Discrete Focus Group Optimization Algorithm for Solving Constraint Satisfaction Problems

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Abstract: We present a new nature-inspired approach based on the Focus Group Optimization Algorithm (FGOA) for solving Constraint Satisfaction Problems (CSPs). CSPs are NP-complete problems meaning that solving them by classical systematic search methods requires exponential time, in theory. Appropriate alternatives are approximation methods such as metaheuristic algorithms which have shown successful results when solving combinatorial problems. FGOA is a new metaheuristic inspired by a human collaborative problem solving approach. In this paper, the steps of applying FGOA to CSPs are elaborated. More precisely, a new diversification method is devised to enable the algorithm to efficiently find solutions to CSPs, by escaping local optimum. To assess the performance of the proposed Discrete FGOA (DFGOA) in practice, we conducted several experiments on randomly generate hard to solve CSP instances (those near the phase transition) using the RB model. The results clearly show the ability of DFGOA to successfully find the solutions to these problems in very reasonable amount of time.

1 INTRODUCTION

A wide variety of real world applications, including scheduling, planning (Mouhoub, 2003), configuration (Mouhoub & Sukpan, 2012) and timetabling (Hmer & Mouhoub, 2016), can be seen as constraint problems. Over the last four decades, researchers have focused on developing effective algorithms including systematic and approximation methods for tackling these problems modeled using the Constraint Satisfaction Problem (CSP) framework (Dechter, 2003). A CSP includes a finite set of variables, $X = \{x_1, \dots, x_n\}$, for every variable x_i, a finite set of values (or domain D_i), and a finite set of constraints $C = \{c_1, \dots, c_k\}$ that restrict the values that variables can simultaneously take. A CSP solution, $S = \{x_1 = d_1, \dots, x_n = d_n\}$ where $d_i \in D_i$, is the assignment of values to each variable such that all constraints are satisfied. When solving a CSP, we might be looking for one, many or all solutions (Solnon, 2002). In the case where a solution does not exist, the problem is inconsistent. As a matter of fact, many of the real world problems are over-constrained and do not have a solution. In this particular case, the goal is to look for an assignment satisfying the largest number of constraints. This latter notion is the generalized definition of CSPs which is called Max-CSPs (Freuder, 1992).

The most well-known systematic search algorithm for solving CSPs is Backtracking (Dechter, 2003). This algorithm incrementally attempts to extend a partial solution toward a complete one by assigning values to variables in a particular sequence. Given that CSPs are NP-hard problems, solving them with systematic search methods requires an exponential time, $O(d^n)$, where n is the number of variables and d their domain size. Despite this limitation, the running time, in practice, of Backtracking can be improved through constraint propagation (Dechter, 2003). However this latter algorithm has limitations for those hard to solve problems (Solnon, 2002).

An alternative is to use incomplete methods like metaheuristic algorithms. Although these algorithms do not guarantee to find a solution to a CSP (nor they can prove the inconsistency of over-constrained problems), they are often capable solving CSPs in a reasonable amount of time. Metaheuristics explore search spaces, using a compromise between exploitation and exploration in order to find a solution. The main inspiration sources of these algorithms are swarm intelligence, biological

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processes as well as chemical and physical systems. Over the last two decades, these algorithms have become very popular due to their successes in dealing with combinatorial problems and CSPs in particular.

For instance, in (Solnon, 2002) a new approach based on Ant Colony Optimization (ACO) is presented for solving CSPs. The basic idea of this work was to keep track of promising areas by laying pheromone on them. This pheromone information is used then as a heuristic for assigning appropriate values to the problem variables. The performance of the proposed ACO algorithm is boosted using local search methods.

The Firefly Algorithm (FA) is another powerful metaheuristic which has been successfully adopted to CSPs as shown in (Bidar, 2018) and (Bidar, Mouhoub, & Sadaoui, 2018). In these works, discrete version of FAs (called discrete FA or DFA) were proposed and evaluated on different CSP instances generated using the model RB. In (Fister, 2013) the applicability of FA for solving graph coloring problems has been investigated. In this work, a heuristic swap local search has been employed to improve the overall search.

In (Breaban, 2007), a new discrete Particle Swarm Optimization algorithm (PSO) is proposed for solving CSPs. The new algorithm is obtained after transforming a continuous PSO into its discrete version as well as adopting important features such as velocity and new positions of the particles. In (Bidar & Mouhoub, 2019), a new discrete PSO was proposed for solving CSPs in dynamic environment (Dynamic CSPs (DCSPs)). In this work all the features of the standard PSO redefined to be able to deal with discrete problems like CSPs. This method successfully applied to DCSPs and achieved very promising results.

