# An Autoregressive Multiple Model Probabilistic Framework for the Detection of SSVEPs in Brain-Computer Interfaces

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Abstract: This work investigates a novel autoregressive multiple model (AR-MM) probabilistic framework for the detection of steady-state visual evoked potentials (SSVEPs) in brain-computer interfaces (BCIs). The proposed method is compared to standard SSVEP detection techniques using a 12-class SSVEP dataset recorded from 10 subjects. The results, obtained from a single-channel analysis, reveal that the AR-MM probabilistic framework significantly improves the SSVEP detection performance compared to the standard single-channel power spectral density analysis (PSDA) method. Specifically, an average classification accuracy of 82.02 $\pm$ 16.21 % and an information transfer rate (ITR) of 48.22 $\pm$ 17.25 bpm are obtained with a 2 s period for SSVEP detection with the AR-MM probabilistic framework. These results are found to be on average only 2.29 % and 3.73 % lower in classification accuracy compared to the state-of-the-art multichannel SSVEP detection methods, specifically the canonical correlation analysis (CCA) and the filter bank canonical correlation analysis (FBCCA) methods, respectively. In terms of training, it is shown that the proposed approach requires only a few seconds of data to train each model. This study revealed the potential of using the AR-MM probabilistic approach to distinguish between different classes using single-channel SSVEP data. The proposed method is particularly appealing for practical use in real-world BCI applications where a minimal amount of channels and training data are desirable.



## 1 INTRODUCTION

Brain-computer interface (BCI) systems offer an alternative means of communication and control, removing dependence on the peripheral nervous system, opening up accessibility to individuals suffering from motor disabilities, and providing alternative access methods to healthy individuals. Non-invasive BCIs are typically based on electroencephalography (EEG) where brain activity is acquired through scalp electrodes. Specific patterns of electrical brain activity are then translated into commands to control specific equipment. There are various brain signals having distinctive characteristics that are suitable as control signals for BCIs. This work focuses on SSVEPs, which are electrical potentials evoked in the brain in response to a repetitive visual stimulus that is flickering at a specific frequency. This neural response consists of oscillatory activity at the fundamental frequency and harmonics of the visual stimulus, and is prominent in the occipital region of the brain (Regan, 1966). An SSVEP-based BCI exploits this response by uniquely associating each flickering visual stimuli to a specific command. These stimuli are presented to the user who may select a command by attending to the corresponding stimulus. The BCI identifies the SSVEP response in the EEG signal and generates the particular command to control a software application or an external device (Vialatte et al., 2010).

Parametric modelling, particularly autoregressive modelling and its variants have been used extensively to analyse EEG signals (Pardey et al., 1996; Cerutti et al., 1988; Fabri et al., 2011). For example, AR models have been used in spectral analysis (Krusienski et al., 2006), feature extraction (Zhang et al., 2015; Chen et al., 2013), artifact/noise rejection (Wang et al., 2014; Friman et al., 2007) and for the estima-

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tion of the cortical functional connectivity (Babiloni et al., 2005). These techniques fit a model to a time series signal, having parameters that characterise the dynamics of the data being modelled. In SSVEP studies, parametric AR models have been typically used to provide a power spectrum from which the SSVEP frequency components are enhanced and used as features for SSVEP detection (Herrmann, 2001; Guger et al., 2012; Safi et al., 2018). The encouraging results of AR models applied to several EEG-based applications, has motivated this work to investigate the use of AR models in a multiple model probabilistic approach to detect SSVEPs.

In a multiple modelling framework, different states or modes of operation are represented by specific models, known as expert models (Liehr et al., 1999). For an SSVEP-based BCI application, each expert model would represent an SSVEP class that is attributed to a specific flickering stimulus. In this work, each expert model is represented as a linear Gaussian state space model (Ghahramani and Hinton, 1998) which is autoregressive in nature. The use of AR parameters as features for classification is common in EEG based BCI related literature (Galka et al., 2010; Zhang et al., 2016; Wenchang et al., 2019; Samdin et al., 2013; Zhang et al., 2016; Miran et al., 2018), however in this work we use the AR models for prediction. The residual between the actual and predicted data is used to determine which model from the available set best represents the data at each point in time. Since in an SSVEP-based BCI the subject is expected to attend to a sequence of stimuli, the dynamics of the EEG signals are expected to switch from one model to another in time. This residual is incorporated within a probabilistic framework in order to provide a probabilistic decision for classification.

The proposed autoregressive multiple modelling probabilistic framework works on a single channel of EEG data and provides classification of SSVEPs in batch mode, that is segmenting EEG data into a specific time window which is then processed and labelled. The framework is thus compared to power spectral density analysis (PSDA) which is also a commonly used single channel SSVEP detection technique (Müller-Putz et al., 2008) as well as to canonical correlation analysis (CCA) (Lin et al., 2007) and filter bank CCA (FBCCA) (Chen et al., 2015), both of which are multi-channel techniques. The results of the proposed framework showed significant improvement in classification performance compared to PSDA for 1.5-3.5 s window lengths and on average decreased slightly in classification performance compared to CCA and FBCCA, with the added advantage of requiring only a single channel. The latter is an important criterion for the practicality of real-time BCIs applications.

The paper is organized as follows. In Section 2, the SSVEP EEG data used for this analysis is presented and a mathematical description of the AR-MM framework is given. This is followed by a brief mathematical description of standard SSVEP detection methods found in the literature and which are used here as reference methods. Section 3 presents the results obtained when incorporating AR models into a multiple model framework for the identification and labelling of SSVEP data. Classification is initially based on the prediction error alone and then based on a probability measure. This is followed by a discussion and conclusion, highlighting the benefits of using the proposed AR-MM approach as an alternative SSVEP detection method.

