\in F_j} x_{jl}^k + \sum_{r\in R}\sum_{t\in Fc_{rj}} y_{jrt}^k \quad \forall\, j\in N, k\in K \qquad (4)$$

$$\sum_{i\in Bi_{rt}} y_{irt}^k + w_{rt-1}^k = \sum_{j\in Fi_{rt}} z_{rtj}^k + w_{rt}^k \quad \forall\, r\in R, t\in T^r, k\in K \qquad (5)$$

To ensure that each service trip is served the conditions (2) are used. The constrains (3) are connected with a condition that during one time interval only one electric bus can be charged at the charger. The constraints (4) are standard flow constraints that ensure the same bus which was assigned to serve the service trip would be assigned to serve the next task. The constraints (5) are flow constraints also, but they are connected to the charging events. That means the bus which arrived to charge during a charging event also leaves to serve a service trip or continue to charge during the next interval after the end of the charging event.

Energy Consumption Constraints

$$e_{D_0}^k = SoC_{max}^k \quad \forall\, k\in K \qquad (6)$$

$$e_i^k \geq SoC_{min}^k + c_i + \sum_{j\in F_i} x_{ij}^k c_{ij} + \sum_{r\in R}\sum_{t\in Fc_{ri}} y_{irt}^k c_{ir} \quad \forall\, i\in N, k\in K \qquad (7)$$

$$e_j^k + c_{rj} + Mq_r(1-z_{rtj}^k) \geq SoC_{min}^k + z_{rtj}^k c_{rj} \quad \forall\, r\in R, t\in T^r, k\in K, j\in Fi_{rt} \qquad (8)$$

$$e_j^k \leq e_i^k - x_{ij}^k(c_i + c_{ij}) + SoC_{max}^k(1-x_{ij}^k) \quad \forall\, j\in N, i\in B_j, k\in K \qquad (9)$$

$$e_j^k \geq e_i^k - x_{ij}^k(c_i + c_{ij}) - SoC_{max}^k(1-x_{ij}^k) \quad \forall\, j\in N, i\in B_j, k\in K \qquad (10)$$

$$\varepsilon_{rt}^k \leq e_i^k - y_{irt}^k(c_i + c_{ir}) + SoC_{max}^k(1-y_{irt}^k) \quad \forall\, r\in R, t\in T^r, k\in K, i\in Bi_{rt} \qquad (11)$$

$$\varepsilon_{rt}^k \geq e_i^k - y_{irt}^k(c_i + c_{ir}) - SoC_{max}^k(1-y_{irt}^k) \quad \forall\, r\in R, t\in T^r, k\in K, i\in Bi_{rt} \qquad (12)$$

The constraints (6) initialize the battery capacity to the maximum before the start of the working day for each electric bus. To ensure the bus has enough energy to drive the service trip and the following transfer the constraints (7) are used. In these constraints, the constant SoC_{min}^k is used as a lower limit of the battery energy state, which can be understood as the energy reserve of the battery. A similar condition is represented by the constraints (8), which ensures the bus has enough energy to transfer to the following service trip after the charging.

One of the conditions which must be satisfied is the energy preservation. To ensure this condition the constraints (9) - (12) are introduced. The preservation of energy between two consecutive service trips is defined by a pair of constraints (9) and (10). The next pair of constraints (11) and (12) preserve the energy between the service trip and the following charging event.

Charging Constraints

$$e_j^k + c_{rj} - \varepsilon_{rt}^k + SoC_{max}^k(1-z_{rtj}^k) \geq 0 \quad \forall\, r\in R, t\in T^r, k\in K, j\in Fi_{rt} \qquad (13)$$

$$\varepsilon_{rt+1}^k - \varepsilon_{rt}^k + SoC_{max}^k(1-w_{rt}^k) \geq 0 \quad \forall\, r\in R, t\in T^r, k\in K \qquad (14)$$

