Interactions of Gap Solitons in Coupled Bragg Gratings with Cubic-quintic Nonlinearity and Dispersive Reflectivity

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Abstract: The interaction of quiescent gap solitons in coupled fiber Bragg gratings with dispersive reflectivity and cubic-quintic nonlinearity in both cores is investigated. It has been found that with low to moderate dispersive reflectivity the interactions have similar characteristics to the nonlinear Schrodinger solitons i.e. in-phase solitons always attract each other and out-of-phase solitons repel. It is found that the interaction of in-phase solitons may result in a number of outcomes such as formation of a quiescent soliton, generation of two separating solitons and formation of a quiescent and two moving solitons. For strong dispersive reflectivity, the interaction outcomes depend on the initial separation.

1 INTRODUCTION

Fiber Bragg gratings (FBG) have found many applications in optical signal processing, dispersion compenation and sensing (Kashyap, 2010; Loh et al., 1996; Cao et al., 2012; Cao et al., 2014). The periodic variation in FBGs leads to linear resonant coupling of counter-propagating waves which induces high effective dispersion. This dispersion can be six order of magnitude greater than the chromatic dispersion of silica fiber (de Sterke and J.E.Sipe, 1994). In the nonlinear regime, the balance between the Kerr nonlinearity and the effective dispersion of the FBG gives rise to the formation of gap solitons (GSs) (Aceves and Wabnitz, 1989) (Christodoulides and Joseph, 1989). The first observation of a GS was reported in a 6-cm long FBG (Eggleton et al., 1996). One of the properties of the GSs is that they can travel at any velocity from zero to the speed of the light in the medium. As a result, theoretical and experimental studies have focused on understanding of the fundamental properties of GSs and how they can be exploited to build novel optical devices (Eggleton et al., 1996) (Barashenkov et al., 1998).

The formation of GSs have also been considered in other nonlinear systems such as coupled FBGs (Mak et al., 1998), semilinear coupled system (Atai and A. Malomed, 2001), cubic-quintic nonlinear system (Atai and Malomed, 2001; Dasanayaka and Atai, 2010), photonic crystal fibers(Skryabin, 2004; Neill and Atai, 2007) and FBGs with dispersive reflectivity (Atai and Malomed, 2005; Baratali and Atai, 2012).

In this paper, we consider the interactions of quiescent GSs in a dual-core system where each core has a FBG with dispersive reflectivity and cubic-quintic nonlinearity.

2 THE MODEL

Light propagation in a linearly coupled Bragg gratings with dispersive reflectivity and cubic-quintic nonlinearity can be described as the following equations (Akter et al., 2019):

$$\begin{aligned}
&iu_{1t} + iu_{1x} + \left(\frac{1}{2}|u_1|^2 + |v_1|^2\right)u_1 - \\
&\eta\left(\frac{1}{4}|u_1|^4 + \frac{3}{2}|u_1|^2|v_1|^2 + \frac{3}{4}|v_1|^4\right)u_1 + \\
&\qquad v_1 + \lambda u_2 + mv_{1xx} = 0, \\
&iv_{1t} - iv_{1x} + \left(\frac{1}{2}|v_1|^2 + |u_1|^2\right)v_1 - \\
&\eta\left(\frac{1}{4}|v_1|^4 + \frac{3}{2}|v_1|^2|u_1|^2 + \frac{3}{4}|u_1|^4\right)v_1 + \\
&\qquad u_1 + \lambda v_2 + mu_{1xx} = 0, \\
&\qquad (\ldots)
\end{aligned} \tag{1}$$

$$(\ldots)$$

$$iu_{2t}+iu_{2x}+\left(\frac{1}{2}|u_2|^2+|v_2|^2\right)u_2-$$
$$\eta\left(\frac{1}{4}|u_2|^4+\frac{3}{2}|u_2|^2|v_2|^2+\frac{3}{4}|v_2|^4\right)u_2+$$
$$v_2+\lambda u_1+mv_{2xx}=0, \quad (1)$$
$$iv_{2t}-iv_{2x}+\left(\frac{1}{2}|v_2|^2+|u_2|^2\right)v_2-$$
$$\eta\left(\frac{1}{4}|v_2|^4+\frac{3}{2}|v_2|^2|u_2|^2+\frac{3}{4}|u_2|^4\right)v_2+$$
$$u_2+\lambda v_1+mu_{2xx}=0.$$

