




# Semisupervised Classification of Anomalies Signatures in Electrical Wafer Sorting (EWS) Maps

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**Keywords:** Semiconductor Manufacturing, Electrical Wafer Sorting Map, Wafer Map Clustering, Anomalies Signature Classification, Yield Analysis.


**Abstract:** We focused onto a very specific kind of data from semiconductor manufacturing called Electrical Wafer Sorting (EWS) maps, that are generated during the wafer testing phase performed in semiconductor device fabrication. Yield detractors are identified by specific and characteristic anomalies signatures. Unfortunately, new anomalies signatures may appear among the huge amount of EWS maps generated per day. Hence, it's unfeasible to define just a finite set of possible signatures, as this will not represent a real use-case scenario. Our goal is anomalies signatures classification. For this purpose, we present a semisupervised approach by combining hierarchical clustering to create the starting Knowledge Base, and a supervised classifier trained leveraging clustering phase. Our dataset is daily increased, and the classifier is dynamically updated considering possible new created clusters. Training a Convolutional Neural Network, we reached performance comparable with other state-of-the-art techniques, even if our method does not rely on any labeled dataset and can be daily updated. Our dataset is skewed and the proposed method was proved to be rotation invariant. The proposed method can grant benefits like reduction of wafer test results review time, or improvement of processes, yield, quality, and reliability of production using the information obtained during clustering process.


## 1 INTRODUCTION


Semiconductor manufacturing requires complex equipment where each machine contains hundreds of components, and thousands of failure points at minimum. Yield across the entire line usually must be very high, and there must be a continuous learning process to keep in place the yield at high levels. New products must be quickly brought up to high yield, as the profit margins of these new products are often a major source of profit in the semiconductor industry, due to the high competitive environment. Given the huge amount of data collected from each facility in the industry, we fall within the context of Big Data. This requires a proper data analysis and management, that can bring to useful insights for increasing yield rate or detect anomalies at early stages.

In this paper, we focus onto a very specific kind

of data called Electrical Wafer Sorting (EWS) maps. These images are generated during the wafer testing phase performed in semiconductor device fabrication. A wafer is a round-shaped support containing several "dies". After testing, good dies are cut out from the wafer and sent to the package phase. Instead, bad ones were literally "inked" to be easily recognized and discarded when dies are extracted from the wafer. Today, defective dies are not inked anymore, as this can be done digitally, employing maps that can be used for masking good and bad portions of the wafer. Maps represent dies on the wafer and, accordingly to a statistical binning approach, they can have several values (i.e., good or failed during test stage 1, stage 2, and so on). For simplicity, we assume to handle binary EWS maps, where white pixels identify failed dies, while black pixels the good ones. Since many yield detractors can impact production at the same time, device engineers have to spend a lot of time analyzing EWS data to identify every yield detractor and relative affected wafers before proceeding to in-depth

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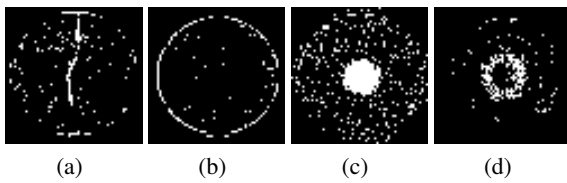


Figure 1: EWS maps showing the following characteristic anomalies signatures: (a) Scratch, (b) Ring, (c) Spot, and (d) Wheel. White pixels identify defective dies.

analysis and data mining to identify root causes. Usually, yield detractors can be identified by specific and characteristic patterns, named EWS map signatures (i.e., scratch, ring, spot, or wheel, as shown in Figure 1). These patterns are useful for investigating the root causes that could be, for instance, related to an equipment component failure, a drifting process, or an integration of processes (May and Spanos, 2006). Unfortunately, new anomalies signatures may appear among the huge amount of EWS maps generated per day (i.e., we are facing an open set problem). Hence, it's unfeasible to define just a finite set of possible signatures, as this will not represent a real use-case scenario. The automatic labeling of old and new anomalies represents an interesting research issue with relevant industrial applications.