$$e_j^k + c_{rj} - Mq_r(1-z_{rtj}^k) \leq SoC_{max}^k \quad \forall\, r\in R, t\in T^r, k\in K, j\in Fi_{rt} \qquad (15)$$

$$\varepsilon_{rt+1}^k - Mq_r(1-w_{rt}^k) \leq SoC_{max}^k \quad \forall\, r\in R, t\in T^r, k\in K \qquad (16)$$

$$e^k_j \leq \varepsilon^k_{rt} + z^k_{rtj}((s_j - t_{rj} - s_{rt})q_r - c_{rj}) + SoC^k_{max}(1 - z^k_{rtj}) \quad \forall j \in N, r \in R, t \in Bc_{rj}, k \in K \tag{17}$$

$$\varepsilon^k_{rt+1} \leq \varepsilon^k_{rt} + w^k_{rt}(s_{rt+1} - s_{rt})q_r + SoC^k_{max}(1 - w^k_{rt}) \quad \forall r \in R, t \in T^r, k \in K \tag{18}$$

$$e^k_j + c_{rj} - \varepsilon^k_{rt} - SoC^k_{max}(1 - z^k_{rtj}) \leq (s_{rt+1} - s_{rt})q_r \quad \forall r \in R, t \in T^r, k \in K, j \in Fi_{rt} \tag{19}$$

The constraints (13) and (14) serves as a limitation that the charged energy must be non-negative. Similarly, the constraints (15) and (16) define the upper bound of the battery energy state, which means the maximal capacity of the battery is not exceeded during the charging. Specifically, constraints (13) and (15) are connected to a charging event which is followed by the service trip and the constraints (14) and (16) are connected to charging event followed by the next charging.

The next constraints (17), (18) and (19) limit the available charging time based on the decisions. It is defined that the ending time of the charging event is variable. But the time is limited by the start of the following service trip if the bus continues to the service trip after charging, which is represented by the constraints (17). The charging time is also limited by the start of the next charging event. This is represented by the constraints (18) and (19). The constraints (18) are applied when the charging event is followed by a next charging event and constraints (19) are used when the charging event is followed by a service trip.

Integrality and Non-negativity Constraints

$$x^k_{ij} \in \{0,1\} \; \forall\, k \in K, i \in N \cup D_0 \cup D_n, j \in F_i \tag{20}$$

$$y^k_{irt} \in \{0,1\} \; \forall\, k \in K, i \in N, r \in R, t \in Fc_{ri} \tag{21}$$

$$z^k_{rtj} \in \{0,1\} \; \forall\, k \in K, r \in R, t \in T_r, j \in Fi_{rt} \tag{22}$$

$$w^k_{rt} \in \{0,1\} \; \forall\, k \in K, r \in R, t \in T_r \tag{23}$$

$$e^k_j \geq 0 \; \forall\, k \in K, i \in N \tag{24}$$

$$\varepsilon^k_{rt} \geq 0 \; \forall\, k \in K, r \in R, t \in T_r \tag{25}$$

The constraints (20), (21), (22) and (23) define the decision variables x^k_{ij}, y^k_{irt}, z^k_{rtj} and w^k_{rt} are binary. Finally, the non-negativity of the variables e^k_j and ε^k_{rt}, that keep track of the energy state, is defined in the constraints (24) and (25).

3.2.2 Discontinuous Charging Model

In this section, we propose changes to the model, which would enable discontinuous charging. That means the electric bus can be unplugged after charging during one time interval at charger, then wait during the following time interval and at last can be plugged again during the next time interval without the need to continue by serving a service trip. The change is shown in figure 2.

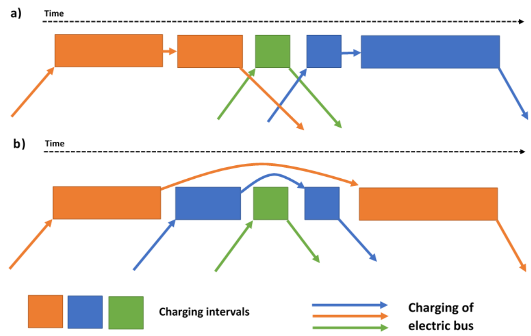

Figure 2: Difference between the continuous (a) and discontinuous (b) charging.