In (Eiben, 1994), Genetic Algorithms (GAs) have been investigated for solving CSPs and their applications including, Graph Coloring Problems (GCPs) and N-Queen problems. In this regard, several experiments have been conducted and their results reported in the paper.

In (Abbasian, 2016) a new parallel architecture, called Hierarchical Parallel Genetic Algorithm (HPGA) has been proposed for solving CSPs. In addition to exploring the search in parallel, through a set of Islands of Parallel GAs (PGAs), this proposed algorithm uses a new operator, called the Genetic Modifier (GM) that injects good solutions to these islands. These good solutions are obtained after gathering useful information from constraint violations in previous runs and ordering variables according to (Mouhoub, 2011).

Other attempts for solving variants of CSPs have also been proposed, such as in (Salari, 2008) where ACO has been proposed for tackling GCPs. Here, the authors present a new Max-Min ACO where at each iteration Kempe Chain local search is applied to boost the search. In (Mouhoub & Wang, 2008) and (Mouhoub & Wang, 2006), the authors adopted the ACO algorithm to quadratic assignment problems using CSPs framework. In these works they proposed new random walks strategies to improve the stochastic local search of the standard ACO in order to address the weakness of the ACO in getting stuck in local optimum solution and immature convergence. They also proposed a new forward look ahead strategy to improve the exploitation feature of the algorithm.

In (Cui, 2008), an improved PSO algorithm is reported for solving GCPs. In this regard, a disturbance factor is used in order to improve the performance of the solving algorithm. The idea behind the disturbance factor is to help the algorithm escape local optimum by choosing some particles (according to a probability function which corresponds to the hardest problems to solve) and resetting their velocities. This addresses one of the main shortcomings of PSOs which consists of immature convergence.

Recently, Fattahi and Bidar have proposed a new metaheuristic, namely the Focus Group Optimization Algorithm (FGOA) based on human collaborating behavior in finding the best solution for a problem through group discussion (Fattahi, 2018). The results of the experiments conducted on different benchmarking functions including the constrained and unconstrained ones, have shown the high performance of FGOA and the potential it has to dealing with problems under constraints. This has motivated us to develop a discrete version of FGOA that we call, Discrete FGOA (DFGOA), in order to deal with CSPs.

To assess the performance of the proposed DFGOA in practice, we conducted several experiments on randomly generate hard to solve CSP instances (those near the phase transition) using the RB model. The results clearly show the ability of DFGOA to successfully find the solutions to these problems in very reasonable amount of the time.

2 FOCUS GROUP OPTIMIZATION ALGORITHM (FGOA)

FGOA is a new metaheuristic algorithm proposed by Fattahi and Bidar (Fattahi, 2018) for global optimization tasks. This algorithm is inspired by collaborative behavior of a group's members in sharing their ideas on a subject in an attempt to develop an appropriate solution for that problem. The pseudo-code of FGOA is presented in Algorithm 1. To get the best solution to a given problem, FGOA works as follows. All members share their solutions through group communication and discussion, in an iterative manner, and under the supervision of an agent called the Note Taker. Each member's solution is getting affected by the other members' solutions. This impact is calculated according to (1):

$$PIm_i^{k+1} = w \times PIm_i^k + \sum_{j=1}^{N}(IC_j \times Rnd \times (PBI_j^k - PI_i^k)) \quad (1)$$

where, PIm_i^k is the impact of other participants' solutions on the solution of participant i in k^{th} iteration, PI_i^k is the solution of the participant i in iteration k, PBI_j^k is the best solution of participant j achieved before iteration k, and IC_j is the impact of participant j which should be calculated based on the cost of the best solution achieved by participant j. w is the inertia weight. It is a real value in the interval [0, 1].

```
Initialization
while (termination criterion is not met)
   for i=1 to N   // N is the population size
      IC = getCindex (PBC)
      Calculate the impact of other solutions on Solution i:
      PIm_i^{k+1} = w × PIm_i^k + Σ_{j=1}^{N}(IC_j × Rnd × (PBI_j^k − PI_i^k))
      Apply impact limits on PIm_i^{k+1}
      Update PI_i^{k+1} based on PIm_i^{k+1}
      Apply limits on PI_i^{k+1} by facilitator
      Evaluate the PC_i^{k+1}
      Update PBI_i^{k+1} based on PC_i^{k+1} and PBI_i^k
end for
Update NBC^k
k=k+1
end while
```