Clearly, we are within the scope of unsupervised learning, and we need to apply a clustering strategy. Since the concept of cluster in data analysis has been introduced, it has spanned through a wide range of disciplines (Tvaronaviciene et al., 2015; Han et al., 2001). Many real world problems, indeed, found their solution on cluster analysis. In this paper, the “objects” to cluster are real EWS maps acquired by STMicroelectronics. We wish to aggregate the EWS maps accordingly to their anomalies signatures. In practical clustering applications, it is not known in advance how many clusters have to be treated, and their number may also change when different days of production are considered. Although in certain cases the number of clusters is already known, this does not apply in our case, as said before. Many studies have been carried out to estimate the optimal number of clusters for a given clustering task, but this field is very challenging and existing methods still have drawbacks (Wang et al., 2018). In our application, the number of clusters is daily increased upon dynamically data analysis. In the proposed method, we have looked into the field of aggregative clustering, one of the earliest and most widely used clustering strategy (Balcan et al., 2014; Bryant and Berry, 2001). We have evidence of aggregative clustering since 1950 (Ackermann et al., 2014).

Once clustering phase is completed, we will have

a dataset of EWS maps where each image will be assigned to a specific and unique cluster. Then, as in a supervised approach, we may leverage the outcome of clustering phase considering the assigned cluster-ID as label for training a classifier. Indeed, this hybrid approach is defined semisupervised learning, and it is particularly meaningful as the manual classification could lead to different results among different operators, who are biased by subjective interpretation of the anomalies signatures. Clustering analysis was leveraged for anomalies signatures classification in recent works (Zhang et al., 2013; Wu et al., 2014; Saqlain et al., 2019; Jin et al., 2019). However, all of these works applied supervised learning assuming a finite set of just 9 anomalies signatures. They employed the WM-811K dataset (WM-811K, 2018), that is made of 811,457 wafer maps collected real-world fabrication. Images in the WM-811K dataset are similar to EWS maps we employed in this work: in the former case images are labeled, while in the latter they are not. One of the most recent related works on anomalies signatures retrieval was focused onto wafer defect maps (WDM) classification (Di Bella et al., 2019). Even if WDM images look really similar to EWS maps, we remark that defectivity analysis is a test phase performed some steps before the EWS one. Differently from the present work, a labeled dataset with a set of finite classes was employed for WDM classification, too.

In sum the contributions of this work are:

- **Semisupervised Classifier:** a new semisupervised approach for classifying anomalies signatures in EWS maps is presented, by combining an unsupervised approach using a Hierarchical clustering algorithm to create the starting Knowledge base, and a supervised one through a classifier trained leveraging clustering phase (Figure 2);
- **Daily Update:** our dataset can be daily increased, and the classifier is dynamically updated considering possible new created clusters. The workflow of our solution can be resumed in: daily arrival of EWS maps, clustering of newcomer images into previously created clusters, possible creation of new clusters, anomalies signatures classification;
- **Variable Number of Anomalies Signatures:** we are not considering a fixed number of anomalies signatures, and the leveraged dataset does not contain any label. This represents the typical scenario of real use-case industrial applications.

The goal of this work is to create a tool to make as automatic as possible the recognition of wafer anomalies signatures. This is meaningful as upon classification the industrial system can be able to automatically

