Decision Support for Shipping Spare Parts in Bundles

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Abstract: A telecommunication equipment company sends spare parts from local hubs to construction sites or other local hubs in mainland China several times a day through parcel delivery services. Depending on the delivery distance, there are various delivery options such as transportation via air, via road, via sea, via rail and via inland waterways. Many choices named service levels are available within each transportation category. There are three parcel delivery pricing policy: price per shipment, weight ranged price, and continuous pricing. Each spare parts delivery usually has a priority level or delivery time requirement. Spare parts to be shipped from the same hub or nearby hubs to the same or nearby destinations are considered being able to ship in bundles. By observing the delivery pricing structure, it is usually beneficial to bundle spare parts together for shipment. The problem is formulated as a mixed integer liner programming model. Numerical experiments are carried out to observe the benefits and also reflect the features of parcel delivery pricing structure.

1 INTRODUCTION

We study a problem where a telecommunication equipment company sends spare parts from local hubs to construction sites or other local hubs in mainland China twice a day. Typically, depending on the depot hub and destinations of the shipment, there are various means of freight transportation, such as transportation via air, via road, via sea, via rail and via inland waterways. Many choices named service levels are available within each transportation category. The service level agreement (SLA) is the guaranteed delivery time of the parcel, e.g. the next day by 18:00. The pricing of each delivery service is different but typically increasing as SLA decreasing.

There are three types of parcel delivery pricing policy. The first one is fixed price per shipment, although there may be a limit on the maximum weight or maximum number of items per shipment. This type of delivery service is usually associated with truck delivery or ship container delivery. The second policy is range pricing with a minimum charge and a unit price rate associated with each calculated weight/volume range. Furthermore, the price can be of continuous charge with a fixed unit price and a base charge. Express delivery services normally adopt this kind of pricing policy. Calculated weight is the maximum of the goods’ physical weight and the volumetric weight. Volumetric or dimensional weight is calculated based on the volume of the package times the throw weight coefficient, and normally international air transportation has a larger throw weight coefficient than domestic road transportation.

One specific thing to take into consideration is the price of shipping dangerous goods, such as liquid, bio-hazardous substances. It requires additional surcharges or charges at a higher unit price. Each spare parts delivery usually has a priority level or delivery time requirement. This requirement should be met on or before SLA, the guaranteed delivery time of the chosen delivery service. Spare parts shipping from the same hub or nearby hubs to the same or nearby destinations are considered to be able to ship in bundles.

The traditional way is to send each spare part separately once it is needed, or to bundle spare parts with the same delivery time requirement. However, this will incur more delivery cost and less profit
margin. Moreover, this needs man hours on dealing with each delivery order (e.g. tedious form filling work). It will be beneficial to bundling spare parts with different delivery time requirements together as long as SLA satisfied all requirements for shipment. For example, good A has a weight of 2kg and good B has a weight of 1kg, both taking a range price delivery option with 0-5kg, 1 per kg and minimum charge of 5. Shipping these two goods separately will cost 10 and bundle them together only cost 5. This simple example demonstrates the cost benefit of shipment bundle. Given a set of spare parts to be shipped, each with a delivery time requirement, from one hub (or nearby hubs) to a destination hub (or nearby hubs) and the available delivery options with known pricing policies, the problem is to determine how to bundle the spare parts to shipments so that the total cost is minimised.

There has been research in the literature on consolidation of shipments to save cost. For example, Wong, et al (2009) and Li et al.(2012) studied the shipment consolidation problems from the logistics providers perspective. They formulate mixed integer programming models to decide the consolidation of shipments in different segments in the shipping network to take advantage of economies of scale while considering delivery target dates and handling capacities. Nguyen et al. (2014) considered a problem in which multiple suppliers consolidate their product in long haul transportation to meet stochastic demands of the perishable products. We have not found previous research with the same settings as the work in this paper which determines bundling of shipments and selection of delivery services with different pricing structures.

Section 2 of this paper demonstrates the features of different delivery pricing policies. The above real-life business problem can be abstracted and translated using mathematical language. We formulate the optimisation problem as a mixed integer linear programming model with the objective of minimising the total delivery cost. The decision variables are the assignment of spare parts to delivery options which reflects the bundles. The constraints are described previously, including delivery time requirement and the logistics of calculated delivery costs. The solution approach and mathematical model is shown in section 3 and section 4. Numerical experiments are carried out and explained in section 5. Section 6 gives some real-life examples. Conclusions are drawn in section 7.

