
O. P. Angwech1, A. S. Alfa2,3 and B. T. J. Maharaj1

1Department of Electrical, Electronic and Computer Engineering, University of Pretoria, Pretoria 0002, South Africa
2Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, Manitoba, Canada
3Department of Electrical, Electronic and Computer Engineering, (CSIR/UP SARChI ASN Chair), University of Pretoria, Pretoria 0002, South Africa

Keywords: Wireless Sensor Networks, Energy Harvesting, Markov Chain.

Abstract: A model that considers energy storage and usage in data transmission in Wireless Sensor Network applications is proposed. The system is modelled as a Geo/Geo/1/k system and analysed using standard finite Markov chain model tools. The stationary distribution of the queue length is obtained. In the model, the harvested energy is stored in a buffer and used as required by the packets. In addition to energy usage by the packets, leakage of energy is captured at each state. A situation that involves high and low priority data transmission is also captured in the model. For evaluation, the effects of the system parameters on the performance measures are analysed. The results show that the model accurately captures the energy usage and it can be used for the management of harvested energy in Wireless Sensor Networks.

1 INTRODUCTION

The growth of Wireless Sensor Networks (WSNs) in the last decade has been aided by their use in various applications some of which are found in areas such as security, military systems, healthcare, transportation systems, agriculture and environment. WSNs consist of several tiny sensor nodes that are densely deployed either within an event or close to it. These sensor nodes consist of communicating, data processing and sensing components which may be required to fit into a match-box sized module (Akyildiz et al., 2002). In addition to size, other constraints on sensor nodes in WSNs include; low power consumption, size depending on the application, able to adapt to the environment, dispensable, have low production costs, operate unattended and be autonomous as they are often inaccessible (Akyildiz et al., 2002). The size constraint of sensor nodes makes power a scarce resource in the nodes as they are usually powered by small lithium cells batteries (usually 2.5cm in diameter and 1cm thick) which have limited capacity (Akyildiz et al., 2002; Vardhan et al., 2000). These batteries are usually affected by the energy usage pattern and active-ness level of the sensor nodes and can sustain the network for long or short periods.

In order to extend the network lifetime, energy harvesting has been one of the exploited areas in WSNs (Tadayon et al., 2013). Energy harvesting models play a very important role in the design and evaluation of energy harvesting systems. The models are primarily divided into three groups, namely, deterministic, stochastic and other models (Ku et al., 2016). Deterministic models are suitable for applications with predictable or slow varying energy sources and depend on the accurate prediction of the energy profile over a long period of time. However, modelling mismatch is encountered as prediction intervals are increased. In other models, RF signals that are artificially generated by external devices are considered (Mouapi et al., 2016). Deterministic models are suitable for applications with predictable or slow varying energy sources and depend on the accurate prediction of the energy profile over a long period of time. However, modelling mismatch is encountered as prediction intervals are increased. In other models, RF signals that are artificially generated by external devices are considered (Mouapi et al., 2017). The RF signals are either random or deterministic. The amount of energy harvested is dependent on two factors, namely, the transmitted power of transmitters and the channels from the transmitters to the harvesting receivers. These factors introduce a trade-off between energy and information transfer in wireless sensor networks. In a stochastic model, the energy renewal process is considered to be random. Non-causal energy state is not needed, hence making it suitable for applications
that have an unpredictable energy state. Some of the stochastic models used in energy harvesting applications include Bernoulli models, the Poisson process and the exponential process (Ku et al., 2016).

Many studies have been done in queueing theory and energy harvesting. Some works have focused on the self-sustainability of energy-harvesting networks with infinite battery capacity, stochastic arrivals of energy and a fixed rate of energy consumption (Guruacharya and Hossain, 2018). Others have focused on the design of the system in terms of the required sizes of the data and energy buffers (Zhang et al., 2013).

In other studies, priority scheduling is employed in the integration of different types of traffic in packet-based networks and is classified in two ways, pre-emptive and non-pre-emptive (Walraevens et al., 2011). In a pre-emptive system, the service is interrupted once a high priority event enters the system. On the other hand, in a non-pre-emptive system, service is completed and thereafter the high priority event is given service. In some works, the low priority events were subjected to a finite buffer and the high priority buffer was infinite (Jolai et al., 2010). In others, a two dimensional Markov chain is used to develop the state-transition matrix for the model (Ma et al., 2013).