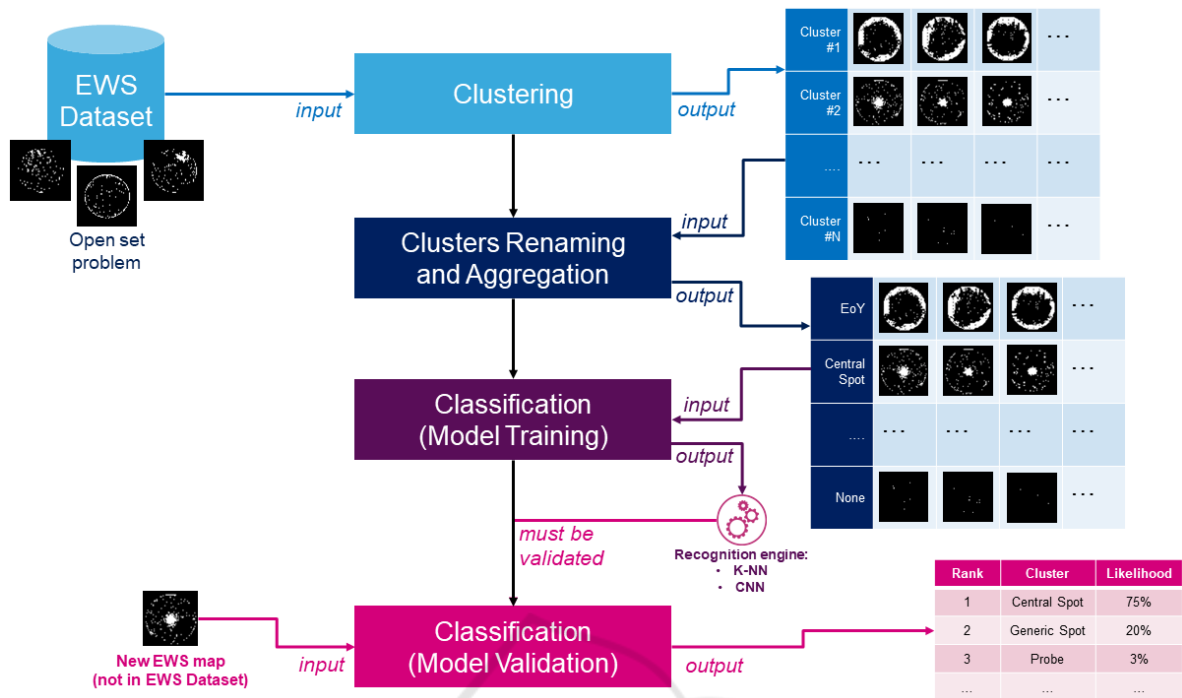


Figure 2: Overview workflow of the proposed approach.

choose (or at least suggest) either to discard a wafer or to ship it to customer. The proposed method can also grant benefits like reduction of wafer test results review time, or improvement of processes, yield, quality, and reliability of production using the information obtained during clustering process.

The remainder of the paper is organized as follows. In Section 2 we define our semisupervised approach for anomalies signatures classification, describing in detail the clustering and classification phases. Some cues about image descriptors, knowledge base (i.e., the number of clusters), daily update, and data augmentation, are given within this Section. Then, in Section 3, we report our experimental results, showing clustering visual assessment, rotation invariance of the proposed method, and performance of both clustering and classification phases. Finally, we conclude the paper with a final discussion and some remarks for possible future works.

## 2 DATA AND METHODS

The proposed method is based onto a semisupervised approach (Figure 2), that can be divided in two main parts: the clustering and the classification phases (i.e., unsupervised and supervised learning). They are described in order. Dataset was gradually incremented

during the several reported phases, so the dataset definition will be given together with the method description.

### 2.1 Clustering

In the clustering phase we firstly defined the descriptors computed from EWS maps. Then, we described a preliminary clustering phase, in which we assess the cluster algorithm to be used in our experiments. In this Section, we also defined the initial dataset and the error measure for determining the number of clusters. Finally, we introduced our knowledge base of anomalies signatures.

#### 2.1.1 Descriptors

There are several descriptors for labeled wafer maps images (Wu et al., 2014). However, given the unlabeled characteristic of our dataset, we decided to investigate two more general effective descriptors: Local Binary Pattern (LBP) and Principal Components (PC).

**Local Binary Pattern (LBP):** the LBP operator is defined as a grey scale invariant texture operator. It has become a popular approach in applications, including visual inspection, and image analysis. Given a grayscale image, the operator compares the  $3 \times 3$

neighborhood of each pixel with this central pixel value, and transforms the result to a binary number. Then, image is represented through the histogram of these computed binary numbers (Hadid et al., 2008).

**Principal Components (PCs):** the PCs are computed through the Principal Component Analysis (PCA). The PCA is an orthogonal linear transformation that modifies the data to a new coordinate system in such a way to highlight their similarities and differences (Mishra et al., 2017). Images with a resolution of  $61 \times 61$  pixels are “vectorized”, that is they are reshaped into a vector of 3,721 pixels (Turk and Pentland, 1991). We decided to keep as many PCs as needed to have at least the 80% of variance retain. Typically, only 15 to 20 PCs survive over the 3,721 original ones. These PCs are used as descriptor for the clustering phase.

### 2.1.2 Preliminary Clustering Phase

During the preliminary clustering phase, we collected a first dataset of 296 images, with a resolution of  $61 \times 61$  pixels. This tiny dataset was the only one to be manually labeled by a team of experts, in order to validate the preliminary clustering outcomes. Among the many available clustering techniques, we focused onto K-means, divisive hierarchical clustering and, particularly, on the aggregative hierarchical clustering algorithm. We clustered the 296 images in 5 clusters. Then, clustering outcomes were validated by experts. Indeed, the main purpose of this phase was to select the clustering algorithm, and we chose the aggregative hierarchical clustering.