Here, $u_{1,2}(x,t)$ and $v_{1,2}(x,t)$ are the amplitudes of the forward and backward-propagating waves in core 1 and 2 respectively. η is a positive real parameter that represents the strength of the quintic nonlinearity, λ is the linear coupling between two cores, and the strength of the dispersive reflectivity is represented by $m>0$. To determine the spectral gap, we substitute $u_{1,2}, v_{1,2}\sim\exp(ikx-iwt)$ into the linearized Eqs. (1). This leads to the following dispersion relation:

$$\omega^2=\left(1-mk^2\right)^2+\lambda^2+k^2\pm 2\lambda\sqrt{\left(1-mk^2\right)^2+k^2} \quad (2)$$

From Eq. 2 it is found that the bandgap is given by $\omega^2<(1-|\lambda|)^2$ for $m\leq 0.5$ and $\omega^2<\left(\frac{\sqrt{4m-1}}{2m}-|\lambda|\right)^2$ for $m>0.5$.

3 SOLITON SOLUTIONS

Soliton solutions can be obtained by substituting $u(x,t)=U(x)e^{-i\omega t}$and $v(x,t)=V(x)e^{-i\omega t}$ into Eqs. (1) which will result in the following system of coupled equations (Akter et al., 2019):

$$-mU_{1xx}=\omega V_1-iV_{1x}+\left(\frac{1}{2}|V_1|^2+|U_1|^2\right)V_1-$$
$$\eta\left(\frac{1}{4}|V_1|^4+\frac{3}{2}|V_1|^2|U_1|^2+\frac{3}{4}|U_1|^4\right)V_1+$$
$$U_1+\lambda V_2,$$
$$-mV_{1xx}=\omega U_1+iU_{1x}+\left(\frac{1}{2}|U_1|^2+|V_1|^2\right)U_1-$$
$$\eta\left(\frac{1}{4}|U_1|^4+\frac{3}{2}|U_1|^2|V_1|^2+\frac{3}{4}|V_1|^4\right)U_1+$$
$$V_1+\lambda U_2,$$
$$(\ldots) \quad (3)$$

$$(\ldots)$$
$$-mU_{2xx}=\omega V_2-iV_{2x}+\left(\frac{1}{2}|V_2|^2+|U_2|^2\right)V_2-$$
$$\eta\left(\frac{1}{4}|V_2|^4+\frac{3}{2}|V_2|^2|U_2|^2+\frac{3}{4}|U_2|^4\right)V_2+$$
$$U_2+\lambda V_1,$$
$$-mV_{2xx}=\omega U_2+iU_{2x}+\left(\frac{1}{2}|U_2|^2+|V_2|^2\right)U_2-$$
$$\eta\left(\frac{1}{4}|U_2|^4+\frac{3}{2}|U_2|^2|V_2|^2+\frac{3}{4}|V_2|^4\right)U_2+$$
$$V_2+\lambda U_1. \quad (3)$$

Equations 3 do not have any analytical solution. Hence, we have to solve Eqs. 3 by a relaxation algorithm. There are two disjoint families of gap solitons known as Type 1 and Type 2 solitons, and each of the soliton families supports symmetric $(u_1=u_2, v_1=v_2)$ and asymmetric type solitons $(u_1\neq u_2, v_1\neq v_2)$. The difference between the soliton families is in their amplitude, phase, and parities. The two families are separated by a border that needs to be determined numerically (Akter et al., 2019).

4 INTERACTION OF QUIESCENT SOLITONS

To analyze the interaction of stable quiescent solitons we have numerically solved Eqs. (1) subject to the following initial conditions:

$$u_{1,2}(x,0)=u_{1,2}\left(x+\frac{\Delta x}{2},0\right)+u_{1,2}\left(x-\frac{\Delta x}{2},0\right)\exp(i\Delta\phi)$$
$$v_{1,2}(x,0)=v_{1,2}\left(x+\frac{\Delta x}{2},0\right)+v_{1,2}\left(x-\frac{\Delta x}{2},0\right)\exp(i\Delta\phi) \quad (4)$$

where Δx and $\Delta\phi$ denote the initial separation and phase difference of the stable zero velocity gap solitons respectively.