PIm_i^k: impact of other solutions on Solution i in k^{th} iteration
PBI_i^k: Best Solution of i^{th} Participant in k^{th} iteration
PI_i^k: Solution of i^{th} Participant in k^{th} iteration
NBC^k: Best Cost in k^{th} iteration
IC_i: Impact Coefficient belong to i^{th} Participant
w: Inertia Weight
PC_i^k: Cost of i^{th} Participant in k^{th} iteration
PBC: Best Cost of all Participants
Rnd: Random Number
getCindex: A function that returns the Impact Coefficient based on PBC

Algorithm 1: Pseudo code of FGOA.

PIm_i^k must be kept within the lower and the upper bounds of impact as shown below in (2), as we need to enforce some constraints to do so.

$$PIm_i^k = \begin{cases} \max(PIm_i^k, lower\ bound\ of\ impact) \\ \min(PIm_i^k, upper\ bound\ of\ impact) \end{cases} \quad (2)$$

Finally, the solution of participant i is updated based on the impact of the other solutions which is calculated using (1):

$$PI_i^{k+1} = PI_i^k + PIm_i^{k+1} \quad (3)$$

The upper and lower bounds are enforced on the participants' solutions by (4) to keep them within the bounds of the problem:

$$PI_i^{k+1} = \begin{cases} \max(PI_i^k, the\ lower\ bound) \\ \min(PI_i^k, the\ upper\ bound) \end{cases} \quad (4)$$

3 DISCRETE FOCUS GROUP OPTIMIZATION ALGORITHM (DFGOA)

The basic version of the FGOA has been developed to deal with continuous problems ($X \in R^n$). To apply it to CSPs where search spaces are discrete ($X \in S_n$), we need a discretization of this algorithm as described in the following subsections.

A. Potential Solution Representation

Let us consider a CSP with 6 variables, $V = \{V_1, V_2, V_3, V_4, V_5, V_6\}$, define on a domain $D = \{1, 2, 3, 4\}$. A candidate solution is represented in Figure 1.

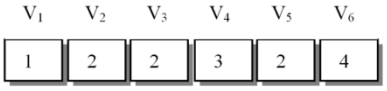


Figure 1: Solution representation.

B. Fitness Function

Given that solving CSPs consists in finding a complete assignment satisfying all the constraints (or the one minimizing the number of constraints in the case of over-constrained problems), we define the fitness function as the total number of violated constraints, for the given potential solution.

C. Solution Update

We define the impact factor parameter for each potential solution based on its quality according to (5).

$$IF^{t+1}(i) = IF^{t}(i) + \sum_{j=1}^{nPop}\left(\left(\frac{rand(1)\times(|F(S_i)-F(S_j)|)\times IC(j)}{Nvar}\right)^{m}\right) \quad (5)$$

Here, *IF(i)* is the impact factor of participant *i* which will take an important role in the next steps to affect the other participants' solutions, $IF^{t+1}(i)$ is the new impact factor of participant *i*, *nPop* is the population size, *Nvar* is the number of variables of the problem, *rand(1)* generates a random number in (0,1) and $F(S_i)$ and $F(S_j)$ are the qualities of solutions *i* and *j* respectively.

$IC(j)$, the impact coefficient, is a random number in (0,1) and is assigned to each solution. In this regard, a set of *nPop* random numbers is generated and is based on the quality assigned to each solution (the more quality a solution has, the larger the value will be assigned to).

As an example, Table 1 shows this process for a minimization problem for a set of given solutions with associated qualities.

Table 1: Assignmnet of Impact Coefficient □□(□) to each Solution for Given Instance Problem (RD are generated random numbers).