2 DELIVERY SERVICE PRICING

There are many different ways of post service charges and different regulations and strategies applied (Crew and Kleindorfer, 2013; Marcus and Petropoulos, 2017; Wilson, 1993). Three main categories of postal service pricing policy are explained in details, which summaries the signed delivery service contracts in the company. The first is price per shipment contract. It computes cost by unit price per container times the number of containers needed, and normally the maximum capacity of a container is big enough for half a day demand from the same locations.

The second type is range pricing for either weight unit or volume unit. With the pricing unit in weight, an example of this type of pricing policy is shown in Table 1. This price policy applies to a certain route and the transportation mode is by air. The guaranteed delivery is within four days. For example, we have a parcel to send with a weight of 21KG and a volume of 0.003 m^3. Firstly, we need to compute the calculated weight, which is the maximum of the
goods’ physical weight and the volumetric weight. Volumetric or dimensional weight is calculated based on the volume of the package times the throw weight coefficient, which is 167. The volumetric weight is $167 \times 0.003 = 0.5$, which is less than the weight so the calculated weight is 21KG for this parcel. 21KG is in the second range, so the unit price is 42. The calculated price is $42 \times 21 = 882$. This price is higher than the minimum charge, so the final charge is 882 for this example. Similar calculation process applies when the pricing unit is volume. The only difference is when calculating the weight converted volume, we use weight divided by the throw rate, which is a different throw rate from previous 167. Furthermore, the throw rate varies from country to country.

The third type of price policy is continuous charge policy such as the one shown in Table 2. The formula is quite different from that in the second one. The calculated weight is equal to the maximum of the goods’ physical weight and the volumetric weight. If the calculated weight is not more than the minimum charge weight, the price is the minimum charge. Otherwise, the amount above the minimum charge weight is rounded up to the nearest half and charged based on the unit rate. For example, if a parcel has a weight of 2.7KG and a volume of 0.05$m^3$, using the parameters in Table 2, the calculated weight is $\max\{2.7, 0.05 \times 167\} = 8.35$. The amount above the minimum charge weight will be $8.35 - 1 = 7.35$ KG and rounded to 7.5KG. The charge for this part is $7.5 \times 45 = 337.5$. Adding the basis charge, the final charge for this parcel is then 519.5.

### 2.1 Features of the Pricing Policy Structures

The weight range pricing policy has been plotted partially for a parcel weight changing from 0KG to 50KG as shown in Figure 1. The bonus zone $[0, A]$ is where you can bundle as many as items into the parcel and the total price would not change, where the weight limit $A = \text{MINI CHARGE RATE}$. Interestingly, the first price range does not take effect as the calculated price will always be less or equal to the minimum charges. In Figure 1, the arbitrage zone $[B, C]$ is where you can bundle more items or even put package materials such as foam into the parcel to reduce the total cost. The existence of an arbitrage zone and a bonus zone verify the potential to reduce total delivery cost by bundling items.

![Figure 1: Partial plot of weight range pricing policy as in Table 1.](image)

The continuous pricing policy has been plotted partially for a parcel weight changing from 0KG to
50KG as shown in Figure 2. It starts from a platform according to the minimum charges and then moves upwards as a staircase line. There are infinity many small bonus zones like the one in Figure 1, but each with a tiny width of 0.5KG. Those bonus zones are created due to the round-to-half structure of the pricing policy. The potential of reducing delivery cost is much less than the range pricing policy. Figure 3 compares the pricing policy in Table 1 and Table 2. When the weight of the parcel is less than or equal to 2KG, it is cheaper to choose continuous pricing policy service; otherwise, it is better to send the parcel with weight range pricing.

Figure 2: Partial plot of continuous pricing policy as in Table 2.

Figure 3: Comparison of weight range pricing and continuous pricing.