Hierarchical clustering is an unsupervised algorithm that subdivides the dataset in partitions leveraging a distance function. It produces a hierarchical representation, with the lowest level of the hierarchy counting  $n$  clusters, where  $n$  is the total number of observations. Instead, at the top level we have a single cluster containing all the observations. Hierarchical clustering comes with many linkage methods to perform the clustering (Li and de Rijke, 2017). Linking methods define how the distance between two clusters is measured. This is important, as it also defines how to assign an observation to one of the many available clusters. The Ward linkage (Murtagh and Legendre, 2014) is the method used in this work. In the Ward linkage, an error function is defined for each cluster. This error function is the average distance of each observation in a cluster to the centroid of the cluster. The distance between two clusters is defined as the error function of the unified cluster minus the error functions of the single clusters. Indeed, Ward linkage is

used to minimize the variance of the clusters being merged.

Finally, the only hyper-parameter we need to set is the number of clusters. Defining this number, one has to find a good compromise between how many clusters to generate and how coherent they should be. There are many techniques for determining the number of clusters (Xu et al., 2016), and calculating the within cluster sum of squared error (WCSSE) is one of them (Thinsungnoena et al., 2015), as defined by the following equation:

$$WCSSE = \sum_{k=1}^K \sum_{i \in S_k} \sum_{j=1}^P (x_{ij} - \bar{x}_{kj})^2 \quad (1)$$

where  $S_k$  is the set of observations in the  $k$ -th cluster, and  $\bar{x}_{kj}$  is the  $j$ -th variable of the cluster for the  $k$ -th cluster found with the clustering algorithm. In our experiments, the value of this parameter was empirically set to 6,000. We visually confirmed that, with this chosen value, we have a good compromise between clusters generated and intra-cluster variance.

### 2.1.3 Knowledge Base Definition

In a typical scenario of real use-case industrial applications, the dataset is daily increased with new EWS maps. Hence, in a process of knowledge base definition, we gradually increased the size of our dataset until it counted 10,000 images, with a resolution of  $61 \times 61$  pixels. It is called “knowledge base” as it represents our core knowledge about the possible anomalies signatures (i.e., the number of clusters) known until each daily update. We therefore dynamically proceeded to test our clustering procedure on the incrementing dataset. Eventually, we obtained 10 clusters. Then, clustering outcomes were once again validated by experts. The dataset of 10,000 unlabeled images has been used as a starting knowledge base for the classification phase.

## 2.2 Classification

In the classification phase we firstly investigated performance of K-Nearest Neighbour (KNN) algorithm. This technique is also employed in a routine for creating new clusters, enabling us to increase our knowledge base with new clusters (i.e., new anomalies signatures). We also investigated a deep learning approach based on the training of a Convolutional Neural Network (CNN), where we introduced a data augmentation procedure need for classes having a few number of samples.



Table 1: Dataset Skewness in Training and Validation sets. We reported the relative quantities (Rel Qty) of the 4 biggest clusters in the dataset. Note that *Cluster 26* is considered by our expert to be the group of wafers without any anomalies signatures (i.e., good wafers), among all the wafers sent to testing phase.

Cluster ID	26	34	12	6	Others
Training Set Rel Qty	32.20%	16.45%	7.13%	4.55%	<4.00%
Validation Set Rel Qty	32.14%	16.53%	7.08%	4.55%	<4.00%

### 2.2.1 KNN and Knowledge Base Update

After the creation of a starting knowledge base using aggregative hierarchical clustering on the initial dataset of 10,000 images, we have to daily classify new images, evaluating if they should be included into a previously created cluster or whether they should be part of some new cluster. To do so, we propose to use the K-Nearest Neighbour (KNN) algorithm. The principle behind this algorithm consists in finding a number  $K$  of samples closest in distance to the query observation. The label of the new sample will depend on the majority of the closest samples (Tsigkritis et al., 2018). We evaluate the distance between the descriptors of the samples using the Euclidean distance. For our implementation, we empirically granted robustness to this procedure setting  $K = 15$ . This means that, for each of the new images, the distance with their 15 nearest neighbours will be evaluated.

**Affinity Percentage:** we put an affinity percentage to discard images that do not clearly belong to a class. If at least 10 over 15 neighbours belong to a certain class, that sample will be assigned to that class as well. Otherwise, it will contribute to a new class, increasing our knowledge base. Then, after running the KNN algorithm, some images are isolated as not clearly belonging to any cluster. We compute aggregative hierarchical clustering on these isolated and unclassified images, defining new clusters and incrementing our knowledge base.

