Guessing Games Experiments in Ukraine: Learning towards Equilibrium

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Abstract: The paper deals with experimental game theory and data analysis. The research question, formulated in this work, is how players learn in complex strategic situations which they never faced before. We examine data from different games, played during lectures about game theory and present findings about players progress in learning while competing with other players. We proposed four "pick a number" games, all with similar-looking rules but very different properties. These games were introduced (in the body of scientific popular lectures) to very different groups of listeners. In this paper we present data gathered during lectures and develop tool for exploratory analysis using R language. Finally, we discuss the findings propose hypothesis to investigate and formulate open questions for future research.

1 INTRODUCTION

Game theory is a field of science which investigates decision-making under uncertainty and interdependence, that is, when the actions of some players affect the payoffs of others. Such situations arise around us every day and we, consciously or unconsciously, take part in them and try to succeed. The struggle to achieve a better result (in some broad sense) is called rationality. Every rational player must take into account the rules of the game, the interests and capabilities of other participants in other words think strategically. Game theory provides a tool for analyzing such situations, which allows you to better understand the causes of conflicts, learn to make decisions under uncertainty, establish mutually beneficial cooperation and much more.

A key element of strategic thinking is to include into consideration what other agents do. Agent here is a person, who can make decisions and his/her actions have influence on the outcome. Naturally, person cannot predict with 100% what will others do, so it is important to include into model beliefs about other person thinking and update them during the game. Also, if we can't know what other player think, we can understand what is his/her best course of action. This is the main research topic of game theory.

All this makes decision making very interesting problem to investigate. In this work we will apply game theory to analyze such problems. Game theory provides mathematical base for understanding strategic interaction of rational players. There is important note about rationality, we should make. As Robert Aumann formulate in his famous paper (Aumann, 1985), game theory operates with "homo rational", ideal decision maker, who is able to define his/her utility as a function and capable of computing best strategy to maximize it. This is the main setup of game theory and one of major lines of criticism. In reality, of course, people are not purely rational in game theory sense. They often do not want to concentrate on a given situation to search for best decision or simply do not have enough time or capabilities for this. Sometimes they just copycat behavior of others or use some cultural codes to make strange decisions. Also (as we see from the experiments) it seems that sometimes homo sapiens make decisions with reasons, one can (with some liberty in formulation) label as "try and see what happens", "make random move and save thinking energy" and even "make stupid move to spoil game for others".

This is rich area of research, where theoretical constructions of game theory seems to fail to work and experimental data shows unusual patterns. However, these patterns are persistent and usually do not depend on age, education, country and other things. During last 25 years behavioral game theory in numerous studies examines bounded rational-

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ity (best close concept to rationality of game theory) and heuristics people use to reason in strategic situations. For example we can note surveys of Crawford et al.; Mauersberger and Nagel (Crawford et al., 2013; Mauersberger and Nagel, 2018). Also there is comprehensive description of the field of behavioral game theory by Camerer (Camerer, 2011).

Also we can note work of Gill and Prowse (Gill and Prowse, 2016), where participants were tested on cognitive abilities and character skills before the experiments. Then authors perform statistical analysis to understand the impact of such characteristics on the quality of making strategic decisions (using p-beauty contest game with multiple rounds). In more recent work of Fe et al. (Fe et al., 2019) even more elaborate experiments are presented. It is interesting that in the mentioned paper experiments are very strict and rigorous (as close to laboratory purity as possible) in contrast to games, played in our research. But in the end of the day the results are not differ very much.

The guessing games are notable part of research because of their simplicity for players and easy analysis of rules from game theoretic prospective. In this paper we present results of games played during 2018–2020 years in series of scientific popular lectures. The audience of these lectures was quite heterogeneous, but we can distinguish three main groups:

- kids (strong mathematical schools, ordinary schools, alternative education schools);
- students (bachelor and master levels);
- mixed adults with almost any background;
- businessmen;
- participants of Data Science School.

We propose framework of four different games, each presenting one idea or concept of game theory. These games were introduced to people with no prior knowledge (at least in vast majority) about the theory. From the other hand, games have simple formulation and clear winning rules, which makes them intuitively understandable even for kids. This makes these games perfect choice to test ability of strategic thinking and investigate process of understanding of complex concepts during the play, with immediate application to the game. This dual learning, as we can name it, shows how players try-and-learn in real conditions and react to challenges of interaction with other strategic players.

1.1 Game Theory Definitions

We will consider games in strategic or normal form in non-cooperative setup. A non-cooperativeness here does not imply that the players do not cooperate, but it means that any cooperation must be self-enforcing without any coordination among the players. Strict definition is as follows.

A non-cooperative game in strategic (or normal) form is a triplet $G = \{\mathcal{N}, \{S_i\}_{i=\in\mathcal{N}}, \{u_i\}_{i\in\mathcal{N}}\}$, where:

- $\mathcal{N}$ is a finite set of players, $\mathcal{N} = \{1,\dots,N\}$;
- S_i is the set of admissible strategies for player i;
- $u_i : S \to \mathcal{R}$ is the utility (payoff) function for player i, with $S = \{S_1 \times \cdots \times S_N\}$ (Cartesian product of the strategy sets).

A game is said to be static if the players take their actions only once, independently of each other. In some sense, a static game is a game without any notion of time, where no player has any knowledge of the decisions taken by the other players. Even though, in practice, the players may have made their strategic choices at different points in time, a game would still be considered static if no player has any information on the decisions of others. In contrast, a dynamic game is one where the players have some (full or imperfect) information about each others' choices and can act more than once.

Summarizing, these are games where time has a central role in the decision-making. When dealing with dynamic games, the choices of each player are generally dependent on some available information. There is a difference between the notion of an action and a strategy. To avoid confusions, we will define a strategy as a mapping from the information available to a player to the action set of this player.