Although research has been done on stochastic models, specifically, queueing models for energy harvesting in WSNs, some of these models do not accurately capture the energy drawing process (Zhang and Lau, 2015; Jeon and Ephremides, 2015; Ashraf et al., 2017; Dudin and Lee, 2016; Ku et al., 2015). The energy needs of nodes met by a combination of energy harvesting and a connection to the mains as a back-up when the batteries have been depleted has been studied (Gelenbe, 2015). This work assumes that the arrival of data packets and energy tokens follow a Poisson process and the rate at which energy is harvested is slow in comparison to the rate at which data is transmitted. In another study, a sensor node capable of energy harvesting was analysed and assumes uncertainty in energy harvesting, depletion and data acquisition. The sensor node was modelled as a two dimensional Markov chain with one queue for the energy and the other for the data. This study assumes that arrival rates of energy and data follow a Poisson process (De Cuypere et al., 2018).

In another study, the effect of energy loss through battery or capacitor leakage and standby operation is studied. The arrival rates of energy and data packets are assumed to follow a Poisson process and the leakage is represented as an exponential decay (Gelenbe and Kadioglu, 2015).

In this paper, the major contributions are as follows. Three models are proposed, two of which have two queues, the data queue and the energy queue. The third model is the priority model and has two data buffers, high priority (HP) and low priority (LP) data packets. The energy units are modelled as tokens required by packets in order to get processed. The arrival of both data packets and energy tokens are considered to be random. Both the data packets and energy tokens are considered to be discrete. One energy token is a discrete unit and is the minimum amount of energy required to transmit one data packet. The proposed model is a three-dimensional Markov chain with leakage imposed on the energy queue. To the author’s knowledge, a leakage of tokens at different transitions has not been implemented. This work aims to capture the accumulation, the leakage and the dissipation of energy in the model.

2 SYSTEM MODEL

The server is modelled as a single node queue in discrete time and the following assumptions are made, the data and energy buffers are finite i.e. the number of tokens and packets that can be queued is limited. One bit of data is referred to as a packet and one unit of energy is referred to as a token.

![Figure 1: System Model.](image)

For the token analysis, the following assumptions are made, each token gives permission for the transmission of one packet. If a token is waiting in the energy buffer, an arriving packet removes it from the buffer and enters the network. If there are no tokens waiting in the energy buffer, the incoming packet waits in the data buffer of a given size, when the buffer is at its maximum capacity the packet is lost. Similarly if the energy buffer is full, the tokens are lost.
3 QUEUEING ANALYSIS

3.1 Model

3.1.1 Description

The model suggests that the discrete inter-arrival times of the packets and tokens follow a geometric distribution with probabilities $a$, arrival of a packet and $b$, arrival of a token. Let $\bar{a} = 1 - a$, $\bar{b} = 1 - b$, $C^T$ be the transpose of matrix $C$ and $B_{01}^0$ be the transpose of matrix $B_{01}$. The system is considered to be first-come-first-serve (FCFS) unless stated otherwise. Figure 1 shows the proposed model.

3.1.2 State Space

The state space of the system in Figure 1 is described by a two-dimensional Markov chain, $I_n, J_n$, $n \geq 0$. $I_n$ is the number of packets in the buffer at time $n$, $0 \leq I_n \leq F$, $J_n$ is the number of tokens in the buffer at time $n$, $0 \leq J_n \leq K$. $K$ represents the token buffer and $F$ represents the packet buffer.