Figure 4 compares the same pricing policy structure with different SLAs (weight range pricing). As the data in Tables 3 and 4 show, the minimum charges of 3 days and 4 days are the same, but for every weight range, 3 days service has a higher unit price rate. So the service with 3 days SLA is in general more expensive than that with 4 days SLA. In the situation where the parcel is small and the price is the minimum charge, a shorter SLA is more preferable. This should be considered into the mathematical model as well. With many different scenarios and combinations of different SLA choices, it is extremely difficult to solve the problem by hand or by searches guided by rules found in this section, even given a long time. As a consequence, we propose to formulate this problem using mixed integer linear programming model. The mathematical model can be solved by exact method within seconds in most of the cases.

3 SOLUTION APPROACH

In the previous section, the features of different delivery options with different SLA are demonstrated.

The previous solution approach applied by the company is to bundle orders by simple rules, which is sending all orders with the same SLA in one parcel. This is the rule based strategy for shipping spare parts in bundles, but this may not lead to optimal solutions. An alternative way to solve this is to formulate the problem as a mathematically rigorous optimisation problem, specifically a mixed integer programming problem. The formulation is presented in section 4. The solution framework is demonstrated in Figure 5.

4 MATHEMATICAL MODEL

A mixed integer programming model can be formulated to demonstrate the problem of interest. The objective is to minimise the total cost of all the shipments after bundling spare parts. The constraints are:

- Orders from nearby depot hubs to nearby destination hubs can be considered to be bundled;
- Delivery time requirement of each spare part shipment must be satisfied;
The pricing policy of each delivery option is strictly followed;

Dangerous goods are normally bundled with dangerous goods and cannot be bundled with ordinary goods.

### 4.1 Notations

**Parameters:**
- \( i \) : index of spare parts that can be bundled together
- \( k \) : index of delivery options
- \( l \) : index of ranges in the range price policy
- \( N \) : total number of spare parts
- \( L_k \) : total number of ranges, \( \forall k \in K_1,K_3 \)
- \( K \) : set of indexes of all delivery options \( k \)
- \( K_1 \) : set of indexes of weight range price policy delivery options
- \( K_3 \) : set of indexes of continuous price policy delivery options
- \( K_2 \) : set of indexes of volume range price policy delivery options
- \( r_k \) : the throw weight coefficient of delivery option \( k \)
- \( d_k \) : the minimum charge of delivery option \( k \)
- \( g_k \) : the conversion rate of delivery option \( k \)
- \( m_k \) : the minimum charge weight of delivery option \( k \)
- \( w_i \) : the weight of spare part \( i \)
- \( v_i \) : the volume of spare part \( i \)
- \( b_k \) : beginning weight/volume of price range \( l \) of delivery option \( k \), for \( k \in K_1,K_3 \)
- \( e_k \) : ending weight/volume of price range \( l \) of delivery option \( k \), for \( k \in K_1,K_3 \)
- \( u_k \) : unit price rate of range \( l \) of delivery option \( k \), for \( k \in K_1,K_3 \)

**Variables:**
- \( X_{ik} \) : 1, if spare part \( i \) is allocated to delivery option \( k \)
- \( 0 \), otherwise
- \( \mu_{lk} \) : 1, if the cost of delivery option \( k \) is in range \( l \)
- \( 0 \), otherwise
- \( \eta_k \) : 1, if at least one item is allocated to delivery option \( k \)
- \( 0 \), otherwise
- \( WL_k \) : bundle pricing calculated weight of delivery option \( k \in K_1,K_3 \), 0 if no spare part is allocated to it
- \( RWL_k \) : bundle pricing calculated weight of delivery option \( k \in K_2 \), 0 if no spare part is allocated to it
- \( C_k \) : total delivery cost of bundled spare parts of delivery option \( k \)

### 4.2 Mixed Integer Programming Model

The mathematical model is formulated as follows

Minimise \( \sum_{k \in K} C_k \) \hspace{1cm} (1)

Subject to:

\( \sum_{k \in K} X_{ik} = 1, \hspace{0.5cm} \forall i \in [1,...,N] \) \hspace{1cm} (2)

\( r_k X_{ik} \geq ts_k X_{ik}, \hspace{0.5cm} \forall i \in [1,...,N], k \in K \) \hspace{1cm} (3)

\( WL_k \geq \sum_{i=1}^{N} w_i X_{ik}, \hspace{0.5cm} k \in K_1,K_2 \) \hspace{1cm} (4)