Based on the assumption that all players are rational, the players try to maximize their payoffs when responding to other players' strategies. Generally speaking, final result is determined by non-cooperative maximization of integrated utility. In this regard, the most accepted solution concept for a non-cooperative game is that of a Nash equilibrium, introduced by John F. Nash. Loosely speaking, a Nash equilibrium is a state of a non-cooperative game where no player can improve its utility by changing its strategy, if the other players maintain their current strategies. Of course players use also information and beliefs about other players, so we can say, that (in Nash equilibrium) beliefs and incentives are important to understand why players choose strategies in real situations. Formally, when dealing with pure strategies, i.e., deterministic choices by the players, the Nash equilibrium is defined as follows:

A pure-strategy Nash equilibrium (NE) of a non-cooperative game G is a strategy profile $s' \in S$ such that for all $i \in \mathcal{N}$ we have the following inequality:

$$u_i(s_i', s_{-i}') \geq u_i(s_i, s_{-i}')$$

for all $s_i \in S_i$.

Here $s_{-i} = \{s_j | j \in \mathcal{N}, j \neq i\}$ denotes the vector of strategies of all players except i. In other words, a strategy profile is a pure-strategy Nash equilibrium if no player has an incentive to unilaterally deviate to another strategy, given that other players' strategies remain fixed.

1.2 Guessing Games

In early 90xx Rosemary Nagel starts series of experiments (Mitzkewitz and Nagel (Mitzkewitz and Nagel, 1993)) of guessing games, summarized in (Nagel, 1995). She wasn't the first one to invent the games, it was used in lectures by different game theory researchers (for example Moulin (Moulin, 1986)). But her experiments were first experimental try to investigate the hidden patterns in the guessing game. Ho et al. (Ho et al., 1998) gave the name "p-beauty contest" inspired by Keynes (Keynes, 1936) comparison of stock market instruments and newspaper beauty contests. This is interesting quote, so lets give it here: "To change the metaphor slightly, professional investment may be likened to those newspaper competitions in which the competitors have to pick out the six prettiest faces from a hundred photographs, the prize being awarded to the competitor whose choice most nearly corresponds to the average preferences of the competitors as a whole; It is not a case of choosing those which, to the best of one's judgment, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligence to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees." (Keynes, 1936, chapter 12.V).

The beauty contest game has become important tool to measure "depth of reasoning" of group of people using simple abstract rules. Now there are variety of rules and experiments presented in papers, so lets only mention some of them.

2 EXPERIMENTS SETUP

The setup is closer to reality then to laboratory and this is the point of this research. All games were played under following conditions:

1. Game were played during the lecture about the game theory. Participants were asked not to comment or discuss their choice until they submit it. However, this rule wasn't enforced, so usually they have this possibility if wanted;
2. Participants were not rewarded for win. The winner was announced, but no more.
3. During some early games we used pieces of paper and we got some percentage of joking or trash submission, usually very small. Later we have switched to google forms, which is better tool to control submission (for example only natural numbers allowed).
4. Google forms gives possibility to make multiple submission (with different names), since we didnt have time for verification, but total number of submission allows to control that.

The aim of this setup was to free participants to explore the rules and give them flexibility to make decision in uncertain environment. We think it is closer to real life learning without immediate rewards then laboratory experiments. Naturally, this setup has strong and weak sides. Lets summarize both.

The strong sides are:

1. This setup allow to measure how people make decisions in "almost real" circumstances and understand the (possible) difference with laboratory experiments;
2. These games are part of integrated approach to active learning, when games are mixed with explanations about concepts of game theory (rationality, expected payoff, Nash equilibrium etc), and they allow participants to combine experience with theory;
3. Freedom and responsibility. The rules doesn't regulate manipulations with conditions. So this setup allows (indirectly) to measure preferences of players: do they prefer cheat with rules, just choose random decision without thinking or put efforts in solving the task.

Weak sides are:

1. Some percentage of players make "garbage" decisions. For example choose obviously worse choice just to spoil efforts for others;
2. Kids has (and often use) possibility to talk out decision with the neighbors;
3. Sometimes participants (especially kids) lost concentration and didn't think about the game but made random choice or just didn't make decisions at all;
4. Even for simplest rules, sometimes participants failed to understand the game first time. We suppose it is due to conditions of lecture with (usually) 30-40 persons around.

2.1 Rules

All games have the same preamble: Participants are asked to guess integer number in range 1 – 100, margins included. Note, that many setups, investigated in references, use numbers starting with 0. But the difference is small.

To provide quick choice calculation we have used QR code with link to google.form, where participants input their number. All answers were anonymous (players indicate nicknames to announce the winners, but then all records were anonymized). The winning condition is specific for every game.

1) p-beauty contest. The winning number is the closest to 2/3 of average;
2) Two equilibrium game. The winning number is the furthest from the average;
3) Coordination with assurance. The winning number is the number, chosen by plurality. In case of tie lower number wins;
4) No equlibrium game. The winning number is the smallest unique.

All these games are well-known in game theory. Lets briefly summarize them. First game is dominance-solvable game. Strategy "to name numbers bigger then 66" is (weakly) dominated, since it is worse then any other for almost all situations and equal in the rest. So rational player will not play it and everybody knows that. Then second step is to eliminate all numbers higher then 44 and so on. At the end rational players should play 1 and all win. In our setup we go further then just give players learn from observation. After first round we explain in detail what is Nash equilibrium and how it affect the strategies. After this explanation all participants actually knew that choosing 1 is the equilibrium option, when everyone wins. We supposed, that this should help to improve strategies in next round, but it is not.

Second game is about mixed strategies. Easy to show that if you want to choose number smaller then 50 – best way is to choose 1, since all other choices are dominated. And if you want to choose number bigger then 50 – best idea is to choose 100. Also it is meaningful to choose 50 – it almost never wins. So if many players will choose 1 – you should choose 100 and visa versa. In this game the best way to play is literally drop a coin and choose 1 or 100.

Third game has many equilibria, basically every number can be winning. But to coordinate players must find some focal points (Schelling (Schelling, 1960)). Natural focal point (but not only one!) is the smallest number since smaller number wins in case of tie. This slim formulation allow nevertheless make successful coordination in almost all experiments.

Finally last game is in a dark waters. As far as we know there is no equilibrium or rational strategy to play it. So sometimes very strange numbers are winners here.

3 RESULTS AND DATA ANALYSIS

In this section we present summary of data, gathered during the games.

3.1 First Game

Summary of results of First game is given in the table 1.