	S1	S2	S3	S4	S5	S6	S7	S8
F(S_i)	*21*	*20*	*18*	*14*	*10*	*7*	*8*	*2*
RD	*0.71*	*0.51*	*0.07*	*0.18*	*0.40*	*0.59*	*0.24*	*0.14*
IC(i)	*0.18*	*0.21*	*0.24*	*0.40*	*0.51*	*0.63*	*0.59*	*0.71*

D. Affecting other participants' Solutions

In a discrete problem space, affecting a solution can be interpreted as replacing its variables' values with the corresponding values of the better solution with an appropriate probability. This is done in order to avoid the immature convergence of the algorithm. In our proposed algorithm, this replacement is done by considering *IF()* as the probability of this replacement. In our experimentation, we normalize the Impact Factor between 0 and 1 according to (6).

$$IF(i)_{Normalized} = 1 - \frac{F(S_i)-F(Bsolution)}{F(Wsolution)-F(Bsolution)} \quad (6)$$

Here, $F(Bsolution)$ and $F(Wsolution)$ are the expected qualities of the best and the worst solutions. In fact, the larger *IF(i),* the more chance participant *i* (S_i) has to impact the other participant' solutions. This replacement is done according to (7).

$$Rep(S_i, S_j) = \{S_j(k) \leftarrow S_i(k), \quad if\ S_j(k) \neq S_i(k) \text{ and } rnd < IF(I) \quad (7)$$

$Rep(S_i, S_j)$ is the replacement equation, *rnd* is a random number in (0,1). Figure 2 indicates the steps through which S_2 is being affected by S_1. According to this figure, the corresponding variables in two solutions with equal values remain unchanged. However, the other variables' values of S_2 are replaced with probability *IF(1)=0.3,* by the corresponding variables' values of S_1 .

	V1	V2	V3	V4	V5	V6
S_1:	1	3	2	4	1	1
S_2:	3	1	3	4	2	3
$S_1 \rightarrow S_2$ ↓						
1st step →	3			4		
2nd step→	3	1		4		
3rd step→	3	1	2	4		
4th step→	3	1	2	4	2	
5th step→	3	1	**2**	4	2	**1**

Figure 2: Steps showing how participant 2 (S_2) is affected by participant 1 (S_1).

At the first, second and forth steps above, the variables' values of S_2 remained unchanged. However, in third and fifth steps, S_2 variables values are replaced by those of S_1, resulting in S_2 = [3 1 2 4 2 1].

E. Solutions Diversification

One of the main challenges when searching for a solution is the risk of being trapped in a local optimum. This immature convergence is caused by the lack of diversity in potential solutions. To overcome this issue, diversification via randomization is adopted to enable the algorithm to search problem spaces more efficiently.

In this regard, solutions that are different from the current ones are generated which results in higher probability of escaping local optimum and, hopefully, get optimal solutions.

In this regard, we use a controlling parameter, called *CP,* to detect if the FGOA has been trapped in local optimum, and this happens when it cannot make further improvements. This parameter, through (8), monitors the progress trend of the algorithm and if, for some iterations, not enough progress has been made by the algorithm, this parameter enables a randomization method to diversify the solutions.

$$CP = \frac{\sum_{i=IN-WS}^{IN} (GB(i) - GB(i-1))}{WS} \quad (8)$$

IN is the current iteration number, *WS* is the window size, and *GB(i)* is the global best solution in iteration *i*. Here, window size determines the number of iterations to be considered to determine if an acceptable progress has been made by the algorithm. If *CP* is less than the user-defined threshold value, the algorithm activates a new randomization method called *IF* Randomization (IFR).

F. IF Randomization (IFR)

We have employed *IF Randomization* method for diversifying the solutions. According to this method based on the Impact Factor (*IF*) of a solution, a variable value of a given solution is replaced with another value which is randomly chosen from its domain with probability $(1 - IF)^2$ (as shown in Figure 3).

The probability $(1 - IF)^2$ causes more quality solutions to be subject to less changes in their variables values. The procedure of IF Randomization is presented in Figure 4.

	V_1	V_2	V_3	V_4	V_5	V_6
S_1:	1	3	2	4	1	1
Select variables with probability $(1 - IF)^2$.		3	2			1
Assign new values for the selected variables.	**4**	3	2	**2**	**3**	1

Figure 3: Process of diversification of a solution considering probability $(1 - IF)^2$.

```
Procedure IF Randomization
  If CP<Threshold
    For k=1:Nvar
      rnd=rand()
      if rnd<(1-IF(k))^2
        Temp← randomly choose value d ∈ D_i
        S_i(k)←Temp
      Endif
    Endfor
  Endif
```

Figure 4: *IF Diversifier* scheme.

4 EXPERIMENTATION

To assess the performance of our DFGOA, we use the model RB (Xu, 2000) to randomly generate binary CSP instances (CSPs with constraints involving only pair of variables).