\( WL_k \geq \sum_{i=1}^{N} v_i X_{ik}, \hspace{0.5cm} \forall k \in K_1,K_2 \) \hspace{1cm} (5)
\[ WL_k \geq \sum_{i=1}^{N} \frac{W_i}{r_k} X_{ik}, \quad \forall k \in K3 \]  \tag{6} \\
\[ WL_k \geq \sum_{i=1}^{N} v_i X_{ik}, \quad \forall k \in K3 \]  \tag{7} \\
\[ \sum_{i=1}^{N} X_{ik} \leq M \eta_k, \quad \forall k \in K \]  \tag{8} \\
\[ \eta_k \leq \sum_{i=1}^{N} X_{ik}, \quad \forall k \in K \]  \tag{9} \\
\[ \sum_{i=1}^{N} \mu_{ik} = \eta_k, \quad \forall k \in K1, K3 \]  \tag{10} \\
\[ b_{ik} - (1 - \mu_{ik})M \leq WL_k \leq b_{ik} + (1 - \mu_{ik})M, \quad \forall k \in K1, K3, \ l \in [1, ..., L_k] \]  \tag{11} \\
\[ C_g \geq u_{ik} WL_k - (1 - \mu_{ik})M, \forall k \in K1, K3, \ l \in [1, ..., L_k] \]  \tag{12} \\
\[ C_k \geq d_k - (1 - \eta_k)M, \quad \forall k \in K \]  \tag{13} \\
\[ C_g \geq d_k + \frac{(RWL_k - m_k)}{g_k} u_c - (1 - \eta_k)M, \quad \forall k \in K2 \]  \tag{14} \\
\[ 2WL_k \leq RWL_k \leq 2WL_k + 0.99999, \quad \forall k \in K2 \]  \tag{15} \\
\[ X_{ik}, \eta_k \in [0,1], \quad \forall k \in K, \ i \in [1, ..., N] \]  \tag{16} \\
\[ \mu_{ik} \in [0,1], \quad \forall k \in K1, K3, \ l \in [1, ..., L_k] \]  \tag{17} \\
\[ WL_k, C_g \geq 0, \quad \forall k \in K \]  \tag{18} \\
\[ RWL_k \text{ is integer} \]  \tag{19} 

The objective (1) of the model is to minimise the sum of delivery cost of all delivery options, and if there is no spare parts allocated to a certain option, \[ C_g = 0. \]  Once there is a tie on price we will choose the fastest delivery option in the post processing check. Constraints (2) indicate that a spare part must be allocated to exactly one delivery option. Constraints (3) ensure that if spare part \( i \) is allocated to delivery option \( k \), then the required time of spare part \( i \) (e.g. 3-day arrival) must not be shorter than the guaranteed delivery time of option \( k \) (e.g. 2-day SLA). In the program, we modelled constraints (3) such that if \[ tr_i < ts_k, \quad \forall i \in [1, ..., N], \ k \in K, \]  then \[ X_{ik} = 0. \]  Constraints (4) and (5) compute the sum of calculated weight of delivery option \( k \in K1, K2 \), which is sum of the maximum weight (max of physical weight or volumetric weight) of all spare parts allocated to it. Constraints (6) and (7) compute the sum of calculated volume of delivery option \( k \in K3 \), which is sum of the maximum volume (max of physical volume or weight converted volume) of all spare parts allocated to it. Constraints (8) and (9) define that \( \eta_k = 1 \) means at least one spare part is allocated to delivery option \( k \). Then constraints (10) require that if delivery option \( k \) is used, the calculated weight/volume of spare parts to be delivered using option \( k \) must fall into one and only one range. Constraints (11) identify the right range \([b_{ik}, e_{ik}]\) of delivery option \( k \in K1, K3 \) which the calculated weight falls in. Constraints (12) and (13) calculate the total cost of delivery option \( k \in K1, K3 \) which is the maximum of the minimum charge and the unit price times the calculated weight/volume. Constraints (13), (14) and (15) calculate the total cost of delivery option \( k \in K2 \) which is the maximum of the minimum charge and the continuous price charge as stated in section 2. Constraints (15) ceil the calculated weight \( WL_k \) to the nearest half. One may notice that \( WL_k \) is defined for \( k \in K1, K3 \) as well, but calculated differently in constraints (15) for \( k \in K2 \), as for continuous price the calculated weight is rounded every 0.5kg. In a word, \( WL_1, WL_2 \) can be viewed as a different variable as \( WL_3 \). The rest constraints state that all decision variables are greater or equal to zero, among them \( X_{ik}, \eta_k, \mu_{ik} \) are binary variables and \( RWL_k \) only take integer values. For dangerous goods, a separate problem will be considered and solved using the same model, as they cannot be shipped together with other spare parts.