Almost all winning numbers are fall (roughly) in the experimental margins, obtained in Nagel (Nagel, 1995) work. With winning number no bigger then 36 and not smaller then 18 in first round. Two exceptions in our experiments were Facebook on-line test (15.32), when players can read information about the game in, for example, Wikipedia. And other is alternative humanitarian school (40.1), where participants seems didn't got the rules from the first time.

Using R statistical visualization tool we can analyze in details how players from different types change their decisions between first and second round (figure 1).

3.1.1 Metrics and Analysis

Interesting metric is the percent of "irrational choices" – choices that can't win in (almost) any case. Lets explain, imagine that all players will choose 100. It is impossible from practice but not forbidden. In this case everybody wins, but if only one player will deviate to smaller number – he/her will win and others will lose. So playing numbers bigger then 66 is not rational, unless you don't want to win. And here we come to important point, in all previous experiments this metric drops in second round and usually is very low (like less than 5%) (Ho et al., 1998). But in our case there are experiments where this metric become higher or changes very slightly. And initially values are much higher then expected. So here we should include factor of special behavior, we can call it "let's show this lecturer how we can cheat his test!". What is more interesting – this behavior more clear in case of adult then kids.

It is also interesting to see distribution of choices for different types of groups. We can summarize choices on the histograms (figure 2). Using models of

Table 1: Summary of first game for types of players.

Type	Round	Average	Winning	Median	Count	Irrationality
Adults	1	40.6	27	40	19	10.5
Adults (facebook online)	1	22.98	15.32	17	102	4.9
Alternative humanitarian	1	60.2	40.1	63	24	45.8
Alternative humanitarian	2	9.67	6.44	4	24	4.17
Alternative humanitarian	3	3.08	2.05	2	13	0
Alternative mathematical	1	41.9	27.9	42	35	17.1
Alternative mathematical	2	20.7	13.8	18	33	0
Business	1	44.4	29.6	41	65	27.7
Business	2	14.1	9.43	12	99	1.01
DS conference attendees	1	35.6	23.7	32.5	142	12.0
DS conference attendees	2	15.9	10.6	9	148	6.08
Math lyceum	1	37.7	25.1	33	148	14.2
Math lyceum	2	19.2	12.8	13	106	4.72
MS students	1	39.0	26.0	30	35	20
MS students	2	8.6	5.75	8.5	8	0
Ordinary school	1	48.7	32.5	46.5	26	23.1
Ordinary school	2	19.8	13.2	22	23	0
Tech School	1	43.4	28.9	45	51	23.5
Tech School	2	46.5	31.0	29	62	33.9

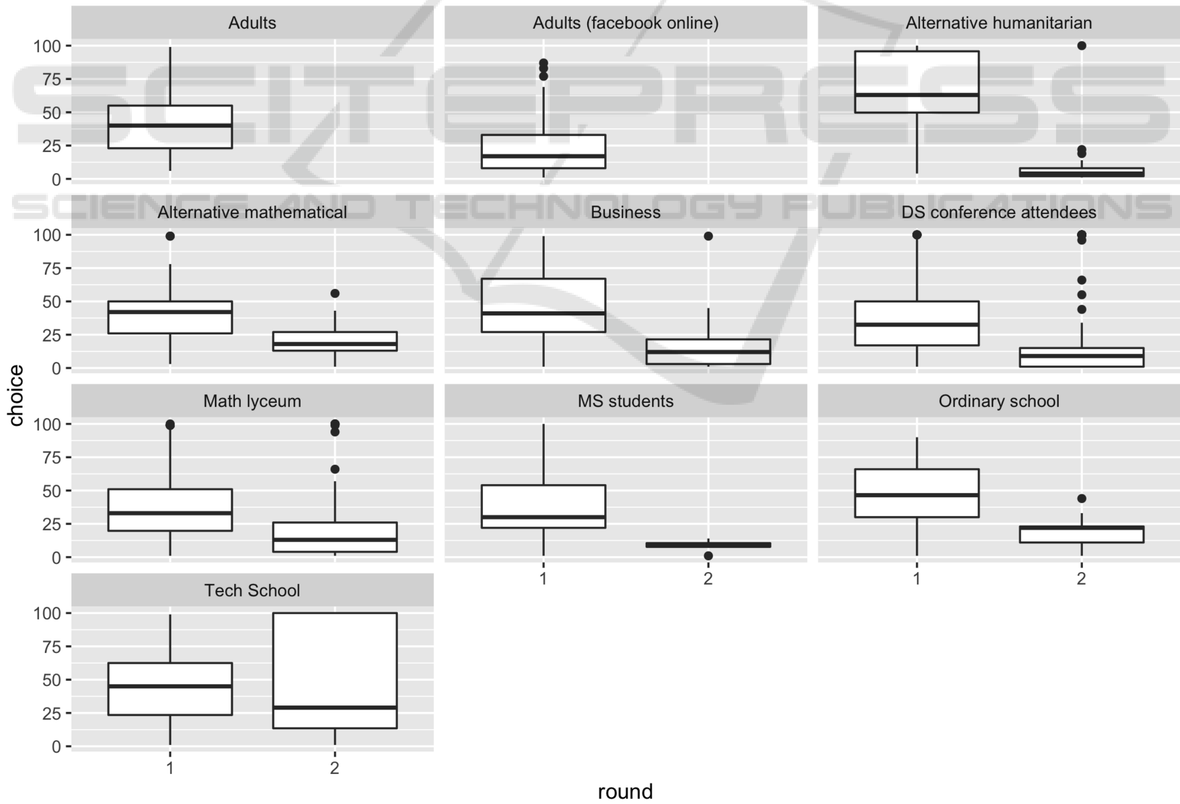

Figure 1: Graphical representation of learning between rounds.

strategic thinking we will adopt the theory of k-levels. According to this idea 0-level reasoning means, that players make random choices (drawn from uniform distribution), and k-level reasoning means that these players use best-response for reasoning of previous level. So 1-level reasoning is to play 33, which is best response to belief that average will be 50, 2-level is best response to belief that players will play 33 and so on.

Highlighting first 4 levels with dotted lines is a good idea, it is showing hidden patterns in strategy choosing of players.