# 2 MATERIALS AND METHODS

The first part of this section presents the SSVEP dataset used for analysis and then presents the theory of the proposed autoregressive multiple modelling (AR-MM) framework as well as that of the SSVEP detection techniques being used as baseline for comparison purposes.

## 2.1 SSVEP Dataset

The dataset used in this work contains frequency and phase modulated SSVEP data from 12 classes, acquired from 10 subjects. This data is used to estimate the performance of a BCI virtual keyboard (Nakanishi et al., 2015) and is freely available online (Nakanishi, 2015). The 12 stimuli were presented simultaneously in matrix form as white squares on a black background, each having different frequencies ranging between 9.25 Hz to 14.75 Hz and phases between $0\ \pi$ to $1.5\ \pi$. EEG data were recorded with eight Ag/AgCl electrodes covering the occipital area. Each subject was asked to gaze at each of the 12 visual stimuli in a random order for 15 repetitions. At the beginning of each trial, a red square appeared for 1 s at the position of the target stimulus. Subjects were asked to shift their gaze to the target within the same 1 s duration. After that, all stimuli started to flicker simultaneously for 4 s. To reduce eye movement artifacts, subjects were asked to avoid eye blinks during the stimulation period. The EEG data was down-sampled to 256 Hz and then band-pass filtered from 6 Hz to 80 Hz. A latency delay in the visual system of 135 ms was considered after each stimulus onset.

## 2.2 Autoregressive Multiple Modelling (AR-MM) Framework

In this section a mathematical description is given of the AR-MM framework, where each model is assumed to be autoregressive, specifically applied to detect SSVEPs from a single channel of EEG data. Classification of the multiple modelling framework is first based on the AR models used for prediction. A different approach is then taken where the AR models are incorporated within a probabilistic MM framework to provide a probability measure.

### 2.2.1 Fitting AR Models for Prediction

In the time domain a univariate $AR(p)$ model of order $p$ is represented by:

$$y_t = -\sum_{j=1}^{p} a_j y_{t-j} + e_t\,, \tag{1}$$

where $y_t$ represents data from a single channel at time $t$, $a_j$ represent the AR parameters, and $e$ represents a zero mean white Gaussian noise process. This is expressed in the $z$-domain by:

$$Y(z) = \frac{E(z)}{A(z)}\,, \tag{2}$$

where $E(z)$ is the error term and $A(z) = 1 + a_1 z^{-1} + \ldots + a_p z^{-p}$. Since all the model parameters lie in the denominator of the rational transfer function, the AR process is also typically known as an all-pole process. SSVEP signals are typically characterised by spectra with narrow peaks at the fundamental and harmonic components of the stimulus frequency. In autoregressive models, these spectra are characterised by poles located close to the unit circle in the $z$-domain. Furthermore, since over short time windows the SSVEP data is relatively stationary it is reasonable to assume non-adaptive AR models, where the model parameters remain fixed over time. Non-adaptive parameter estimation techniques, such as the Yule-Walker (YW), Burg, and Least Squares (LS) methods (Pardey et al., 1996; Stoica and Moses, 2005), can be used to find the optimal parameters that best fit the EEG data.

The parameters of the AR models that have been fit to SSVEP data may be used as features in a typical SSVEP classification approach. However, here the AR models will be used to forecast future data and analyse the residual between the actual data and the predicted data.

If $\hat{y}_t$ denotes the predicted value of $y_t$, then Equation (1) may be expressed as (Pardey et al., 1996):

$$\hat{y}_t = -\sum_{j=1}^{p} a_j y_{t-j}\,. \tag{3}$$

The error between the actual value and the predicted value is known as the forward prediction error and is given by:

$$r_t = y_t - \hat{y}_t = y_t + \sum_{j=1}^{p} a_j y_{t-j}\,. \tag{4}$$

This residual error $r$ may be used to determine the fit of the model to the actual data. Specifically, a window of SSVEP data can be labelled to one of the possible classes by estimating and comparing the root mean square error (RMSE) of each respective model.

### 2.2.2 AR-MM Probabilistic Framework

Let $\{y_1, \ldots, y_T\}$ be a sequence of observed EEG data having dynamics that depend upon the $p$-dimensional latent state variable $\boldsymbol{x}_t$. At time $t$, the joint distribution between $\boldsymbol{x}_t$ and $y_t$ of a linear Gaussian state space model that follows a Bayesian network is given by (Ghahramani, 2001):

$$p(\boldsymbol{x}_t, y_t) = p(\boldsymbol{x}_1)\prod_{t=2}^{T} p(\boldsymbol{x}_t|\boldsymbol{x}_{t-1})\prod_{t=1}^{T} p(y_t|\boldsymbol{x}_t)\,, \tag{5}$$

where $p(\cdot)$ denotes the probability density function and $\boldsymbol{x}_t$ is assumed to be a continuous real-valued hidden state variable. Assuming linearity and Gaussianity, the state space model is represented by the following state and measurement equations:

$$\boldsymbol{x}_t = \boldsymbol{\Phi}\boldsymbol{x}_{t-1} + \boldsymbol{w}_t \tag{6}$$