3.1.3 Transition Matrix

The transition matrix, $P$, is a classical Quasi-Birth-Death (QBD) matrix. An entry in the matrix represents the transition from one state given in the row to the next state given in the corresponding column. An absence of an entry in the matrix implies that the two states are not accessible to each other. By applying the lexicographical order for the state space, the probability transition matrix $P$ is obtained as follows.

$$ P = \begin{bmatrix} B & C \\ E & A_1 & A_0 \\ A_2 & A_1 & A_0 \\ \vdots & \vdots & \vdots \\ A_2 & A_1 & A_0 \end{bmatrix}, \quad (1) $$

where

$$ B = \begin{bmatrix} ab + \tilde{a}\tilde{b} & \tilde{a}b & ab + \tilde{a}\tilde{b} \\ ab & ab + \tilde{a}\tilde{b} & \tilde{a}b \\ \vdots & \vdots & \vdots \\ \tilde{a}b & ab + \tilde{a}\tilde{b} \end{bmatrix}, $$

$$ C = \begin{bmatrix} \tilde{a}b & 0 & \cdots & 0 \end{bmatrix}^T, $$

$$ E = \begin{bmatrix} \tilde{a}b & 0 & \cdots & 0 \end{bmatrix}, $$

$$ A_1 = ab + \tilde{a}\tilde{b}, A_0 = \tilde{a}b, A_2 = \tilde{a}b. $$

For the stable system $x$ is obtained such that

$$ x = xP, x1 = 1, \quad (2) $$

where

$$ x = [x_{00}, x_{01}, x_{02}, \ldots, x_{0K}, x_{10}, x_{20}, \ldots, x_{1K}]. $$

In the analysis of the performance of a queuing system, key system characteristics are specified namely, the arrival and service times (Alfa, 2010). The performance measures are obtained using Equation 3. The mean number of packets and tokens in the system are given as follows respectively.

$$ E_p[x] = \sum_{j=1}^{N} jx_{1j}, \quad E_t[x] = \sum_{j=0}^{K} jx_{0j}. \quad (3) $$

3.2 Model Including Leakage

To cater for energy leakage in the system, a parameter $\theta$ is introduced in the system. Energy leakage is expected when there is at least one token in the system. An assumption is made, when a token arrives, it stays until the next transition (at least one unit) before it leaks. The probability of leakage is $\theta$ and that of no leakage is $1 - \theta$. $K$ is the token buffer. The probability of having $k$ tokens leak when there are $n$ tokens at time $n$ in the system is a binomial distribution given as follows.

$$ I_{n,k} = \binom{n}{k} \theta^k (1 - \theta)^{n-k}, n \geq k. \quad (4) $$

The state space of this model is described in section 3.1.2.

3.2.1 Transition matrix

The probability transition matrix, $P$, for this model is obtained using equation 1.

where

$$ B = \begin{bmatrix} B_{00}^0 & B_{01}^0 & B_{02}^0 & \cdots & B_{0K}^0 \\ B_{00}^1 & B_{01}^1 & B_{02}^1 & \cdots & B_{0K}^1 \\ B_{00}^2 & B_{01}^2 & B_{02}^2 & \cdots & B_{0K}^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ B_{00}^K & B_{01}^K & B_{02}^K & \cdots & B_{0K}^K \end{bmatrix}, $$

with

$$ B_{00}^0 = (ab + \tilde{a}\tilde{b})I_{n,n} + abI_{n,n-1}, $$

$$ B_{01}^0 = \tilde{a}bI_{n,n} + (ab + \tilde{a}\tilde{b})I_{n,n-1} + abI_{n,n-2}, $$

$$ B_{02}^0 = \tilde{a}bI_{n,n-1} + (ab + \tilde{a}\tilde{b})I_{n,n-2} + abI_{n,n-3}, $$

$$ B_{0K}^0 = \tilde{a}bI_{K,1} + (ab + \tilde{a}\tilde{b})I_{K,K-1} + abI_{K,K-2}, $$

where

$$ C = \begin{bmatrix} B_{10}^0 & B_{10}^1 & B_{10}^2 & \cdots & B_{10}^K \end{bmatrix}^T, $$

with

$$ B_{10}^0 = \tilde{a}bI_{n,n}, E = \begin{bmatrix} \tilde{a}b & 0 & \cdots & 0 \end{bmatrix}, $$

$$ A_1 = ab + \tilde{a}\tilde{b}, A_0 = \tilde{a}b, A_2 = \tilde{a}b. $$

The performance measures are obtained using equations 2 and 3.
3.3 Priority Model

3.3.1 Description

A system with two classes of packets arriving according to the Bernoulli process is considered with probabilities $a_{H}$, arrival rate of High priority (HP) and $a_{L}$, arrival rate of Low priority (LP) packets. Figure 1 is modified to cater for the priority. The priority model has two separate packet buffers, one for the HP packets and the other for LP packets and is modelled as a Geo/Geo/1 pre-emptive system which considers the pre-emptive resume discipline.