As we can see from the diagram 2, some spikes in choices are predicted very good, but it depends on the background of players. The best prediction is for attendees of Data Science conference, which presumes high level of cognitive skill and computer science background.

Next two figures show the learning process from different angles. On figure 3 we can see points, defined by number of players with 0-level and "irrational" (choices with big numbers) versus "too smart" choices – choices from [1,5], which is not good for first round. The players, who choose small rounds probably knew about this game or they thought that everyone are as smart as they are. It is also possible, that some part of them were 0-level players, who just pick small number randomly. In any case, we can see two distinct clusters: first round (round dots) and second round (triangles). The explanation about equilibrium concept created this transition in choices, when choices from [50,100] decreasing, and choices from [1,5] increasing.

Interesting hypotheses, that need to be tested in details, can be formulated: **Higher number of choices from [50,100] in first round leads to higher number of choices from [1,5] in second round and vice verse.**

Another metric (Güth et al., 2002) is how much winning choice in second round is smaller then in first. Due to concept of multi-level reasoning, every player in this game trying to its best to win but cant do all steps to winning idea. So there are players, who just have 0-level reasoning, they choose random numbers. First-level players choose 33, which is best response for players of 0-level and so on. Based on result of first round and, in fact, explanation about the Nash equilibrium, players must know that it is better to choose much lower numbers. But graph shows that decrease is quite moderate. Only students shows good performance in this matter. And tech school shows increase in winning number in second round! (figure 4)

3.1.2 Levels of Reasoning Analysis

Another point about the process of learning in this game is how players decision are distributed over the space of strategies. We claim that there is distinct difference in changes between first and second round for different groups. To perform this analysis we apply the idea of k-level thinking.

To find differences we need to simplify this approach. First, we define **b-level** players players who choose numbers from the range [50,100]. It is beginner players, who do not understand rules (play randomly) or do not expect to win or want to loose intentionally (for reasons discussed above). The substantiation for such range is that numbers higher then 50 did not win in any game. Second level we call **m-level**, it is for range [18,50]. It is for players with middle levels of reasoning, usually first round winning number is in this range (and in part of second rounds also).

Third level is **h-level**, it is for range [5, 18]. It is for high level reasoning and finally **inf-level** ([1,5] range) is for "almost common knowledge" level of thinking.

Calculating the number of levels for each game we can estimate change (in percentage of number of players) in adopting different strategy levels.

There are some limitation of this approach:

- number of players changed with rounds, since not everyone participated (it was option, not obligation);
- limits of ranges are not defined by model or data. It can be future direction of research – how to define levels in best way.

Results are presented in table 2.

What conclusions we can draw from this data? There are no clear difference in changing, but at least we can summarise few points:

- Usually after first round and equilibrium concept explanation there is decrease in **b-level** and **m-level**;
- Symmetrically, there is increase in two other levels, but sometimes it is more distributed, sometimes it is (almost) all for **inf-level**;
- Last situation is more likely to happen in schools, were kids are less critical to new knowledge;
- Usually second round winning choice in the realm of **h-level**, so groups with biggest increase in this parameter are the ones with better understanding.

3.1.3 Size and Winning Choice

This game is indeed rich for investigation, let us formulate last (in this paper) finding about this game.

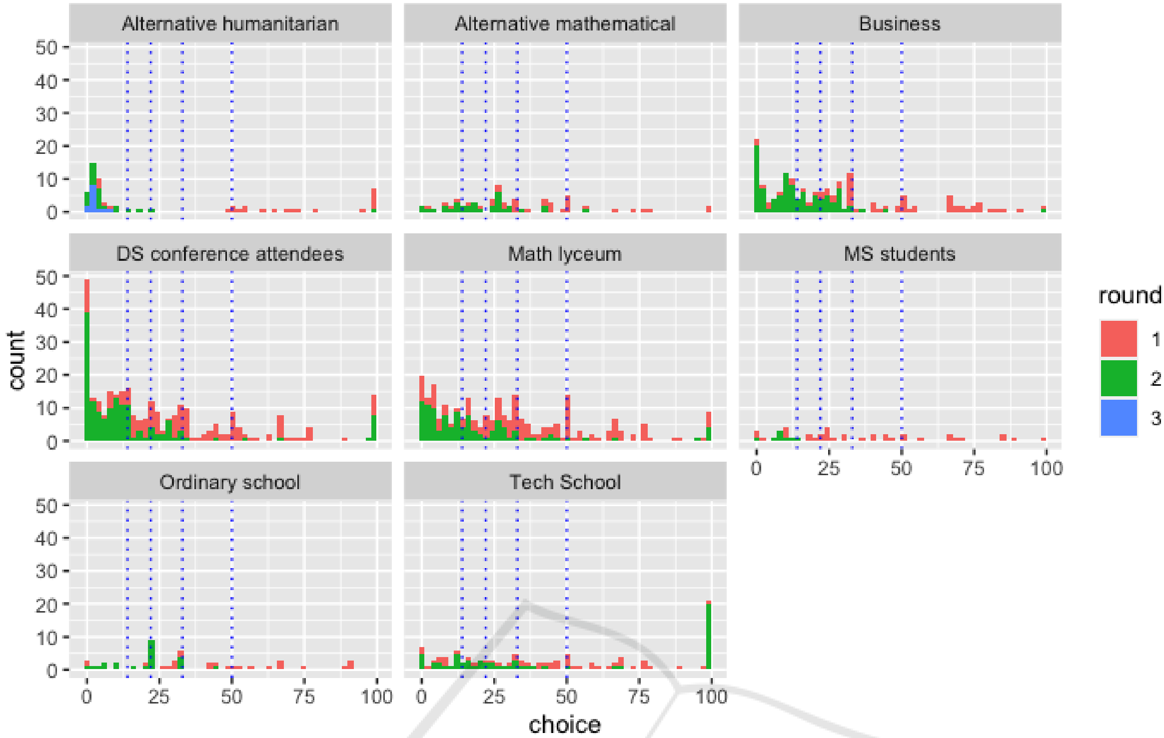

Figure 2: Histogram of choices for each round.