$$y_t = \boldsymbol{H}_t\boldsymbol{x}_t + v_t\,, \tag{7}$$

where $y_t$ is the observed EEG data from a single channel at time $t$, $\boldsymbol{x}_t = [-a_1, \ldots, -a_p]^{\mathrm{T}}$ is the hidden state vector made up of $p$ autoregressive parameters, $\boldsymbol{H}_t = [y_{t-1}, \ldots, y_{t-p}]$ is the observation vector made up of $p$ past EEG data samples, $\boldsymbol{\Phi}$ represents the state transition matrix and $\boldsymbol{w}_t$ and $v_t$ are two independent Gaussian noise processes assumed to have zero mean and covariance $\boldsymbol{Q}$ and variance $R$, respectively. In this work, $\boldsymbol{H}_t$ is taken to consist of the past $p$ narrow band pass filtered EEG data, specifically filtered around the first $H$ harmonic frequency components corresponding to the SSVEP class being modelled by that state space model, and $y_t$ is made up of unfiltered EEG data at time $t$. Apart from the unknown AR parameters $\boldsymbol{x}_t$, the unknown parameters that characterise the model are collectively represented by $\boldsymbol{\Theta} = [R, \boldsymbol{Q}, \boldsymbol{\Phi}, \boldsymbol{\mu}, \boldsymbol{\Sigma}]$, where $\boldsymbol{\mu}$ represents the initial hidden state vector and $\boldsymbol{\Sigma}$ its corresponding covariance.

In a MM system, a set of $N$ expert models each represented by unique state space equations (6) and (7) and having system parameters $\boldsymbol{\Theta}^i$ for $i = 1, \ldots, N$ need to be trained. The training process to learn these system parameters $\boldsymbol{\Theta}$ shall be described in Section 3.3. Once the different models are trained to represent each of the states or classes in the system, a probabilistic approach may be used to identify which of the candidate models best represents the data.

Under this framework, the posterior probability for each model $M^i$ given $\boldsymbol{Y}^t$, which represents the observed EEG data up till time $t$, is given by Bayes' rule as follows (Fabri and Kadirkamanathan, 2001):

$$\begin{aligned} Pr(M^i|\boldsymbol{Y}^t) &= Pr(M^i|y_t, \boldsymbol{Y}^{t-1}) \\ &= \frac{p(y_t|M^i, \boldsymbol{Y}^{t-1})Pr(M^i|\boldsymbol{Y}^{t-1})}{\sum_{j=1}^{N} p(y_t|M^j, \boldsymbol{Y}^{t-1})Pr(M^j|\boldsymbol{Y}^{t-1})}, \end{aligned} \tag{8}$$

where $Pr(M^i|\boldsymbol{Y}^{t-1})$ is the prior probability and $p(y_t|M^i, \boldsymbol{Y}^{t-1})$ is the likelihood function which is assumed to be Gaussian and hence estimated as follows:

$$p(y_t|M^i, \boldsymbol{Y}^{t-1}) = -\frac{1}{(2\pi)^{\frac{1}{2}}|\boldsymbol{C}_t^i|^{\frac{1}{2}}} \exp^{-\frac{1}{2}(y_t-\hat{y}_t^i)'(\boldsymbol{C}_t^i)^{-1}(y_t-\hat{y}_t^i)}. \tag{9}$$

In Equation (9), $(y_t - \hat{y}_t^i)$ represents the difference between the observation $y_t$ and its mean estimate $\hat{y}_t^i$, referred to as the residual. The mean estimate of the observation $\hat{y}_t^i$ is given by:

$$\hat{y}_t^i = \boldsymbol{H}_t^i \hat{\boldsymbol{x}}_{t|t-1}^i, \tag{10}$$

where $\hat{\boldsymbol{x}}_{t|t-1}^i$ denotes the conditional expectation of state $\boldsymbol{x}_t$. In Equation (9) $\boldsymbol{C}_t^i$ is the variance of $\hat{y}_t^i$ estimated as follows:

$$\boldsymbol{C}_t^i = \boldsymbol{H}_t^i \boldsymbol{P}_{t|t-1}^i \boldsymbol{H}_t^{i^{\mathrm{T}}} + R^i, \tag{11}$$

where $\boldsymbol{P}_{t|t-1}^i$ denotes the corresponding state estimation error covariance.

When evaluating the posterior probability $Pr(M^i|\boldsymbol{Y}^t)$ in Equation (8), the prior probability of the model $Pr(M^i|\boldsymbol{Y}^{t-1})$ is taken into consideration. The prior probabilities are here set to be uniformly distributed, that is, each model has equal chance of modelling new data. This was done by setting all prior probabilities equal to $1/N$, where $N$ represents the number of models.

Given the assumption of stationarity within a fixed time window, a non-adaptive approach is considered, where the hidden state vector and its covariance are not updated but remain constant over time. Therefore, the initial state vector and its corresponding covariance remain the same $\boldsymbol{\mu} = \boldsymbol{x}_t$ and $\boldsymbol{\Sigma} = \boldsymbol{Q}$. Consequently in Equations (10) and (11), the initial state vector and corresponding covariance are used to calculate the posterior probability, $\hat{\boldsymbol{x}}_{t|t-1}^i = \boldsymbol{\mu}^i$ and $\boldsymbol{P}_{t|t-1}^i = \boldsymbol{\Sigma}^i$ for all $t$. It follows that $\boldsymbol{\Phi} = I$ in Equation (6).

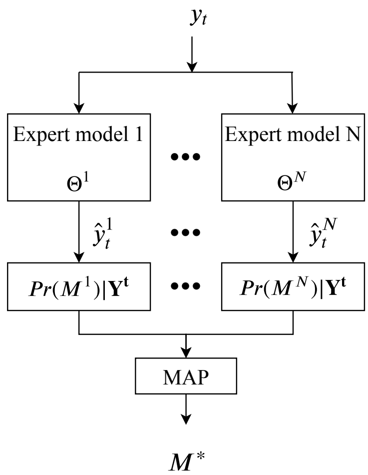



Figure 1: AR-MM probabilistic framework with $N$ expert models acting as candidates to model the observed sequence $y_t$. The model $M^*$ that has the highest posterior probability is considered as the model that best represents the data.