The service discipline is as follows. No LP priority can start receiving service unless there is no HP packet in the system and, if a LP packet is receiving service (in the absence of a HP packet in the system), the service of this LP packet will be interrupted at the arrival of a HP packet occurring before the completion of the LP service. There is a possibility of having up to two packets of each type arriving since we are dealing with discrete time. Hence, the probability of arrivals of $i$ type HP and $j$ type LP is defined as $a_{i,j}$, $i = 0, 1; j = 0, 1$. For example, $a_{0,0} = (1 - a_{H})(1 - a_{L})$, $a_{0,1} = (1 - a_{H})a_{L}$, $a_{1,0} = a_{H}(1 - a_{L})$, $a_{1,1} = a_{H}a_{L}$.

3.3.2 State Space

The state space of the model is described by a three-dimensional Markov chain, $(I_{n}, J_{n}, K_{n})$, $n \geq 0$. $I_{n}$ is the number of HP packets in the buffer at time $n$, $0 \leq I_{n} \leq M$. $J_{n}$ is the number of LP packets in the buffer at time $n$, $0 \leq J_{n} \leq N$. $K_{n}$ is the number of tokens in the buffer at time $n$, $0 \leq K_{n} \leq K$. $k$ represents the token buffer, $M$ represents the HP packet buffer and $N$ represents the finite LP buffer. To cater for leakage of tokens in the system, equation 5 is used.

3.3.3 Transition Matrix

The transition matrix, $P_{t}$, is given as follows.

$$
P_{t} = \begin{bmatrix}
B & C \\
E & A_{1} & A_{0} \\
A_{2} & A_{1} & A_{0} \\
A_{2} & A_{1} & A_{0} \\
\vdots & \vdots & \vdots \\
A_{2} & A_{1} & A_{0}
\end{bmatrix}, \tag{5}
$$

where

$$
B = \begin{bmatrix}
B_{00} & B_{01} & B^{0} \\
B^{2} & B^{1} & B^{0} \\
\vdots & \vdots & \vdots \\
B^{2} & B^{1} & B^{0}
\end{bmatrix},
$$

with

$$
B_{00} = \begin{bmatrix}
B_{000}^{K} & B_{010}^{K} & B_{002}^{K} & \cdots & B_{00K}^{K} \\
B_{010}^{K} & B_{011}^{K} & B_{012}^{K} & \cdots & B_{01K}^{K} \\
B_{020}^{K} & B_{021}^{K} & B_{022}^{K} & \cdots & B_{02K}^{K} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
B_{020}^{K} & B_{021}^{K} & B_{022}^{K} & \cdots & B_{02K}^{K}
\end{bmatrix},
$$

with

$$
B_{00n}^{K} = (a_{1,0}b + a_{0,0}b)l_{n,n} + a_{1,1}b_{n,n-1} + (a_{1,0}b + a_{0,0}b)l_{n,n-2} + a_{1,1}b_{n,n-2},
$$

$$
B_{01n}^{K} = a_{0,0}b_{n,n} + (a_{1,0}b + a_{0,0}b)l_{n,n-1} + a_{1,1}b_{n,n-2} + (a_{1,0}b + a_{0,0}b)l_{n,n-3},
$$

$$
B_{02n}^{K}(K - 1) = a_{0,0}b_{K,K}(K - 1) + (a_{1,0}b + a_{0,0}b)l_{K,K}(K - 1) + a_{1,1}b_{K,K}(K - 3),
$$

$$
B_{010}^{K} = a_{0,0}b_{n,n} + (a_{1,0}b + a_{0,0}b)l_{n,n-1} + a_{1,1}b_{n,n-2} + (a_{1,0}b + a_{0,0}b)l_{n,n-3} + a_{1,1}b_{n,n-2},
$$