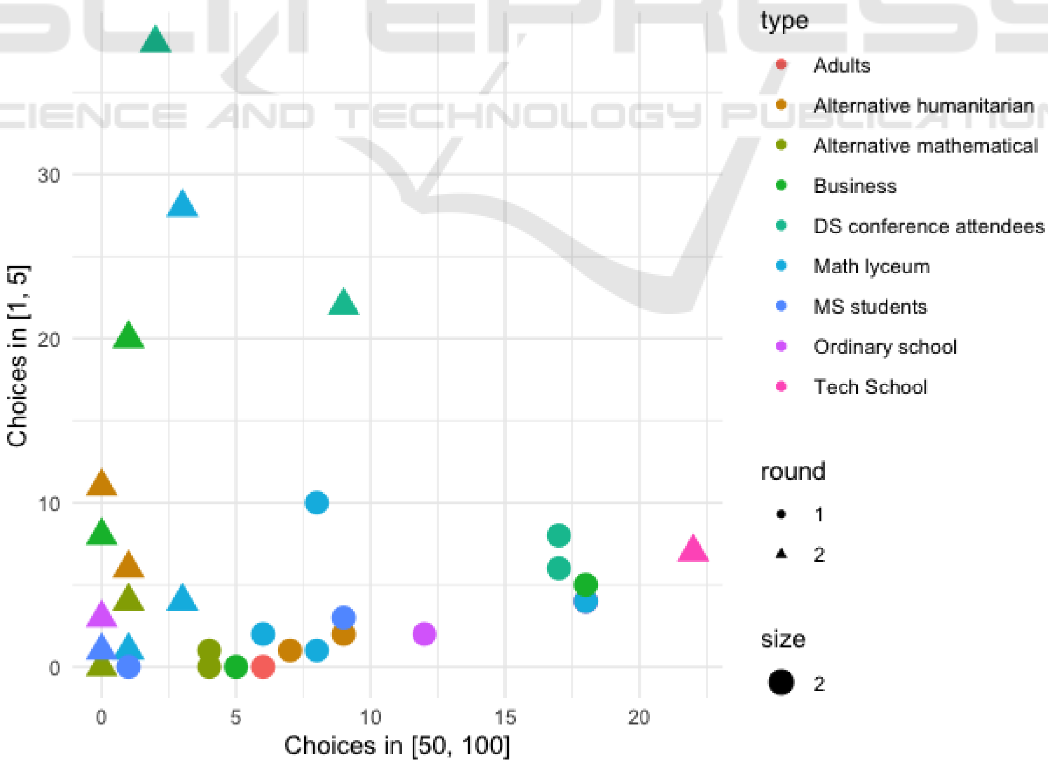

Figure 3: Comparing choices for different levels.

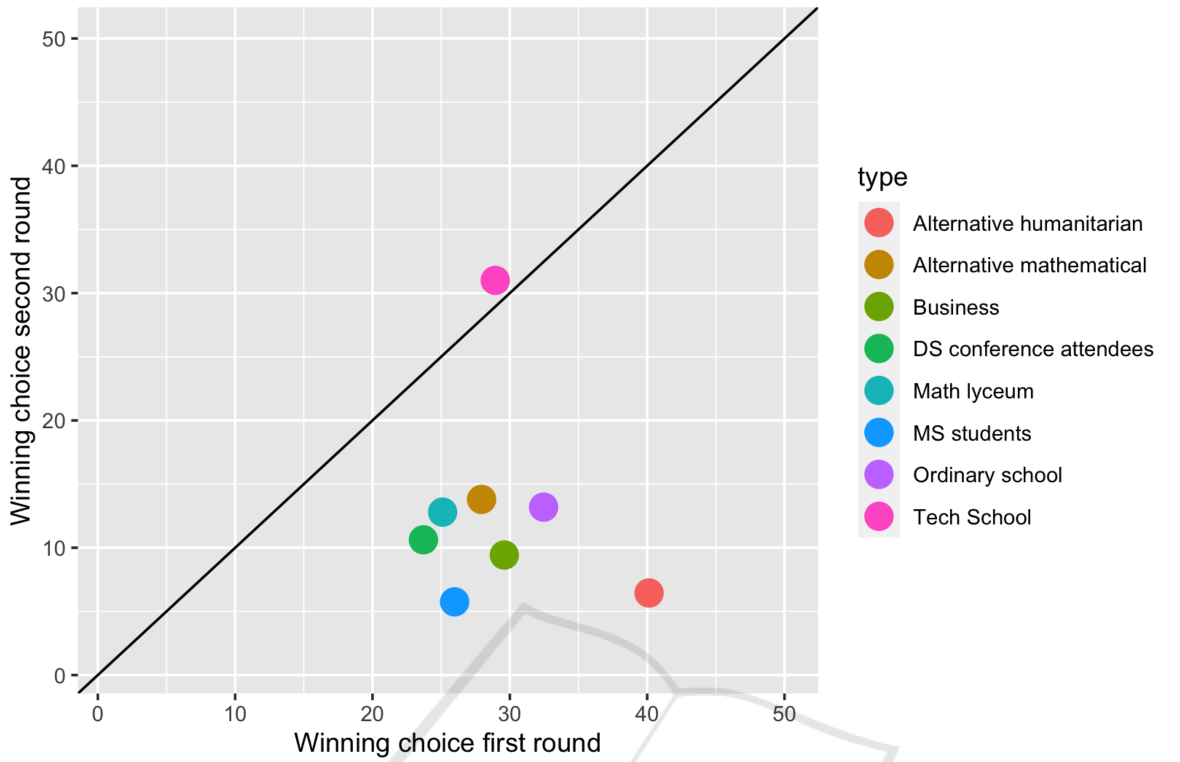

Figure 4: Change in winning number for rounds.

Table 2: Summary of change in strategy levels.