Under this framework, $N$ probabilities corresponding to the $N$ models are obtained at each time instance with Equation (8). The model $M^*$ that has the maximum a posteriori (MAP) probability is then considered as the model that best represents the incoming data from the set of available models. Figure 1 shows a block diagram of the AR-MM probabilistic framework.

## 2.3 Standard SSVEP Detection Methods

For the purpose of creating a reference against which the new proposed method can be evaluated, a number of standard SSVEP detection techniques found in the literature (Zerafa et al., 2018) were also implemented. Similar to the proposed AR-MM probabilistic framework, the following methods exploit only the frequency information of the SSVEP signal, whereas the phase information is not considered. The PSDA method was chosen as a reference for the univariate case since this is the simplest and most widely used single-channel feature extraction method. For the multivariate case, the CCA and FBCCA were selected as reference methods. In the literature, the CCA method is generally used as a reference method against which new SSVEP methods are compared to, while the FBCCA method, which is an extension of CCA, has been shown to improve the frequency detection of SSVEPs. A brief mathematical description of these methods is presented below to better covey the requirements of each technique and the framework on which each is based.

#### 2.3.1 Power Spectral Density Analysis (PSDA)

In PSDA a Fourier representation is used to transform an EEG signal from the time domain to the frequency domain and extract specific SSVEP features from the resulting spectral content. The fast Fourier transform (FFT) is performed on a single-channel EEG signal, from which the one-sided power spectral density (PSD) (Müller-Putz et al., 2008) is estimated. A feature vector is constructed from the PSD values at the fundamental frequencies and harmonics corresponding to the flickering stimuli. A harmonic sum decision (HSD) method (Müller-Putz and Pfurtscheller, 2008) is adopted to construct SSVEP feature vectors, where the sum of harmonic power values for each stimulus frequency $f_s$ is computed as:

$$\sum_{h=1}^{H} \max_{f}(\mathrm{PSD}(f \mid f \in [hf_s - 0.125, hf_s + 0.125])) , \quad (12)$$

where $h$ represents the harmonic number being considered and $H$ represents the number of harmonic frequencies considered. The feature vector is log normalised to make the features normally distributed. The largest power amplitude is expected to correspond to the frequency of the target stimulus, representing the SSVEP response.

#### 2.3.2 Canonical Correlation Analysis (CCA)

CCA is a multivariable statistical method used to find the underlying correlation between two sets of data (Lin et al., 2007), used to detect SSVEPs in multichannel EEG data. Let $\boldsymbol{Y}$ and $\boldsymbol{X}_f$ be two multidimensional variables representing the multi-channel EEG signals of length $T$ samples and a set of SSVEP reference signals which have the same length as $\boldsymbol{Y}$, respectively. The sine-cosine reference signals $\boldsymbol{X}_f$ for the target stimulus frequency $f$ are defined by:

$$\boldsymbol{X}_f = \begin{pmatrix} \sin(2\pi hf\frac{t}{F_s}) \\ \cos(2\pi hf\frac{t}{F_s}) \\ \vdots \\ sin(2\pi Hf\frac{t}{F_s}) \\ \cos(2\pi Hf\frac{t}{F_s}) \end{pmatrix}^{\mathrm{T}} , \quad \begin{matrix} t = 1,\dots,T \\ h = 1,\dots,H \end{matrix} , \quad (13)$$

where $F_s$ is the sampling frequency, $T$ is the number of samples and $H$ is the number of harmonics. CCA finds the linear combinations $y = \boldsymbol{Y}^{\mathrm{T}}\boldsymbol{W}_y$ and $x_f = \boldsymbol{X}_f^{\mathrm{T}}\boldsymbol{W}_{x_f}$, such that the correlation between the two canonical variants $y$ and $x_f$ is maximized (Lin et al., 2007). The weight vectors $\boldsymbol{W}_y$ and $\boldsymbol{W}_{x_f}$ are found by solving the following optimization problem (Bin et al., 2009):

$$\begin{aligned} \max_{\boldsymbol{W}_y,\boldsymbol{W}_{x_f}} \rho_f(y,x_f) &= \frac{\mathrm{E}[y^{\mathrm{T}}x_f]}{\sqrt{\mathrm{E}[y^{\mathrm{T}}y]\mathrm{E}[x_f^{\mathrm{T}}x_f]}} \\ &= \frac{\mathrm{E}[\boldsymbol{W}_y^{\mathrm{T}}\boldsymbol{Y}\boldsymbol{X}_f^{\mathrm{T}}\boldsymbol{W}_{x_f}]}{\sqrt{\mathrm{E}[\boldsymbol{W}_y^{\mathrm{T}}\boldsymbol{Y}\boldsymbol{Y}^{\mathrm{T}}\boldsymbol{W}_y]\mathrm{E}[\boldsymbol{W}_{x_f}^{\mathrm{T}}\boldsymbol{X}_f\boldsymbol{X}_f^{\mathrm{T}}\boldsymbol{W}_{x_f}]}} . \end{aligned} \quad (14)$$

For each reference signal, a maximum canonical correlation $\rho_f$ is obtained and used as an SSVEP feature. The hypothesis is that the reference signal with the largest correlation contains SSVEP at the same frequency as the stimulus signal.