$$
B_{01n}^{K} = (a_{1,0}b + a_{1,1}b)l_{n,n} + (a_{1,0}b + a_{0,0}b)l_{n,n-1} + (a_{1,0}b + a_{0,0}b)l_{n,n-2} + a_{1,1}b_{n,n-2} + (a_{1,0}b + a_{0,0}b)l_{n,n-3} + a_{1,1}b_{n,n-3},
$$

$$
B_{02n}^{K}(K - 1) = a_{0,0}b_{K,K}(K - 1) + (a_{1,0}b + a_{0,0}b)l_{K,K}(K - 1) + a_{1,1}b_{K,K}(K - 3),
$$

$$
B_{00n}^{K} = (a_{1,0}b + a_{1,1}b)l_{n,n} + (a_{1,0}b + a_{0,0}b)l_{n,n-1} + a_{1,1}b_{n,n-1} + (a_{1,0}b + a_{0,0}b)l_{n,n-2} + a_{1,1}b_{n,n-2} + (a_{1,0}b + a_{0,0}b)l_{n,n-3} + a_{1,1}b_{n,n-3}.
$$

74
\[ C^2 = a_{1,0}b, C^1 = a_{1,0}b + a_{1,1}b, C^0 = a_{1,1}b, \]

\[ E = \begin{bmatrix} E^1 & \cdots & 0 & E^0 \\ \vdots & \ddots & E^1 & E^0 \\ \vdots & \ddots & \ddots & \ddots \\ 0 & \cdots & 0 & E^1 + E^0 \end{bmatrix}, \]

with

\[ E^1 = a_{0,0}b, E^0 = a_{0,1}b, \]

\[ A_1 = \begin{bmatrix} A_1^1 & A_1^0 \\ A_1^0 & A_1^1 + A_1^0 \end{bmatrix}, \]

\[ A_0 = \begin{bmatrix} A_0^1 & A_0^0 \\ A_0^0 & A_0^1 + A_0^0 \end{bmatrix}, \]

\[ A_2 = \begin{bmatrix} A_2^1 & A_2^0 \\ A_2^0 & A_2^1 + A_2^0 \end{bmatrix}, \]

with

\[ A_2^1 = a_{0,0}b, A_2^0 = a_{0,1}b. \]

For a stable system we want to obtain \( x \) such that equation 3 is satisfied.

where

\[ x = [x_{000}, x_{001}, x_{002}, \ldots, x_{00K}, x_{100}, x_{010}, x_{020}, \ldots, x_{000}, x_{100}, x_{200}, x_{300}, \ldots, x_{M00}]. \]

The performance measures are obtained using Equations 6 and 7. The mean number of tokens \( E_t[x] \), LP packets \( E_{lp}[x] \) and HP packets \( E_{hp}[x] \) in the system are given as,

\[ E_t[x] = \sum_{k=0}^{K} jx_{0,0,k} \] (6)

\[ E_{lp}[x] = \sum_{j=1}^{N} jx_{0,j,0} \]

\[ E_{hp}[x] = \sum_{i=1}^{M} ix_{i,0,0} \] (7)

4 RESULTS

Having developed the models and established the numerical analysis, we now evaluate the performance of a sensor node. We carry out simulations and obtain results with the following parameters, \( a \), the arrival rate of packets, \( b \), the arrival rate of tokens, \( F \), the packet buffer and \( K \) token buffer.

For the priority model the following parameters are used, \( a_H \), the arrival rate of HP packets, \( a_L \), the arrival rate of LP packets, \( M \), HP packet buffer, \( N \), LP packet buffer and \( K \) token buffer.

4.1 Model

Figure 2 shows the mean number of tokens and packets in the system as the arrival rate of packets is varied. The mean number of tokens decreases with an increase in the mean number of packets in the system.

![Figure 2: Effect of arrival rate of data packets on the mean number of tokens and packets in the system. Here \( a \) is varied, \( b = 0.6 \), \( F = 200 \) and \( K = 100 \).](image)

4.2 Model Including Leakage

Figure 3 shows the mean number of tokens and packets in the system as the rate of leakage is varied. The mean number of tokens decreases with an increase in the rate of leakage as packets are transmitted by the available tokens and the unused ones leak.