Type	b-difference	m-difference	h-difference	inf-difference
Alternative humanitarian	-72	-8	0	72
Alternative mathematical	-24	-6	30	-6
Alternative humanitarian	-52	0	17	43
Math lyceum	-9	-36	24	34
Math lyceum	-10	-24	28	7
Ordinary school	-49	12	20	4
DS conference attendees	-14	-32	14	27
MS students	-12	-50	50	12
Alternative mathematical	-17	-34	23	23
DS conference attendees	-17	-30	23	35
Business	-32	-17	21	28

Can we in some way establish connection between number of players and winning number (actually with strategies, players choose during the game)? To clarify our idea see at 5. It is scatter plot of two-dimensional variable, x-axis is for number of participants in the game and y-axis is for winning choice per round. Different color are for different types of group, where games was played.

Summarise findings about this plot:

- First and second rounds form two separate clusters. This is expected and inform us that players learned about the equilibrium concept between rounds and apply it to practice;
- There are two visible groups inside each round – undergraduates (schoolchildren, masters) and adults. Inside each group there is mild tendency that bigger group has bigger winning number.

This is yet too bold to formulate connection between size of the group and winning number, but probably the reason is that when size of the group is bigger, number of “irrational” players increases. It can be due to some stable percentage of such persons in any group or other reasons, but it is interesting connection to investigate.

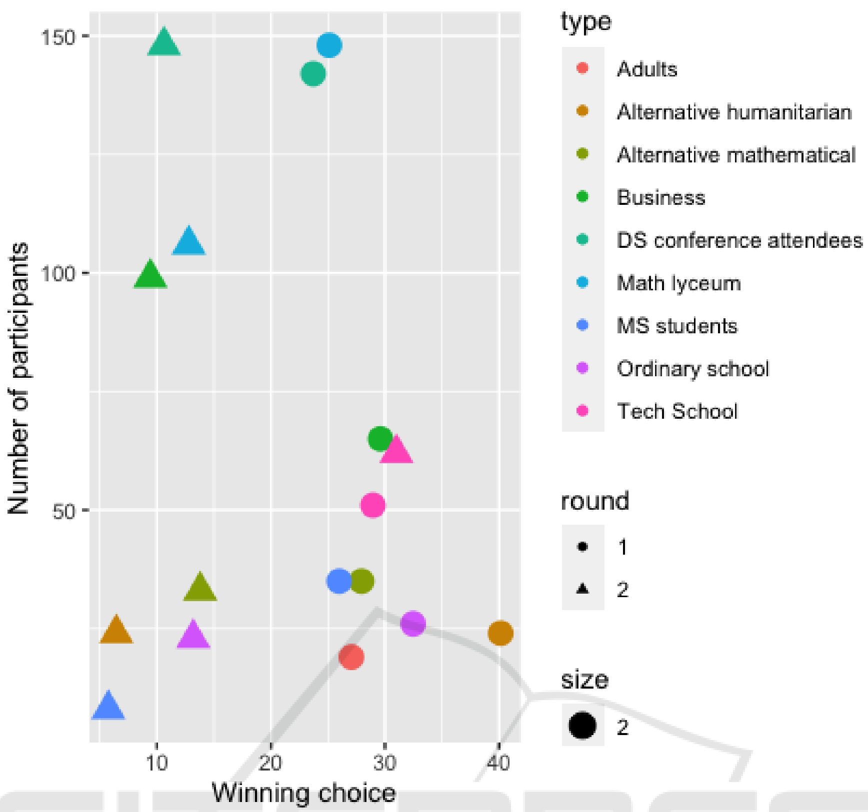

Figure 5: Change in winning number for rounds.

3.2 Second Game

In second game the key point is to understand that almost all strategies are dominated. The results are presented on figure 6 and we can see that average can be bigger or smaller then 50, and accordingly winning choice will be 1 or 100. It is worth to note, that popular nature of these experiments and freedom to participate make the data gathering not easy. For example many participants just didn't take any decision in second game. Results are summarised in the table 3.

We refine players decisions to see how many players made choices with rationalizability (Bernheim (Bernheim, 1984)), which are best response for some strategy profile of other players. In this game there are only two best responses possible (in pure strategies), literally 1 and 100.

This is remarkable result, players without prior communications choose to almost perfect mixed equilibrium: almost the same percentage choose 1 and 100. This is even more striking taking into account no prior knowledge about mixed strategies and mixed equilibrium, kids play it intuitively and without any communication. To illustrate the mixed Nash learning by groups, put dependency of percent of 1 choices and 100 choices on plot (figure 7).

3.3 Third Game

Third game is simpler then first two, it is coordination game where players should coordinate without a word. And, as predicted by Schelling (Schelling, 1980), they usually do. Date presented on figure 8 shows that 1 is natural coordination point, with one exception – Tech school (id = 1 here) decided that it would be funny to choose number 69 (it was made without single word). Probably, it is the age (11th grade) here to blame. Also we can note attempt to coordinate around 7, 50 and 100.

Interesting and paradoxical result, which is expected from general theory, that with fewer options coordination in fact is more difficult. Lets consider (figure 9), where players decision was to choose integer from [1,10], only 10 choices. Comparing to previous game with 100 possible choices, coordination was very tricky – two numbers got almost the same result.