The model RB is based on the model B and has the advantage of generating those hard instances that are close to the phase transition.

The model RB has two controlling parameters *p* and *r*, and two critical values P_{cr} and r_{cr}. The relation between these two parameters and their corresponding critical values determines if a generated CSP instance is solvable or not. More precisely, if $P < P_{cr}$ and $r < r_{cr}$, a random CSP instance generated using the model RB is solvable with a high probability (close to 1) as the number of the variables approach the infinity. If $P > P_{cr}$ and $r > r_{cr}$, a CSP instance is unsolvable with probability close to 1.

Each CSP instance is generated as follows using the parameters *n, p, α* and *r* where *n* is the number of CSP variables, *p* (*0 < p < 1*) is the constraint tightness (ratio of the number of eligible tuples over the Cartesian product of the domains of the involved variables), and *r* and α (*0 < r, α < 1*) are two positive constants used by the model RB.

1. Select with repetition $t = r \times n \times ln\,(n)$ random constraints. Each random constraint is formed by selecting k of n variables (without repetition). *r* is the number of constraints for each CSP.
2. For each constraint, we uniformly select without repetition $q=p \times d^k$ incompatible pairs of values, where $d = n^\alpha$ is the domain size of each variable and each constraint involves $(1 - p) \times d^k$ compatible tuples of values ($k = 2$ for binary CSPs).

All the variables have the same domain corresponding to the first *d* natural numbers (*0 ... d−1*). According to (Xu, 2000), the phase transition P_{cr} is calculated as follows: $P_{cr} = 1 - e^{-\alpha/r}$. Solvable problems are therefore generated with $P<P_{cr}$.

The proposed method and Model RB have been implemented by MATLAB R2013b and all experiments have been performed on a PC with Intel Core i7-6700K 4.00 GHz processor and 32GB RAM.

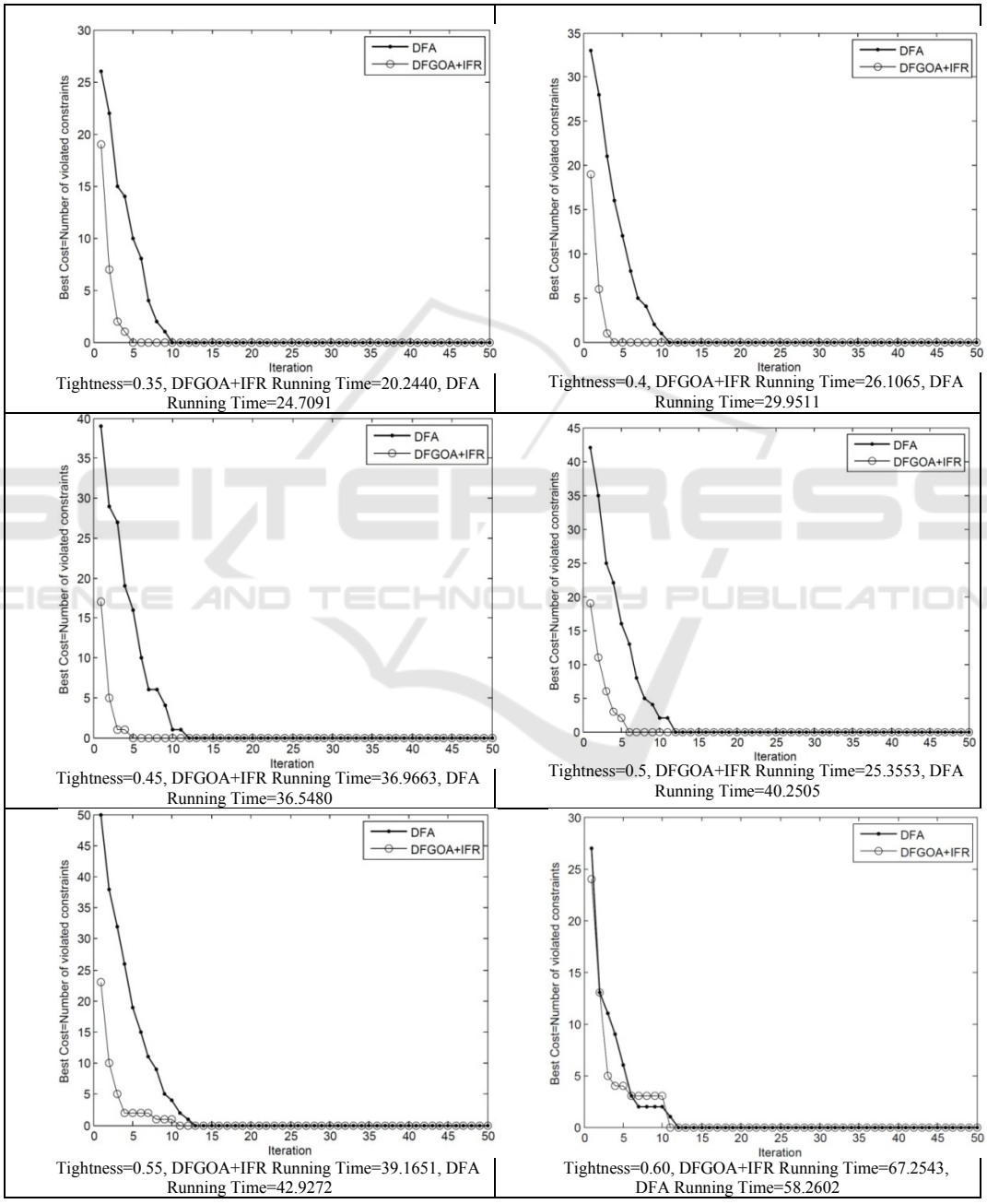
We compare our algorithm with the DFA presented in (Bidar, 2018) on the same test bed in terms of population size (30) and considering the best tuned parameters for both algorithms.