### 2.2.2 CNN and Data Augmentation

We trained a classifier through a deep learning approach using a Convolutional Neural Network (CNN) with a ResNet-18 architecture (He et al., 2015). ResNet stands for residual network, in which first layers are connected to deeper ones through the so called shortcut-connections. We choose ResNet-18 as it is proved to be an architecture fitting the scope of the proposed issue (Saqlain et al., 2019).

When training our CNN, the daily-updated dataset was counting 58,038 images, with a resolution of  $61 \times 61$  pixels. We splitted our dataset in 46,431 (80%) images for the training set, and 11,607 (20%) images for the validation set. After several iterations of the knowledge base increment, we were considering 85 possible clusters (i.e., anomalies signatures). The number of EWS maps for each class is not balanced (Table 1). Notice that data skewness can be considered a quality of our dataset, and it is a common characteristic for real use-case industrial applications (i.e., some anomalies are more common than others).

When in presence of anomalies signatures characterized by a very few number of EWS maps (lesser than 100), we leveraged some well known data augmentation techniques. We created synthetic images until every clusters reached the minimum number of 100 images. Starting from existing images of poorly populated clusters, the augmentation consists on a combination of one or more of the following augmentation techniques (randomly applied):

- **Noising:** some white pixels were randomly put inside the image (Gaussian noise).
- **Rotation:** the image were rotated randomly of 90, 180 or 270 degrees, clockwise or counter-clockwise.
- **Flipping:** the image were horizontally or vertically flipped.

## 3 EXPERIMENTAL RESULTS

Starting from the clustering phase, we firstly compared the goodness of selected descriptors (i.e., Local Binary Pattern - LBP, and Principal Components - PCs) while changing the clustering method (i.e., K-Means, divisive and aggregative hierarchical cluster). Outcomes are reported in Table 2. We observed that the quality of hierarchical clustering when combined with PCs outperformed the quality of K-means. We also found that aggregative hierarchical clustering performs better than divisive one.

A visual assessment of the clustering is given comparing Figures 3 and 4, where clusters were obtained through K-Means with PCs and divisive hierarchical clustering with LBP, respectively. As shown, PCs outperform LBP even if we employ K-Means instead of hierarchical clustering. The LBP operator clearly performs better on images having a well-defined pattern, but fails to find other kind of defects. Moreover, the classification through PCs performs very quickly, as the process runs in no more than five minutes per day, while the LBP operator sig-

Table 2: Clustering outcomes when changing clustering algorithm and EWS maps descriptor (LBP: Local Binary Pattern, PCs: Principal Components). Precision is computed as  $TP/(TP + FP)$ . The best result is highlighted in bold.

Clustering	Descriptor	Precision %
KMeans	LBP	48.98
KMeans	PCs	70.6
Hierarchical clustering divisive	LBP	58.44
Hierarchical clustering divisive	PCs	79.05
Hierarchical clustering aggregative	PCs	<b>90.20</b>

Table 3: Classification outcomes comparing K-Nearest Neighbours (KNN) and Convolutional Neural Network (CNN). EWS maps selected descriptors are the Principal Components (PCs). Precision is computed as  $TP/(TP + FP)$ . The best result is highlighted in bold.

Classification	Descriptor	Precision %
KNN	PCs	85.33
KNN with affinity percentage	PCs	90.55
CNN	PCs	<b>95.87</b>

nificantly increases the processing time to few hours a day, which makes it not feasible for real use-cases industrial applications. Another good quality of the proposed method is its proved rotation invariance (Figure 5).

Classification outcomes are reported in Table 3. KNN is proved to be good enough for classification purposes, reaching more than 90% of precision. As expectable, CNN was able to improve this result, reaching 95.87% of precision. Since we are dealing with skewed data, we also computed for CNN the more robust F1-score, that is equal to 92.18%. Considering we started from a not labeled dataset, our results sound comparable with the ones shown in (Saqlain et al., 2019), where they obtained 96.93% of precision and 96.71% of F1-score with only 9 classes (WM-811K, 2018), instead of the 85 employed in this work.