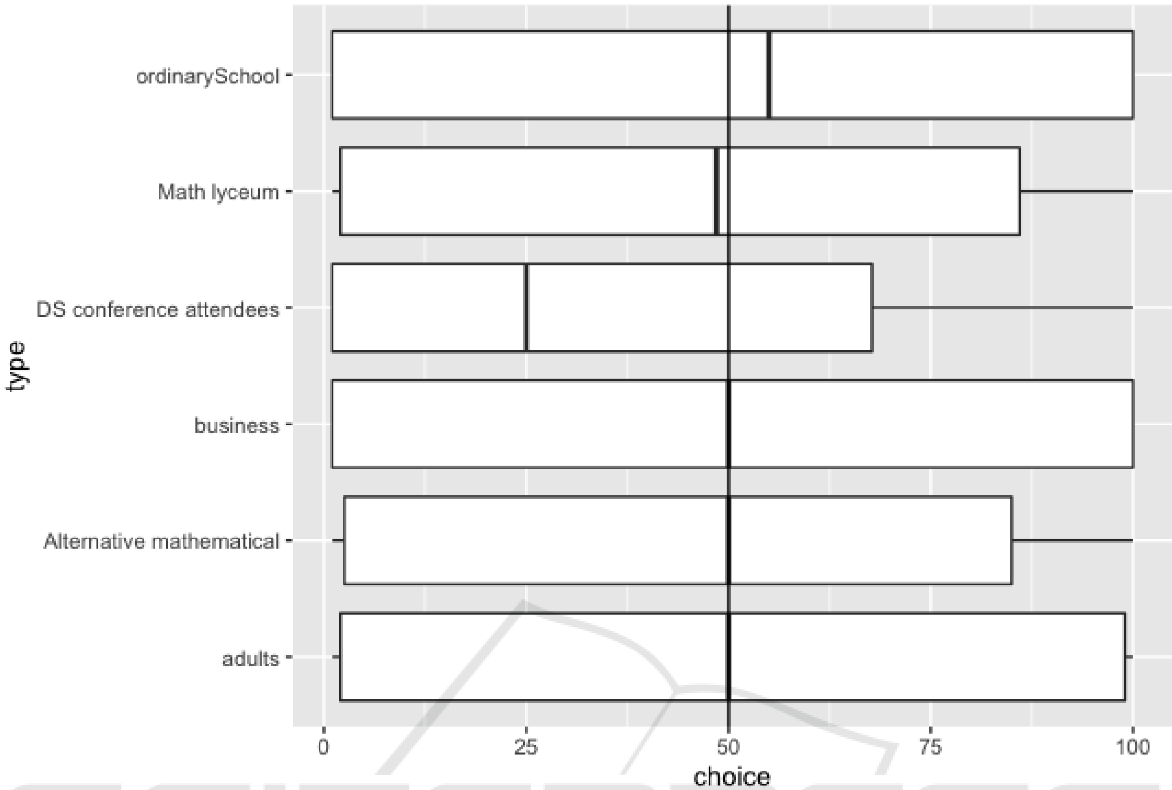

Figure 6: Statistics for choices.

Table 3: Second game. Rationalizable choices summary.

Type	Average	Choose 100	Choose 1	Count
Adults	46.5	24.3%	24.34%	115
Alternative mathematical	43.8	25.9%	27.9%	27
Business	50.6	29.3%	29.3%	99
DS conference attendees	37.4	15.8%	36.8%	114
Math lyceum	48.5	22.7%	24.7%	154
Ordinary school	51.2	30.4%	30.4%	23

3.4 Fourth Game

Here we just note, that the winning numbers were: 12, 2, 4, 20. Since no equilibrium here was theoretically found, we can only gather data at this stage and formulate hypothesis to found one.

All experimental data and R file for graphs can be accessed in open repository (Ignatenko, 2021).

4 CONCLUSIONS

In this paper we have presented approach to make experimental game theory work for learning in educational process and be a research tool at the same time. Our result show classical pattern in decision making – actually every group behave in almost the same way dealing with unknown game. Some tried to deviate for unusual actions (like choosing 100 or choosing 69), and this is interesting point of difference with more "laboratory" setup of existing research. The main findings of the paper are following:

1. To learn the rules you need to break them. Participants have chosen obviously not winning moves (> 66) partly because of new situation and trouble with understanding the rules. But high percent of such choices was present in second round also, when players knew exactly what is going on. This effect was especially notable in the cases of high school and adults and almost zero in case of special math schools and kids below 9th grade. We can formulate hypothesis that high school is the

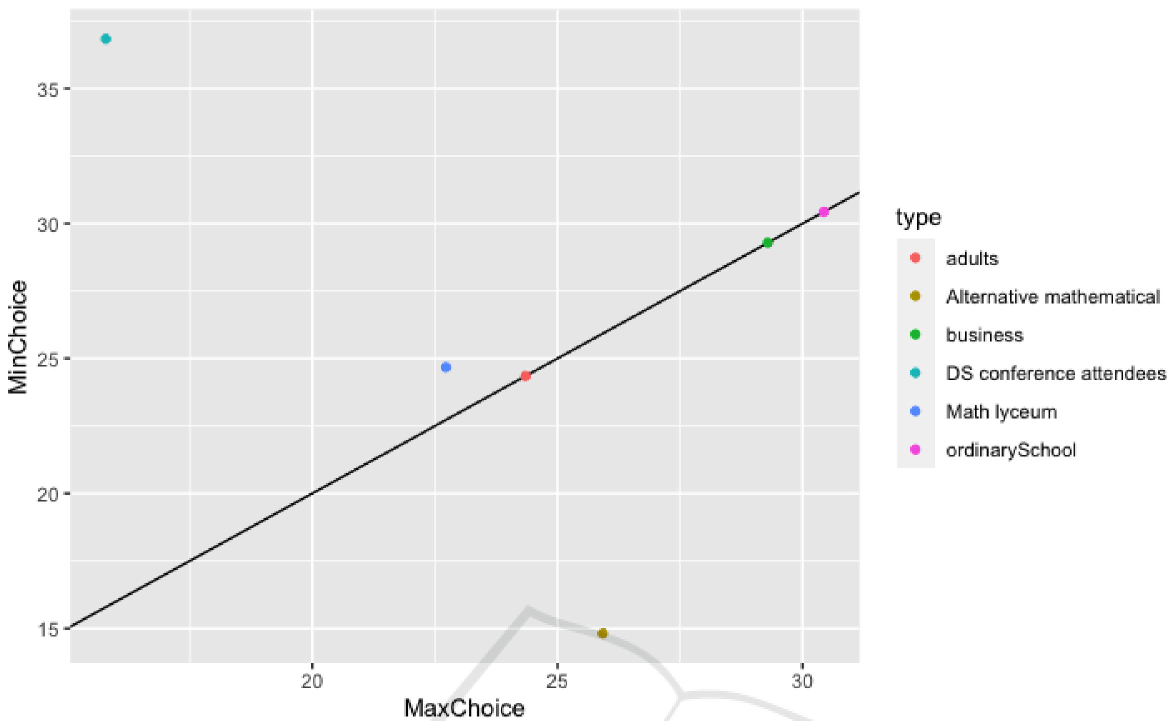

Figure 7: Difference in percent of rationalizable choices.