CSP instances are generated with different tightness value ranging from 0.1 to 0.6. We consider CSPs with 100 variables. The results are compared in terms of Success Rate (SR), Running Time (RT) and the Number of the Violated Constrained (NVC). The window size for our DFGOA is 3 and the threshold value is 0. Therefore, if for 3 successive iterations no improvement has been made, DFGOA activates the *IF Diversifier*. To normalize *IF* (see (6)) the worst solution (the one that violates all the constraints) has a fitness value equal to the total number of

constraints and subsequently the best solution is the one with fitness equal to zero. Figure 5 compare the convergence trend of the proposed DFGOA+IFR and DFA on CSPs with 100 variables. From the figures, we can see how both metaheuristic algorithms converge to the best solutions in a very good amount of the time.

We also compare the results of the experiments achieved by DFGOA, DFA and variants of systematic search methods, namely, Backtracking (BT), Forward Checking (FC), and Full Look Ahead (FLA) (Dechter, 2003). The results are reported in Tables 2.

Since all methods achieved the best solution in all experiments, this table only report the running time of different methods for achieving the best solution.



Tightness=0.35, DFGOA+IFR Running Time=20.2440, DFA Running Time=24.7091

Tightness=0.4, DFGOA+IFR Running Time=26.1065, DFA Running Time=29.9511

Tightness=0.45, DFGOA+IFR Running Time=36.9663, DFA Running Time=36.5480

Tightness=0.5, DFGOA+IFR Running Time=25.3553, DFA Running Time=40.2505

Tightness=0.55, DFGOA+IFR Running Time=39.1651, DFA Running Time=42.9272

Tightness=0.60, DFGOA+IFR Running Time=67.2543, DFA Running Time=58.2602

Figure 5: Comparing convergence trend of the DFGOA and DFA on CSPs with 100 variables and a tightness from 0.3 to 0.

FC and FLA have been developed to improve the performance of BT, in practice, by reducing the domain sizes (and consequently the search space) through constraint propagation (Dechter, 2003).

Table 2: Achieved Results (RT) by DFGOA and Systematic Methods on CSPs with 100 Variables.

P	DFGOA +IFR	DFA	BT	FC	FLA
0.1	2.694	6.0216	9.370	9.433	5.1778
0.15	9.012	8.418	20.321	16.428	15.3742
0.2	12.273	17.628	40.067	39.443	26.3828
0.25	14.483	14.026	49.523	47.175	39.3856
0.3	18.001	15.416	91.067	61.410	56.7726
0.35	19.830	21.849	141.969	101.679	100.783
0.4	19.903	30.539	158.854	149.838	124.532
0.45	26.443	31.810	196.688	148.484	145.761
0.5	34.699	38.109	306.486	198.877	165.945
0.55	39.739	43.627	296.960	239.307	213.962
0.6	46.565	55.126	416.111	323.019	252.535

The results show that although all methods are successful in getting the complete solutions, DFA and DFGOA outperform the systematic search techniques.