#### 2.3.3 Filter Bank Canonical Correlation Analysis (FBCCA)

The FBCCA method is a CCA variant that decomposes SSVEPs into multiple sub-band components. FBCCA performs separate CCAs between each of the sub-band components and sine-cosine reference signals (Chen et al., 2015). The filter bank used consists of sub-bands covering multiple harmonic frequency bands. The correlation values between the sub-band components $\boldsymbol{Y}_{SB_k}$, $k = 1,...,K$ from the original EEG signals $\boldsymbol{Y}$ and the reference signals $\boldsymbol{X}_f$ corresponding to all stimulation frequencies $f$ are estimated to form a correlation vector $\boldsymbol{\rho}_f = [\rho_f^1,\dots,\rho_f^K]^{\mathrm{T}}$, where $K$ is the number of sub-bands. A weighted sum of squares of the correlation values corresponding to all sub-band components is then calculated as the feature for SSVEP detection as follows:

$$\tilde{\rho}_f = \sum_{k=1}^{K} \boldsymbol{w}(k) \cdot (\rho_f^k)^2 , \quad (15)$$

where $\boldsymbol{w}(k)$ are the weights of the sub-band components. These weights are set by the observation that the signal to noise ratio (SNR) of SSVEP harmonics decreases as the response frequency increases:

$$\boldsymbol{w}(k) = k^{-a} + b, \quad k = 1,\dots,K , \quad (16)$$

where $a$ and $b$ are constants that maximize the classification performance. The frequency of the reference signals having the maximum correlation is then considered to be the target stimulus. The FBCCA method requires three parameters to be optimized, namely the number of harmonics to be considered in the reference signals, the weight vector for the sub-band components and the number of sub-bands in the filter bank. Since these may interact with each other in influencing the BCI performance (Chen et al., 2015; Nakanishi et al., 2018), a grid search involving the simultaneous optimization of these parameters is conducted using training data from various subjects. In

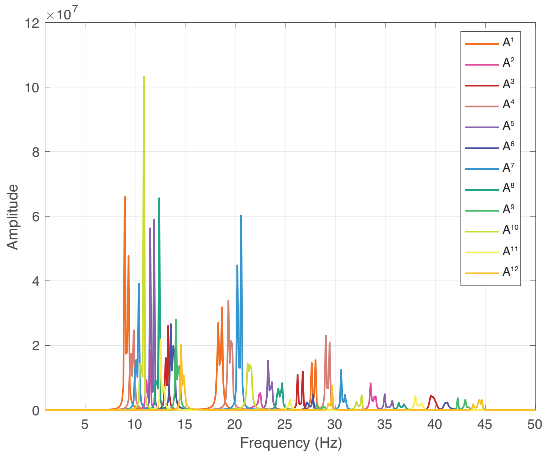



Figure 2: Spectra of the 12 AR models.

this work, based on the findings in the literature, the number of harmonics, the weight vector, i.e. $a$ and $b$ in Equation (16), and the number of sub-bands were fixed to 3, 1.25, 0.25 and 5, respectively.

# 3 RESULTS

In this section the application of AR models to SSVEP data is first described. Classification results of the AR-MM framework are then presented. This is followed by a description of the training requirements of the probabilistic multiple model framework and its classification results compared to that of standard SSVEP detection methods.

## 3.1 AR Models Applied to SSVEP Data

The SSVEP data described in Section 2.1 consisted of 12 different classes, hence an AR model was trained for each respective class. Training data was narrow band pass filtered around the first $H = 3$ harmonic frequency components corresponding to the actual SSVEP response such that background activity not corresponding to the SSVEP response is filtered out and hence not modelled. An AR model of order 20 was then fit to the data. Given the narrow band filtering done at the pre-processing stage, this model order was found to give a good representation of the data irrespective of the stimulus class. A set of AR parameters $A^i = \{a_1, ..., a_p\}$, for each model $i = 1, .., N_f$ was then obtained using Burg's method, where $N_f = 12$ is the number of stimuli classes. Figure 2 shows the spectra of 12 trained AR models, as an example.

When assigning a class to a new test trial $y_t$, the data is first narrow band pass filtered around the first three harmonic frequency components corresponding to each of the 12 stimuli classes. Each trained AR model, having AR parameter vectors $\boldsymbol{x}_t^i$, is then used to estimate $\hat{y}_t$, the predicted value of $y_t$ using Equation (3). This process is shown in Figure 3.

As an example, Figure 4a shows the original unfiltered signal and the narrow band pass filtered signals estimated by each AR model for a 10.25 Hz test trial. The errors between the original test signal $y_t$ and the predicted signals $\hat{y}_t^i$ were estimated and the lowest RMSE across the 2 s window was found. In this case the minimum error was that between $y_t$ and $\hat{y}_t^7$, shown in Figure 4b. A difference between the two signals can still be observed since the estimated signal is narrow band pass filtered around the expected frequency while the original signal is unfiltered. Hence any activity outside the bands of interest will not be modelled. The correct model, however, is expected to best represent the SSVEP related activity and hence result in the smallest residual error. In the case being shown here, this trial was correctly labelled as a class 7 trial, corresponding to the 10.25 Hz SSVEP response.