4.3 Priority Model

Figure 4 shows the mean number of tokens and packets in the system as the arrival rate of the HP packets is varied. The mean number of HP packets increases with an increase in the arrival of HP packets, this is as the rate of arrival of tokens is kept constant and will get depleted after a specific period.

75
Figure 3: Effect of leakage on the mean number of tokens and packets in the system. Here θ is varied, \( a = 0.52 \), \( b = 0.6 \), \( F = 72 \) and \( K = 18 \).

Figure 4: Effect of arrival rate of HP packets on the mean number of tokens and packets in the system. Here \( a_H \) is varied, \( a_L = 0.4 \), \( b = 0.6 \), \( θ = 0.4 \), \( M = 50 \), \( N = 20 \) and \( K = 24 \).

Figure 5: Effect of arrival rate of tokens on the mean number of tokens and packets in the system. Here \( b \) is varied, \( a_H = 0.2 \), \( a_L = 0.4 \), \( θ = 0.4 \), \( M = 50 \), \( N = 20 \) and \( K = 24 \).

Figure 6: Effect of leakage on the mean number of tokens and packets in the system. Here \( θ \) is varied, \( a_H = 0.2 \), \( a_L = 0.4 \), \( b = 0.6 \), \( M = 50 \), \( N = 20 \) and \( K = 24 \).

5 DISCUSSION

An ideal system is modelled to have only packets and tokens and the results are presented in Figure 2. It is observed that both the token and packet buffers cannot be full at the same time. When one buffer is full the other is empty. In addition, as the arrival rate of data packets increases the mean number of packets in the system is observed to increase and the mean number of tokens decreases. This is as each packet requires a token to be processed, thereby decreasing the mean number of tokens in the system as the arrival rate of packets increases.

However, in a practical system, unused tokens often leak. This is captured in the model whose results are presented in Figure 3. In this model, as the rate of leakage increases the mean number of tokens in the systems decreases. None of the buffers are full when the rate of leakage is 0 as neither the arrival rate of packets or tokens is 0. When the probability of leakage is 0, this implies that the token buffer is full, however, this is not the case as there are packets (\( a = 0.52 \)) in the system that use some of the tokens. The token buffer is therefore not full and the mean number of tokens in the system is 97.4.

In addition to leakage, a practical system will have emergency data (referred to as HP packets in this paper) which has to be transmitted immediately. Leakage of energy and priority are captured in the model whose results are presented in Figure 4, 5 and 6. As
the arrival rate of HP packets increases, the mean number of tokens in the system decreases and the mean number of HP and LP packets increases.

Finally we study the effect of leakage on the mean number of tokens, HP and LP packets in the system shown in Figure 6. As the rate of leakage increases, the mean number of tokens in the system decreases and the mean number of LP and HP packets increases.

The proposed models developed as Geo/Geo/1/K systems shows that the proposed model illustrates the effect of leakage on the mean number of tokens and packets in the system.

6 CONCLUSIONS

In this paper, the performance of an energy harvesting sensor node assuming data transmission and energy leakage was analysed. To this end, three models were investigated. Two of the models had an energy harvesting node which was modelled as a stochastic system with two queues, one for data packets and the other energy packets. We investigated the node when a leakage is imposed on the energy buffer. To further investigate the node, a third model was developed to observe the effect of priority. We showed that the each of proposed systems can be described by a Quasi-Birth-Death process (QBD). This allowed us to obtain the performance measures using the matrix-geometric methods. The simulations carried out revealed the effect of leakage on the mean number of tokens and packets in the system. Future work will address the threshold case, when a threshold is imposed on the token buffer. When the token buffer is below a specified level then the transmission of low priority packets will be halted and only high priority packets will be transmitted.

ACKNOWLEDGEMENTS

The authors would like to thank the South African Research Chairs Initiative (SARChI) in Advanced Sensor Networks (ASN) and SENTECH for their financial support in making this work possible.

REFERENCES