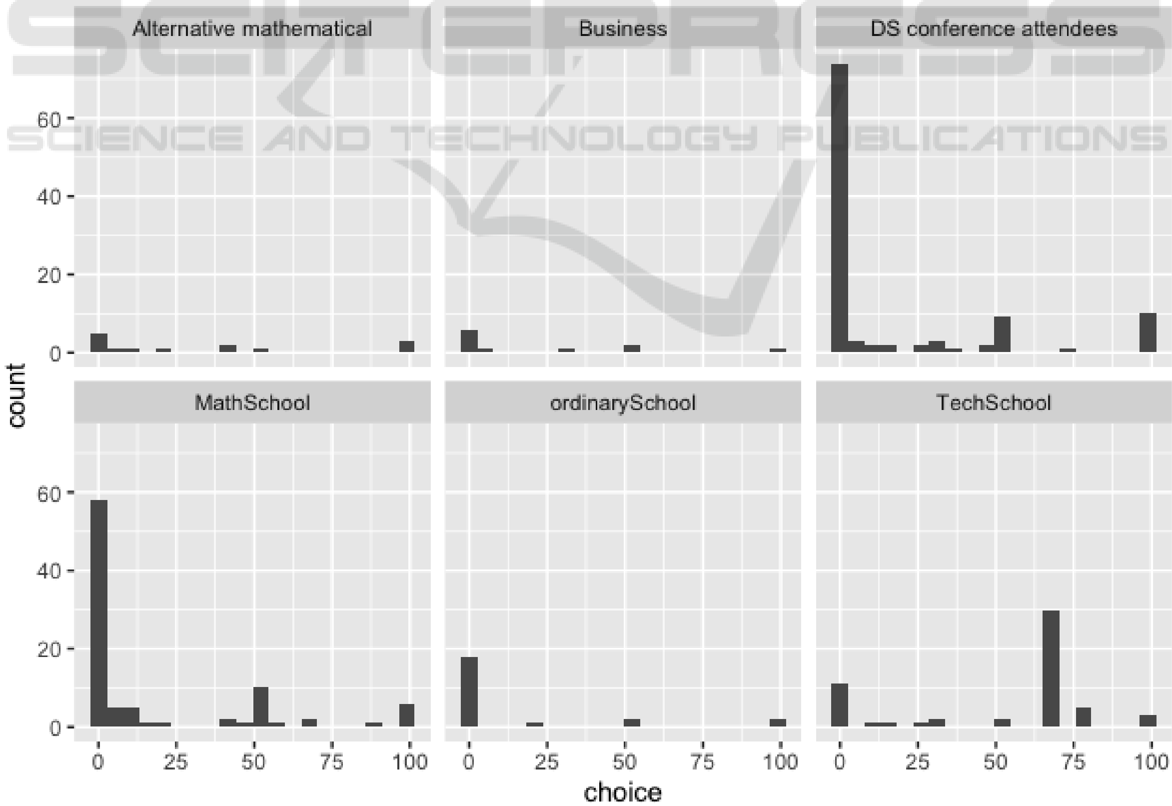

Figure 8: Histogram of choices.

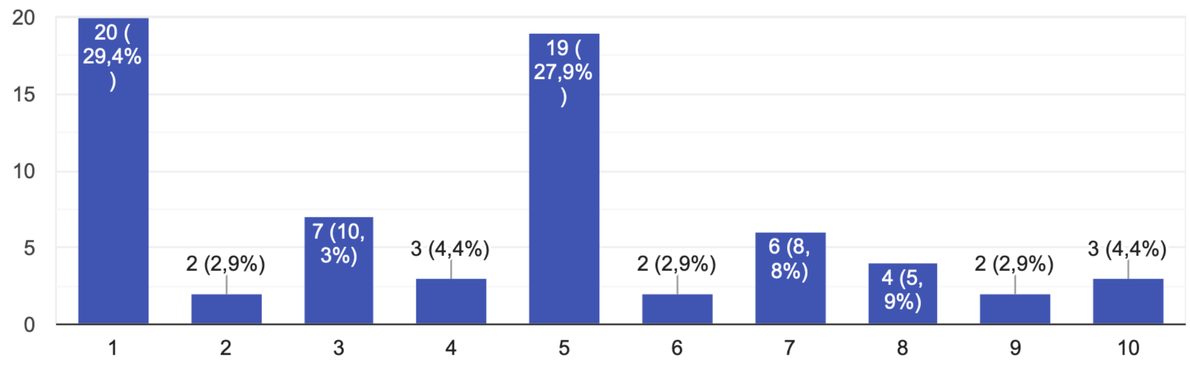

Figure 9: Histogram of choices for 1-10 game.

age of experimentation when children discover new things and do not afraid to do so.

2. If we considered winning number as decision of a group we can see that group learning fast and steady. Even if some outliers choose 100, mean still declines with every round. It seems that there is unspoken competition between players that leads to improvement in aggregated decision even if no prize is on stake. Actually, it is plausible scenario when all participants choose higher numbers. But this didn't happen in any experiment. The closest case – Tech school, when bunch of pupils (posible coordinating) switch to 100 still only managed to keep mean on the same level.

3. In second game the surprising result is that players use mixed strategies very well. It is known (from experiments of Colin Camerer) that chimpanzee can find mixed equilibrium faster and better than humans. It seems that concept of mixed strategies is very intuitive and natural. But still in quite unfamiliar game players made almost equal number of 1 and 100, so each player unconsciously randomized his own choice.

4. In third game players coordinates to 1, as expected, because of condition that from numbers with equal choices – lesser wins. Also we can note attempts of coordination around 7, 50 and 100. What is interesting is that in practice the condition was never applied – majority chooses 1 and that's it. If we decrease the numbers range to 1-10, other numbers has chance to win (5 or 7 for example). So this is unexpected result – increasing of number of choices leads to bigger uncertainty when players trying to find slightest hint what to do, and this is condition of "lesser wins". When players apply this condition to big area, they probably think – "1 is perfect choice, and other will think in that way also, this increase chances of winning".

The results have multiple applications:

- to provide kids with first hand experience about strategic interactions and explain their decisions;
- to demonstrate how game theory experiments can be used in educational process;
- to understand difference in decision making among groups;
- to compare results with classical experiments and replicate them in current Ukrainian education system.

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