## 3.2 Classification Results of the AR-MM Method

This section presents the classification accuracy results when using the lowest RMSE of the 12 trained models to classify each trial into one of the 12 available classes. The classification accuracy was estimated by considering only 1 trial per class as training data to generate the AR models, with the remaining 14 trials per class being used as testing data. Cross validation was then carried out by repeating the process such that each of the 15 trials was used once for training purposes, and finally the average classification accuracy was computed. This analysis was carried out on each bipolar channel combination and the highest performance associated with the best

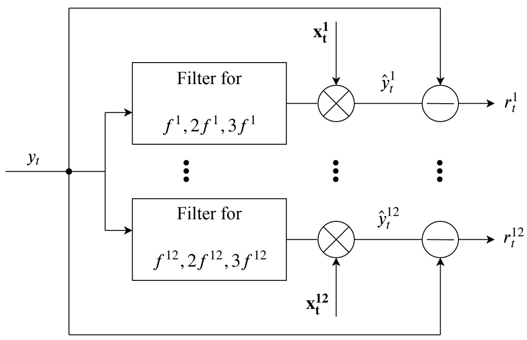
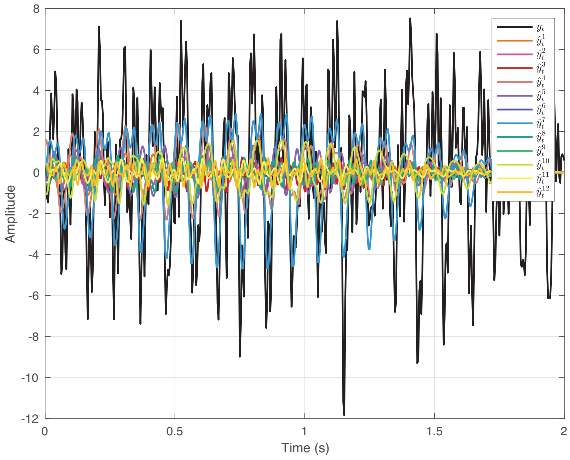
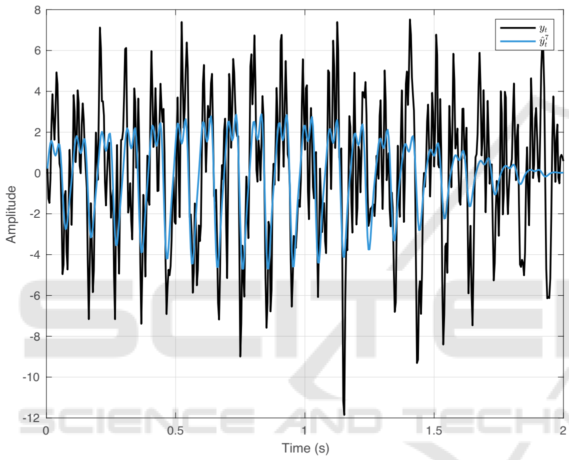



Figure 3: Process to find residual errors between original signal $y_t$ and estimated signals $\hat{y}_t^i$ for $i = 1, \ldots, 12$ using 12 different AR models having parameter vectors $\boldsymbol{x}_t^1$ to $\boldsymbol{x}_t^{12}$.




(a) Test data $y_t$ and estimated $\hat{y}_t^i$ in the time domain.




(b) Test data $y_t$ and estimated $\hat{y}_t^7$ in the time domain.

Figure 4: A 2 s SSVEP test trial at 10.25 Hz, represented by class number 7 in the 12-class dataset.

bipolar channel (BBC) was found. The classification accuracy was also evaluated using different window lengths for SSVEP detection. Apart from classification accuracy, the information transfer rate (ITR) in bits per minute (bpm) (Wolpaw et al., 2002) was used as a performance measure and was calculated as:

$$ITR = \frac{60}{T}\left(\log_2 N_f + P\log_2 P + (1-P)\log_2\left(\frac{1-P}{N_f-1}\right)\right), \quad (17)$$

where $P$ is the classification accuracy, and $T$ is the average time for selection in seconds. An additional gaze shifting time of 1 s was included in the estimation of the ITR.

Figure 5 shows the average accuracy obtained with the AR-MM framework, across all subjects and sessions, for different data lengths ranging between 0.5 s to 4 s, in steps of 0.5 s. The proposed approach was compared to the PSDA method for which the performance obtained with the BBC is also being reported for fair comparison. The PSDA results obtained are in line with those found in literature (Lin et al., 2007; Bin et al., 2009), although it must be noted that direct comparison is difficult due to variations in BCI setups, such as the number of stimuli being used. As expected, for both methods, the SSVEP classification increases with the length of time window for detection. Paired t-tests were conducted to analyse the difference in performance between the two methods across all the time windows. These revealed that there is a statistically significant ($p < 0.05$) increase in performance for the AR-MM method compared with the PSDA for time windows between 1.5 s and 3 s.

Table 1: AR-MM results for 12 classes (8.33 % chance classification accuracy) with 2 s window and 1 s gaze shift.

| Subject | Accuracy (%) | ITR (bpm) |
|---|---|---|
| 1 | 47.78 | 15.60 |
| 2 | 78.33 | 41.63 |
| 3 | 55.00 | 20.71 |
| 4 | 71.11 | 34.37 |
| 5 | 93.33 | 60.02 |
| 6 | 83.89 | 47.81 |
| 7 | 96.11 | 64.27 |
| 8 | 90.56 | 56.14 |
| 9 | 98.89 | 69.17 |
| 10 | 93.89 | 60.83 |
| **Mean ± STD** | **80.89 ± 16.88** | **47.05 ± 17.61** |

To provide insight on individual subjects' performance, the results in Figure 5 for a time window of 2 s were expanded in Table 1.