5 NUMERICAL EXAMPLES

Three numerical examples are generated and selected from real life data. The mathematical models are solved using open source optimiser COIN-OR’s COIN Branch and Cut Solver (CBC) under the Eclipse Public License (Forrest and Lougee-Heimer, 2005). All test cases are run on a 2.11GHz Intel Core i7-8650U (8 cores, 16GB) laptop.

5.1 Example One

The first example from real life data is shown in Table 5. The suppliers and price list are shown in Tables 1-4. Among the five different options (suppliers and price policies), the first three are selected. It is
interesting that item 2, 3, 4 required 4 days to arrive are allocated to delivery option three with 3 days SLA, and by doing this is the minimum-cost delivery plan. The minimum total cost of sending these six items are 2292.21 with a computational time of 0.23 seconds. If the items are sent out separately each in one parcel, the total cost are 2593 (11.6%). If the items are bundled by the same SLA, for example, items 2, 3, 4 can be bundled together and sent with 4 days service, the total cost are 2367.45 (3.18%). The percentage of the reduction in delivery cost of our plan is shown in bracket.

### Table 5: Example one item details.

<table>
<thead>
<tr>
<th>INDEX</th>
<th>SLA</th>
<th>WEIGHT</th>
<th>VOLUME</th>
<th>Bundle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>3.8</td>
<td>0.05</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>0.05</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2.2</td>
<td>0.05</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>14.2</td>
<td>0.14</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>9.5</td>
<td>0.07</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>10.25</td>
<td>0.06</td>
<td>2</td>
</tr>
</tbody>
</table>

### 5.2 Example Two

This example demonstrates the importance of the second objective functions, once there is a tie on the delivery cost of different options. As all the items require the same SLA, it is obvious to bundle all items and send them with the four days SLA delivery options. The delivery cost of the bundle with four days delivery option is 400. However, the delivery cost of the bundle to be shipped with three days SLA is also 400, as demonstrated in Figure 4, the bundle weight is in the bonus zone. With the second objective function, when optimizing for this objective, we only consider solutions that would not degrade the objective values of delivery cost objectives.

### Table 6: Example two item details.

<table>
<thead>
<tr>
<th>INDEX</th>
<th>SLA</th>
<th>WEIGHT</th>
<th>VOLUME</th>
<th>Bundle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0.001</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1</td>
<td>0.001</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>1</td>
<td>0.001</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>1</td>
<td>0.001</td>
<td>1</td>
</tr>
</tbody>
</table>

### 5.3 Example Three

This example shows an interesting case, where we can add packing material into the parcel to increase its weight and get a cheaper deal. The calculated weight of the bundle is 44kg, corresponding to the delivery option in Table 1. The delivery cost of the bundle with 44kg is 1848, while we could add a little bit weight to the current bundle and push it to the arbitrage zone as shown in Figure 1. The optimal cost of the bundle shipment is just above 1710, while we augmented the parcel weight to just above 45kg. This optimal cost is also found by solving the MIP problem.

### Table 7: Example three item details.

<table>
<thead>
<tr>
<th>INDEX</th>
<th>SLA</th>
<th>WEIGHT</th>
<th>VOLUME</th>
<th>Bundle</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.001</td>
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<tr>
<td>4</td>
<td>4</td>
<td>0.001</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

### 6 NUMERICAL EXPERIMENTS

The program is applied into one department’s daily business since earlier this year and achieved around 17% savings on delivery cost every month comparing to the same time period of last year. The program is run several times daily and we selected 20 examples from real life business to demonstrate the benefits of applying this program. One example is one batch of a particular day. The ORDERS column is the total number of orders to be dispatched at that time period of that day, and OPT_GROUPS column is the optimal parcel numbers after we bundled shipment. The ratio column is the bundle ratio, which is calculated as the number of groups divided by the number of original orders. The TIME column is the computational time of the optimization problem.