To define the connection between different charging events we introduce new decision variable u^k_{rts}, which represent decision that the electric bus k will continue charging at charger r during the charging event s after the charging during the charging event t. To reduce the number of decision variables and also to define only variables which are feasible from the time point of view the set is introduced. The charging event s to become a part of a set set Cf_{rt} defined for charger r and charging event t the charging event is s must be on the same charger as event t and the condition $s_{rt} <= s_{rs}$ must hold. In our case we have the charging events sorted by the starting time at each charger, that means in the set Cf_{rt} are all the events that follow the charging event t, or we can write $s \in t+1, \ldots |T^r|$. Similarly, the set of previous charging events Cb_{rt} is defined. It includes all the charging events that have a start time before the start of the charging event t at charger r.

With variable u^k_{rts} we replace the variable w^k_{rt} from the continuous model and also adjust some of the constraints of the continuous charge model to consider not only the following, respectively previous charging event, but all the following, respectively previous charging events. The adjustments are shown and described below.

$$\sum_{k \in K} \sum_{j \in Bi_{rt}} y^k_{jrt} + \sum_{k \in K} \sum_{s \in Cf_{rt}} u^k_{rts} \leq 1 \quad \forall\, r \in R, t \in T^r \tag{26}$$

$$\sum_{i \in Bi_{rt}} y_{irt}^k + \sum_{s \in Cb_{rt}} u_{rst}^k = \sum_{j \in Fi_{rt}} z_{rtj}^k + \sum_{p \in Cf_{rt}} u_{rtp}^k$$
$$\forall\, r \in R, t \in T^r, k \in K \quad (27)$$

The constraints (26) replace the constraints (3), which ensures that during one time interval only one electric bus can be charged. The adjustment lies in the fact we need to consider not only the previous charging event but all of the previous charging events. With this idea we adjusted also the flow constraints (5) into constraints (27), where we added into the constraints all the connections between the current charging event and previous, respectively following charging events.

We have not changed the energy consumption constraints, because they do not depend on the connections between the charging events.

$$\varepsilon_{rs}^k - \varepsilon_{rt}^k + SoC_{max}^k (1 - u_{rts}^k) \geq 0$$
$$\forall\, r \in R, t \in T^r, s \in Cf_{rt}, k \in K \quad (28)$$

$$\varepsilon_{rs}^k - Mq_r (1 - u_{rts}^k) \leq SoC_{max}^k$$
$$\forall\, r \in R, t \in T^r, s \in Cf_{rt}, k \in K \quad (29)$$

$$\varepsilon_{rs}^k \leq \varepsilon_{rt}^k + u_{rts}^k (s_{rt+1} - s_{rt}) q_r + SoC_{max}^k (1 - u_{rts}^k)$$
$$\forall\, r \in R, t \in T^r, s \in Cf_{rt}, k \in K \quad (30)$$

Constraints (14) were changed into constraints (28), where we need to consider all combinations of following charging events to set the charged energy is non-negative. Similarly, the constraints (16) were changed into constraints (29) and define the battery does not exceed its maximal capacity during charging. The constraints (16) are added for each possible combination of charging events. Lastly the constraints (18) is changed into constraints (30) for each possible combination of charging events. These constraints limit the charging time by the start of the immediately following charging event. In this case, the way we count the charged energy is changed.

$$u_{rts}^k \in \{0,1\} \;\forall\, k \in K, r \in R, t \in T_r, s \in Cf_{rt} \quad (31)$$

The integrality constraints (31) define that the decision variable u_{rts}^k is binary and also replace the obligatory constraints (23) of variable w_{rt}^k.

4 NUMERICAL EXPERIMENTS

To test the adjustments of the model we performed a number of experiments. Also to compare the performance of both models we compared the computation time of the exact solution made by the standard IP solver Xpress IVE. The experiments were performed on the machine with Intel Core i5-7200U 2,5Ghz, 16GB of RAM.