This demonstrates that there is a large variation in performance between subjects. In fact, the classification accuracy for 5 subjects is above 90%, for 3 subjects this is between 70% and 90% and for 2 subjects

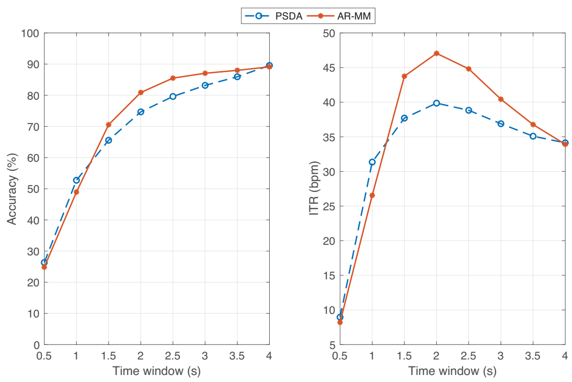



Figure 5: Performance comparison of PSDA and AR-MM methods. (a) Average classification accuracy (%) and (b) ITR (bpm) across all subjects for different time windows (s).

this is in the range of 50%.

These initial findings demonstrate that AR models provide a good fit for the SSVEP data and may be used as a model structure in the multiple modelling probabilistic framework.

### 3.3 Training for the Probabilistic AR-MM Framework

Given the results obtained with the AR-MM approach presented in the previous section, the AR multiple models were incorporated within a probabilistic multiple modelling framework. The process of learning the AR parameters $\boldsymbol{x}_t^i$ and system parameters $\boldsymbol{\Theta}$ in a multiple modelling framework requires SSVEP training data for each class. Let $z_t^i$ be a vector of a single channel SSVEP EEG data, with $i = 1, \ldots, N_f$ representing the number of target stimulus frequencies. This data is then narrow band pass filtered around the fundamental frequency $f$ and the first $H$ harmonic components corresponding to the SSVEP class $i$ and is represented here by $z_{f_t}^i$. The estimation of parameters $\boldsymbol{x}_t^i$ and $\boldsymbol{\Theta}$ is then being carried out as follows:

- With reference to Section 3.1, the AR parameters $\boldsymbol{x}_t^i$ for the expert model $M^i$ are found using the filtered training data $z_{f_t}^i$. AR parameters are estimated using Burg's method and the length of $\boldsymbol{x}_t^i$ represents the AR model of order $p$ which was here set to 20.

- The unknown variance $R$ is estimated from Equation (7), where the observation vector $y_t$ is replaced with training data $z_t$, i.e. $v_t = z_t - \boldsymbol{H}_t \boldsymbol{x}_t$. In this case $\boldsymbol{H}_t$ represents $p$ past narrow bandpass filtered training data, i.e. $\boldsymbol{H}_t = [z_{f_{t-1}}, \ldots, z_{f_{t-p}}]$. The variance $\sigma_l^i$ of the resulting $v_t$ is obtained from each training trial of all the classes $i = 1, \ldots, N_f$, where $l = 1, \ldots, L$ represents the number of training trials. The mean variance across all the training trials gives $R$, i.e. $R = 1/(N_f L) \sum_{i=1}^{N_f} \sum_{l=1}^{L} \sigma_l^i$.

- The unknown covariance $\boldsymbol{Q}^i$ for model $M^i$ is estimated from Equation (6), where for the non-adaptive approach $\boldsymbol{x}_t^i = \boldsymbol{I}\boldsymbol{x}_t^i + \boldsymbol{w}_t$. Therefore, the covariance of $\boldsymbol{x}_t^i \in \mathbb{R}^p$ obtained from AR parameters across different training trials gives $\boldsymbol{Q}^i \in \mathbb{R}^{p \times p}$.

- As discussed earlier, following the non-adaptive approach, the rest of the unknown system parameter are defined as $\boldsymbol{\mu} = \boldsymbol{x}_t$, $\boldsymbol{\Sigma} = \boldsymbol{Q}$ and $\boldsymbol{\Phi} = I$.

### 3.4 AR-MM Probabilistic Framework

In order to compare the performance of the AR-MM probabilistic approach with that of the state-of-the-art methods, batch mode classification is considered in which each trial is first segmented and then passed through the AR-MM probabilistic process. Labelling of one trial is done by finding $\arg\max_{M} \sum_{t=1}^{N_t} Pr(M^i | \boldsymbol{Y}^t)$, where $N_t$ is the total number of samples in the time window.

Figure 6 shows the average classification accuracy and ITR obtained with the AR-MM probabilistic framework across all subjects and sessions for different data lengths from 0.5 s to 4 s, in steps of 0.5 s.

The classification accuracy was estimated by considering 2 trials per class as training data to generate the AR models $\boldsymbol{x}_t$, $R$ and $\boldsymbol{Q}$ and the remaining were used for testing. A minimum of 2 training trials were necessary to estimate the covariance $\boldsymbol{Q}$ of the state vector. This process was then repeated seven times such that all trials were considered for training and an average classification accuracy was computed. The performance of the AR-MM probabilistic approach is compared with that of the PSDA, CCA and FBCCA methods. In the case of both the AR-MM probabilistic approach and the PSDA method, the performance for each bipolar channel combination was computed and the highest performance associated with the BBC was found. On the other hand, all 8 channels were used in the estimation of the CCA and FBCCA multivariate methods.