In Table 8, we selected 20 batches of orders to be dispatched. The average bundle ratio for this example by the proposed optimisation program is 0.387. The average computational time is 1.58 minutes. The comparison between the new solution approach and the traditional solution approach is shown in Table 9. We increased the bundle ratio by 53.6%, which means we largely reduced the packing time and efforts for parcels. More importantly, the unit price for sending those parcels before optimisation is 8.5 and after optimisation is 7, which indicates a 17.49% reduction in delivery cost.

### 7 CONCLUSIONS

A telecommunication equipment company sends spare parts from local hubs to construction sites or other local hubs in mainland China several times a day through parcel delivery services. Depending on the delivery distance, there are various delivery options such as transportation via air, via road, via
sea, via rail and via inland waterways. Many choices named service levels are available within each transportation category. There are three parcel delivery pricing policies: price per shipment, weight ranged price, and continuous pricing. Each spare parts delivery usually has a priority level or delivery time requirement. Spare parts to be shipped from the same hub or nearby hubs to the same or nearby destinations are considered to be able to ship in bundles. By observing the delivery pricing structures, it is beneficial to bundle spare parts together for shipment. The company used to bundle shipment by hand, following the rules of sending orders with the same delivery time requirement in one parcel. We proposed a new solution approach to tackle this problem. A mixed integer programming problem is proposed based on the delivery requirements as well as the various ways to compute delivery cost based on different delivery modes. Numerical experiments have been carried out to observe the benefits and also reflect the features of parcel delivery pricing structures. Then 20 real life business examples are selected. The average computational time is 1.58 minutes. Comparing to the traditional solution approach, we are able to increase the bundle ratio or in other words reduce the total number of parcels sent by 53.6% while keeping the same number of orders. This means that the time and efforts spent packing parcels are greatly reduced. Furthermore, the total delivery cost is reduced by 17% by using the new solution approach.

### Table 8: Real life examples.

<table>
<thead>
<tr>
<th>ID</th>
<th>ORDERS</th>
<th>OPT_GROPS</th>
<th>Ratio</th>
<th>TIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>210</td>
<td>100</td>
<td>0.476</td>
<td>0.933</td>
</tr>
<tr>
<td>2</td>
<td>790</td>
<td>235</td>
<td>0.297</td>
<td>2.383</td>
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<tr>
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<td>145</td>
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<td>0.248</td>
<td>0.367</td>
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<tr>
<td>4</td>
<td>161</td>
<td>69</td>
<td>0.429</td>
<td>0.667</td>
</tr>
<tr>
<td>5</td>
<td>158</td>
<td>75</td>
<td>0.475</td>
<td>0.717</td>
</tr>
<tr>
<td>6</td>
<td>307</td>
<td>121</td>
<td>0.394</td>
<td>1.267</td>
</tr>
<tr>
<td>7</td>
<td>640</td>
<td>219</td>
<td>0.342</td>
<td>2.250</td>
</tr>
<tr>
<td>8</td>
<td>94</td>
<td>39</td>
<td>0.415</td>
<td>0.400</td>
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<td>145</td>
<td>51</td>
<td>0.352</td>
<td>0.500</td>
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<td>136</td>
<td>69</td>
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<td>0.650</td>
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<td>147</td>
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<td>1.250</td>
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<td>0.500</td>
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<td>15</td>
<td>353</td>
<td>146</td>
<td>0.414</td>
<td>1.250</td>
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</tbody>
</table>

### Table 9: Comparison with previous solutions by hand.

<table>
<thead>
<tr>
<th></th>
<th>Orders</th>
<th>Groups</th>
<th>Ratio</th>
<th>Unit Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>By hand</td>
<td>7171</td>
<td>5378</td>
<td>0.75</td>
<td>8.5</td>
</tr>
<tr>
<td>By program</td>
<td>7171</td>
<td>2493</td>
<td>0.387</td>
<td>7</td>
</tr>
</tbody>
</table>

### REFERENCES