DFGOA shows however better performance than DFA and this is mainly due to its ability in diversifying the solutions thanks to our *IF Randomization*.

For further investigations, we compared the results achieved by FGOA and DFA with those achieved by the following GA variants (Abbasian, 2016). These comparisons are based on success rate (SR) and number of violated constraints (NVC).

- MPC: GA with multi parent crossover (Abbasian, 2016).
- OPC: Standard Genetic Algorithm with one point crossover (Abbasian, 2016).
- PSC: GA with Parental Success Crossover proposed in (Abbasian, 2016).
- HPGA+PSC: Hierarchical Parallel Genetic Algorithm.
- HPGA+GM+PSC: Hierarchical Parallel Genetic Algorithm with proposed GM operator in (Abbasian, 2016) and PSC crossover.

The results of these experiments are presented in Table 3. As we can see, DFGOA and DFA achieved the complete solutions (solutions that satisfy all constraint) in all experiments.

HPGA+GM+PSC achieved the best performance and was able to find the solutions with tightness ranging from 0.1 to 0.55 with 100% success rate. For those with a tightness of 0.6, the success rate is 79%, meaning complete solutions are found 79% of time.

The other versions of GA were unable to solve CSP instances near the phase transition.

For example, MPC's success rate in dealing with CSPs with tightness equal to 0.6 is 0% with 78 fitness average of its best achieved solutions.

Table 3: Achieved Results by DFGOA, DFA and variants of Genetic Algorithms on CSPs with 100 Variables.

	DFGOA+IFR	DFA	HPGA+PSC	HPGA+GM+PSC	MPC	OPC	PSC
P	**SR, NVC**	**SR, NVC**	**SR, NVC**	**SR, NVC**	**SR, NVC**	**SR, NVC**	**SR, NVC**
0.1	100%, 0	100%, 0	100%, 0	100%, 0	100%,0	100%, 0	100%, 0
0.15	100%, 0	100%, 0	100%, 0	100%, 0	0, 4	100%, 0	100%, 0
0.2	100%, 0	100%, 0	100%, 0	100%, 0	0, 9	27%, 0	100%, 0
0.25	100%, 0	100%, 0	100%, 0	100%, 0	0, 17	0, 3	100%, 0
0.3	100% ,0	100% ,0	100% ,0	100% ,0	0, 25	0, 5	100%, 0
0.35	100%, 0	100%, 0	100%, 0	100%, 0	0, 28	0, 10	100%, 0
0.4	100%, 0	100%, 0	100%, 0	100%, 0	0, 37	0, 12	62%, 0
0.45	100%, 0	100%, 0	100%, 0	100%, 0	0, 45	0, 27	0, 5
0.5	100%, 0	100%, 0	100%, 0	100%, 0	0, 53	0, 30	0, 9
0.55	100%, 0	100%, 0	73%, 0	100%, 0	0, 69	0, 42	0, 15
0.6	100%, 0	100%, 0	0, 3	79%, 0	0, 78	0, 62	0, 17

5 CONCLUSION

We propose the discrete version of the Focus Group Optimization Algorithm (FGOA) for solving CSPs. In this regard, we described in details all the necessary steps needed for DFGOA. Moreover, in order to deal with local optimum, we devised and proposed a new method for diversifying the potential solutions. The performances of DFGOA when augmented with this diversification method have been assessed by conducting experiments on random CSP instances generated by the model RB. Comparing to other metaheuristics as well as systematic search methods, DFGOA shows better running time even for the hardest instances.

In the near future, we plan to apply the DFGOA for solving different variants of the CSP. First, we will tackle over-constrained CSPs. In this particular case, a solution does not exist and the goal is to find one that maximizes the total number of solved constraints. This latter problem is called the max-CSP.

We will also consider the case where CSPs are solved in a dynamic environment. In this regard, the challenge is to solve the problem, in an incremental way, when constraints are added or removed dynamically.

Finally, we will consider the case where constraints are managed together with quantitative preferences. This problem is captured with the weighted CSP (Schiex, Fargier, & Verfaillie, 1995), where two types of constraints are considered: soft constraint that can be violated with associated costs and hard constraints that must be satisfied. The goal here is to find an optimal solution satisfying all the hard constraints while minimizing the total cost related to soft constraints.

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