The interaction of two zero velocity gap solitons is depends on the strength of the dispersive reflectivity. For lower values of dispersive reflectivity $(0<m\leq 0.3)$, the interaction outcomes behave similar to Nonlinear Schrödinger (NLS) solitons, i.e. the in-phase zero velocity solitons will initially attract each other and out-of-phase solitons will always repel. Interaction of in-phase asymmetric Type 1 may

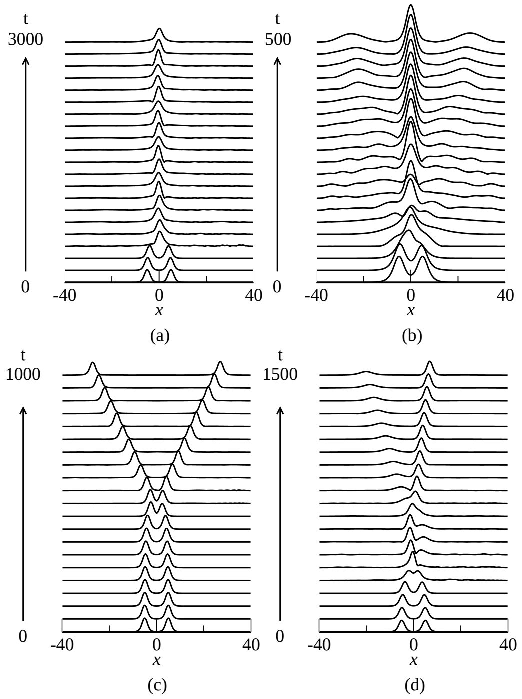

Figure 1: Interaction outcomes of Type 1 asymmetric quiescent solitons for $m = 0.2$, $\lambda = 0.1$, $\Delta x = 10.0$. (a) Merger into a quiescent soliton for $\omega = 0.3$, $\eta = 0.2$, $\Delta\phi = 0.0$, (b) generation of three solitons (one quiescent and two moving solitons) for $\omega = 0.81$, $\eta = 0.77$, $\Delta\phi = 0.0$, (c) symmetric separation for $\omega = 0.2$, $\eta = 0.14$, $\Delta\phi = 0.0$, and (d) asymmetric separation after multiple collision for $\omega = 0.45$, $\eta = 0.21$, $\Delta\phi = 0.0$. In this and other figures, only $u_1$component is shown.

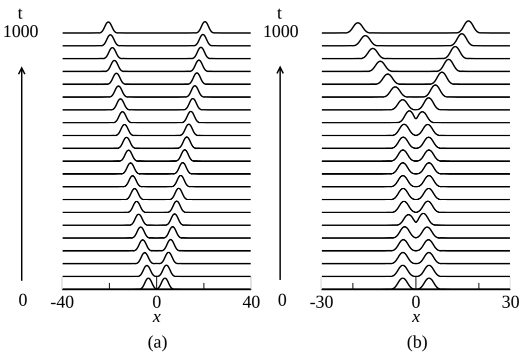

Figure 2: Examples of interaction of Type 1 asymmetric quiescent solitons for $m = 0.4$, $\lambda = 0.1$, $\omega = 0.40$, $\eta = 0.2$; (a) Repulsion of both solitons for $\Delta x = 7.0$, $\Delta\phi = 0$; (b) multiple collisions followed by formation of two separating solitons for $\Delta\phi = \pi$ and $\Delta x = 8.35$.

lead to a variety of outcomes, namely formation of a quiescent soliton Fig. 1(a), generation of three solitons (one quiescent and two moving ones) (Fig. 1(b)), generation of two symmetrically separating solitons (Fig. 1(c)), two separating solitons with different velocities (Fig. 1(d)) and repulsion of solitons.

For larger values of dispersive reflectivity (i.e., $0.3 < m \leq 0.5$), the outcomes of the interactions become dependent on the initial separation. As is shown in Fig. 2(a), when $\Delta x = 7.0$ the in-phase asymmetric Type 1 solitons repel each other. However, when the phase difference between the solitons is π, and $\Delta x = 7.75$, the solitons undergo multiple collisions and eventually two separating solitons are generated. Similar behavior has also been observed in the case of Type 2 solitons.

5 CONCLUSIONS

Interaction properties of stable quiescent Gap solitons in a coupled Fiber Bragg gratings have been investigated, where both cores have cubic-quintic nonlinearity with dispersive reflectivity. A noteworthy result is that for low to moderate dispersive reflectivity the interactions have similar charactersitics of NLS solitons, namely the in-phase solitons attract and π-out-of-phase solitons repel. On the other hand, for strong dispersive reflectivity, the outcomes are affected by the initial separation of the solitons.

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