Two-way repeated measures ANOVA were performed to compare the performance of the proposed

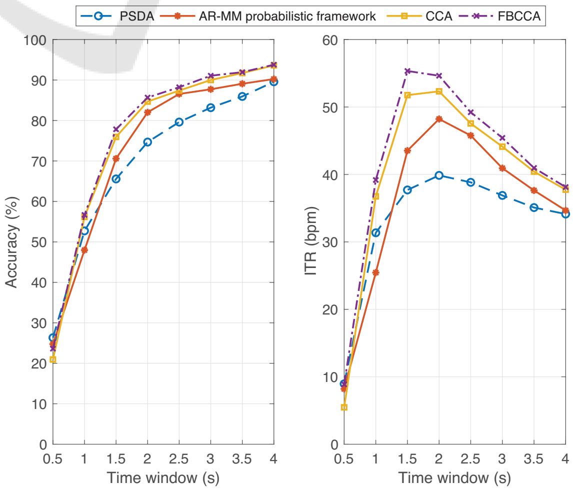



Figure 6: Performance comparison of PSDA, AR-MM probabilistic framework, CCA and FBCCA methods. (a) Average classification accuracy (%) and (b) ITR (bpm) across all subjects for different time windows (s).

AR-MM probabilistic framework with each of the reference SSVEP detection methods across all the different time windows. This indicated that there was a statistically significantly ($p < 0.05$) improvement in the overall SSVEP classification accuracy with the AR-MM probabilistic framework when compared with the PSDA method. Post-hoc paired t-tests, which considered the results from different time windows, revealed that the AR-MM probabilistic method obtained statistically significantly ($p < 0.05$) higher accuracies compared to PSDA method for time windows 1.5 s - 3.5 s. The overall classification accuracies of the multichannel methods, CCA and FBCCA, were slightly higher than that of the single-channel AR-MM probabilistic framework. In the case of the FBCCA method, the accuracy was found to be statistically significantly higher ($p < 0.05$) than the AR-MM probabilistic approach for time windows of 1 s, 1.5 s and 4 s. These results indicate that the AR-MM probabilistic approach can significantly enhance the performance of the single-channel PSDA method. These results also show a slight decrease in performance, specifically 2.29 % compared to the multichannel CCA method and 3.73 % compared to its extension the FBCCA method on average over all time windows, with the advantage of requiring only a single-channel for SSVEP detection.

# 4 DISCUSSION

The results presented in Section 3 demonstrate that the multiple model probabilistic framework is a promising technique for the detection of SSVEPs in BCIs.

Autoregressive models that capture the dynamics of the underlying SSVEP signal in the EEG data were used as a model structure for the AR-MM framework. Initially an investigation was conducted to evaluate whether AR models provide a good fit for the data. The standard approach of using AR models in EEG analysis is to find AR parameters for incoming EEG data to form features which can then be used for classification (Guger et al., 2012; Safi et al., 2018). In this work however, AR expert models were trained for each SSVEP class, each of which has a distinct frequency, and then used for prediction on new EEG data. The residual between the true and the reconstructed signals was evaluated and used for classification. This was done since the MM framework is based on the analysis of this residual which is specifically used to calculate the likelihood function in Equation (9). The initial classification results of the AR-MM framework using the RMSE as a basis for classification were compared to the PSDA method. The latter is the simplest and one of the most commonly used SSVEP detection methods, which similarly to the proposed approach, operates on bipolar EEG channels. It is worth pointing out that the classification of the PSDA method is done using features in the frequency domain, specifically by analysing the power amplitude of respective harmonics, while the classification of the AR-MM approach is done in the time domain, specifically by analysing the residual between the original and the predicted signals. The results obtained by the AR-MM approach showed a significant improvement in performance compared to the PSDA method.

The second part of the analysis involved the restructuring of the AR-MM approach within a probabilistic framework. In this case, a probability measure for each model in representing incoming data was obtained at each time instance and this result was then evaluated to find which model gave the best representation within a fixed time window, allowing batch mode classification as done with other state-of-the-art techniques. In this case, apart from comparing the performance of the AR-MM probabilistic framework to a similar single bipolar channel PSDA method, the framework was also compared to the multivariate CCA and FBCCA methods. Similar to the proposed AR-MM probabilistic framework, these methods exploit only the frequency information of the SSVEP signal, whereas the phase information is not considered. This is particularly useful in BCI setups that do not have phase coded stimuli. It must be highlighted that multichannel SSVEP detection methods are known to benefit from an optimized combination of multiple signals. In fact it has been shown that this results in a greater robustness against noise and hence improved performance compared to single channel methods (Friman et al., 2007; Bin et al., 2009). The results obtained for the single-channel AR-MM probabilistic framework showed significant improvement over the single-channel PSDA method but even more promising, the results obtained were only slightly lower compared to the multichannel methods for most time windows of SSVEP detection. This means that a BCI requires only a pair of electrodes to function with the AR-MM probabilistic approach, making it more practical for the user.

Various studies have shown that incorporating a user's training data significantly improves the detection performance (Nakanishi et al., 2015). However, the amount of training data significantly varies between methods and this may result in long and tedious training sessions (Zerafa et al., 2018). In this work it was shown that the AR-MM probabilistic method requires only 2 trials per class of training data to model the AR models, hence only a few seconds of training are actually required for a new BCI user. From a preliminary analysis we have carried out, we identified that increasing the number of training trials for the AR-MM framework did not show statistical improvements in performance. The transferability of training data in the AR-MM probabilistic approach from one subject to another will also be addressed in future work.

## 5 CONCLUSIONS

A novel autoregressive multiple model (AR-MM) probabilistic framework for the detection of SSVEPs for BCIs was presented in this paper. Through this work we have shown that the univariate AR-MM probabilistic approach can yield a significant improvement in performance over PSDA, a standard single-channel SSVEP detection method and is only 2.29 % and 3.73 % lower in classification performance compared to CCA and FBCCA, respectively, two standard multichannel SSVEP detection methods. The proposed framework also provides a measure of probability for each SSVEP class, which can be used as a measure of certainty in the decision making process.



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