4.1 Data Description

For the experiments, we used data provided by the public transport system provider DPMŽ in the city of Žilina. This data contains information about the service trips of diesel buses performed during one day of operation. To test the models we generated six datasets that cover different bus lines and contain a different number of service trips.

The first dataset, denoted as DS1, contains 49 service trips performed on line 26. The second dataset (DS2) contains 77 trips served on line 27. The third dataset (DS3) is a union of trips served on lines 26 and 29 and contains 83 trips. The fourth dataset (DS4) covers lines 20, 29, 30 and 31 with 105 trips. The fifth dataset (DS5) covers lines 20, 26, 29 and 30 with 133 trips and the last dataset (DS6) is a union of 160 trips served at lines 26, 27 and 29.

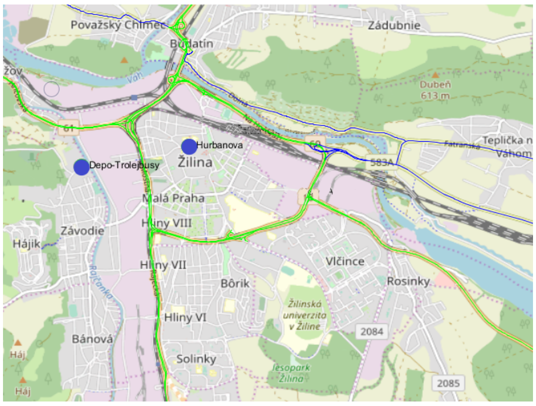

Figure 3: Locations of charger in the city of Žilina.

The second needed part of the experiments is the location of the chargers. In our experiments, we use the current locations of chargers. There are two chargers at the trolleybus depot and one charger at the center of the city. These locations can be seen in figure 3 as blue dots.

Table 1: Energy consumption and battery capacity scenarios.

Scenario	**Energy consumption**	**Battery capacity**
Spring	0,8 kWh/km	140 kWh
Summer	1,08 kWh/km	140 kWh
Winter	1,08 kWh/km	105 kWh

The last part of the experiments were different scenarios based on the time of the year, which represent different maximal energy states and different energy consumption. The scenarios are summed up in the table 1. The first scenario represents the basic setup. It is connected to the spring and autumn season. The battery capacity was set to 140kWh based on the literature (ZeEUS project, 2016). The second scenario is a summer scenario with increased energy consumption per km by 35%, which is caused by the running of air condition. In the last scenario, the energy consumption is increased by 35% and the capacity is decreased by 25%. This scenario represents the winter season, where the energy is also consumed on the heating and the battery has lower capacity due to the low temperature (Wood et al., 2012; Millner, 2010).

4.2 Results

In the tables 2, 3 and 4 we can see results of the experiments performed on all the datasets with each scenario. In each table, we have two columns - Continuous and Discontinuous. The column Continuous represents the results of continuous charging model and column Discontinuous shows the results of the discontinuous charging model. For each model we have the solution (Sol) obtained by the IP solver, then column BB represents best bound of the solution and column Time is the computational time in seconds. Also, the results obtained by the continuous model were the same as the solution of the classic VSP problem.

Table 2: Results of experiments with spring scenario.

Data-set	Continuous			Discontinuous		
	Sol	BB	Time	Sol	BB	Time
DS1	4	4	3	4	4	4,4
DS2	4	4	12,2	4	4	36,6
DS3	5	5	35,6	5	5	49,9
DS4	6	6	29,7	6	6	64,3
DS5	8	8	169,2	8	8	222,6
DS6	9	9	281,6	9	9	507,7

Table 3: Results of experiments with summer scenario.