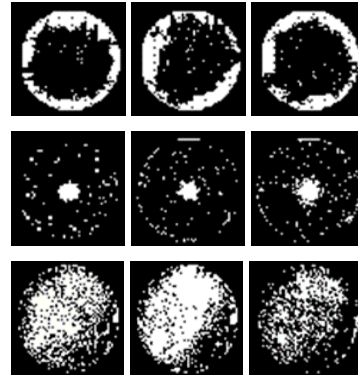


Figure 3: Three clusters (one per row) obtained through K-Means and Principal Components (PCs). This is an example of good clustering.

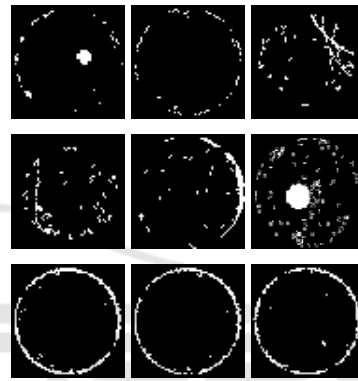


Figure 4: Three clusters (one per row) obtained through divisive hierarchical clustering and Local Binary Pattern (LBP). This is an example of bad clustering: first and second clusters contains different kind of anomalies signatures. Only the third cluster looks fine.

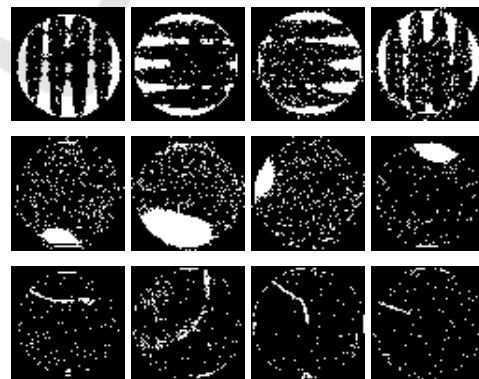


Figure 5: Proposed clustering method is proved to be rotation invariant.

## 4 CONCLUSION

In this work, we focused onto a very specific kind of data from semiconductor manufacturing called Elec-

trical Wafer Sorting (EWS) maps. These images are generated during the wafer testing phase performed in semiconductor device fabrication. We assumed to handle binary EWS maps, where white pixels identify failed dies, while black pixels the good ones. Usually, yield detractors are identified by specific and characteristic patterns, named anomalies signatures. These patterns are useful for investigating the root causes that could be, for instance, related to an equipment component failure, a drifting process, or an integration of processes (May and Spanos, 2006). Unfortunately, new anomalies signatures may appear among the huge amount of EWS maps generated per day. Hence, it's unfeasible to define just a finite set of possible signatures, as this will not represent a real use-case scenario. For the same reason, we did not gathered a labeled dataset.

In this paper, we presented a new semisupervised approach for classifying anomalies signatures in EWS maps, by combining an unsupervised approach using a Hierarchical clustering algorithm to create the starting Knowledge base, and a supervised one through a classifier trained leveraging clustering phase. The knowledge base represents our core knowledge about the possible anomalies signatures (i.e., the number of clusters) known until each daily update. We therefore dynamically proceeded to test our clustering procedure on the incrementing dataset. Indeed, our dataset can be daily increased, and the classifier is dynamically updated considering possible new created clusters. The workflow of our solution can be resumed in: daily arrival of EWS maps, clustering of newcomer images into previously created clusters, possible creation of new clusters, anomalies signatures classification.

We compared several clustering and classification techniques. We found that aggregative hierarchical clustering leveraging Principal Components computed through the Principal Component Analysis can be a robust clustering method. Then, we trained a Convolutional Neural Network with ResNet-18 architecture, reaching performance comparable with other state-of-the-art technique. We remark that our method does not rely on any labeled dataset and can be daily updated, differently by compared literature. Our dataset is skewed, a common characteristic in real use-case industrial scenario. Moreover, we proposed a method that was proved to be rotation invariant.

The goal of this work was to create a tool to make as automatic as possible the recognition of wafer anomalies signatures. This is meaningful as upon classification the industrial system can be able to automatically choose (or at least suggest) either to discard a wafer or to ship it to the customer. The pro-

posed method can also grant benefits like reduction of wafer test results review time, or improvement of processes, yield, quality, and reliability of production using the information obtained during clustering process.

As future works, we are planning to investigate performance of other CNN architectures. We are also designing a comparison study with a two-fold purpose: consolidate outcomes shown in this proposal employing the WM-811K dataset, and exploring the existence of any correlation with test phases before the EWS (e.g., relatively to Wafer Defect Maps - WDM).

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