Data-set	Continuous			Discontinuous		
	Sol	BB	Time	Sol	BB	Time
DS1	4	4	2,9	4	4	5,4
DS2	4	4	12	4	4	36
DS3	5	5	36,5	5	5	50,2
DS4	6	6	31,3	6	6	65,1
DS5	8	8	170,7	8	8	211,8
DS6	9	9	282,8	9	9	544,9

The table 2 represent the results of the spring scenario. We can see that the results obtained by both models are the same. However, the solution time is different. The computational time needed for the discontinuous charging model is always higher than for the continuous charging model. This is caused by the fact, that the discontinuous charging model is more complex, therefore more time is needed to solve this model.

Table 4: Results of experiments with winter scenario.

Data-set	Continuous			Discontinuous		
	Sol	BB	Time	Sol	BB	Time
DS1	4	4	3,3	4	4	10,6
DS2	4	4	33,8	4	4	131,8
DS3	5	5	42,9	5	5	286,8
DS4	6	6	773,5	6	6	920
DS5	10	8	57600	11	8	57600
DS6	11	9	57600	13	9	57600

The results of the summer scenario are listed in table 3. Similar to spring scenario the results of both models are the same and the computational time is also higher for discontinuous charging model. If we compare the computation time between the spring and summer scenario, the times are not much different. Therefore we can conclude that the energy consumption does not have a high impact on the computation time.

The last scenario is the winter scenario, which results are listed in table 4. There, for the smaller datasets, the optimal solutions were obtained for both models. In the case of datasets DS5 and DS6, the optimal solution was not found in the time limit, which was set to 16 hours. There we can see that the solution obtained by the continuous charging model is

Table 5: Number of used charging intervals (UCI), multiple interval charging events (MIC) and interrupted charging events (IC) for continuous and discontinuous charging in spring (Sp), summer (Su) and winter(Wi) scenario (SC).

Data-set	SC	Continuous		Discontinuous		
		UCI	MIC	UCI	MIC	IC
DS1	Sp	3	0	6	1	0
	Su	3	0	6	1	0
	Wi	9	0	13	0	0
DS2	Sp	22	1	31	0	0
	Su	22	1	31	1	0
	Wi	36	4	51	13	0
DS3	Sp	42	3	32	1	0
	Su	42	3	32	1	0
	Wi	41	5	60	13	1
DS4	Sp	43	5	59	9	4
	Su	43	5	59	9	4
	Wi	84	17	70	14	0
DS5	Sp	62	4	65	10	2
	Su	62	4	66	14	2
	Wi	113	23	122	32	9
DS6	Sp	77	5	88	15	7
	Su	77	5	88	15	7
	Wi	157	36	146	35	16

better. As we mentioned before this is caused by the increased complexity of the discontinuous charging model. From the point of comparison of the solution time between different scenarios, the winter scenario has increased computation time for both models. This is caused by the decreased capacity of the battery.

In the table 5, there is a comparison of the count of the charging intervals that were used for charging (UCI) during a specific scenario for each dataset. From the results, we cannot say which type of charging uses less charging intervals. However, a change can be seen in the number of charging events that took multiple intervals (MIC). There the continuous charging uses usually less number of multiple interval charging events than the discontinuous charging, but the count of the used interval during a multiple interval charging was less in the case of discontinuous charging. The last column IC shows the number of interrupted charging events for discontinuous charging. We can see that the number of interrupted charging events is higher with the winter scenario.

5 CONCLUSION

In this paper we propose changes to the linear mathematical model, that would enable the discontinuous charging. The new model was tested by IP solver Xpress IVE and the results of the discontinuous charging model were compared to the results of the continuous charging model. From the results, we can conclude, that in the case of minimizing the number of the used electric vehicles on the selected datasets the discontinuous charging model does not give better results, moreover the computational time is higher.

Despite the obtained results, we see a potential of the discontinuous charging model with the use of different objective functions, for example minimizing the length of deadheading trips between the service trips respectively service trips and chargers. Therefore, more experiments need to be conducted with the presented model in the future, but with different objective functions. On the other hand, the proposed models are complex and the solution time indicates that the use of these models is not possible on large scale problems. Therefore, the use of heuristics is advised on the larger-scale problems.

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