ME2: A Scalable Modular Meta-heuristic for Multi-modal Multi-dimension Optimization

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Abstract: Map, Explore & Exploit (ME2) is a scalable meta-heuristic for problems in the field of multi-modal, multi-dimension optimization. It has a modular design with three phases, as reflected by its name. Its first phase (Map) generates a set of samples that is mostly uniformly distributed over the search space. The second phase (Explore) explores the neighbourhood of each sample point using an evolutionary strategy, to find a good - not necessarily optimal - set of neighbours. The third phase (Exploit) optimizes the results of the second phase. This final phase applies a simple gradient descent algorithm to find the local optima for each and all of the neighbourhoods, with the objective of finding a/the global optima of the whole space. The performance of ME2 is compared, on a fair basis, with the performance of benchmark optimization algorithms: Genetic Algorithms, Particle Swarm Optimization, Simulated Annealing and Covariance Matrix Adaptation Evolution Strategy. In most test cases it finds the global optima earlier than the other algorithms. It also scales-up, without loss of performance, to higher dimensions. Due to the distributed nature of ME2's second and third phase, it can be comprehensively parallelized. The search & optimization process during these two phases can be applied to each sample point independently of all the others. A multi-threaded version of ME2 was written and compared to its single-threaded version, resulting in a near-linear speed-up as a function of the number of cores employed.

1 INTRODUCTION

Optimization is a demanding field with a multitude of industrial applications. Optimization problems range from engineering design applications to logistics, and from scheduling applications to food production, just to mention a few (Bäck et al., 2013). For multi-modal multi-dimension optimization problems evolutionary methods are most relevant and effective instead of methods which are purely analytic.

In the following paragraphs we discuss the evolutionary methods that are currently the standard in the field of optimization. There after we present a new modular algorithm framework for optimization, consisting of three step: Map (2.1), Explore (2.2) and Exploit (2.3) (ME2). Following a detail description of this algorithm and each of its steps we present its comparison (3) results and analysis (5) with other standard algorithms using benchmark optimization problems. We conclude by presenting how ME2 can be scaled using multi-threaded implementation in a multi-core system (5.4).

Genetic Algorithm (GA). First conceived by John Holland as a means for studying adaptive behaviour (Holland, 1992), Genetic Algorithms have come to have a wide set of applications. If a problem can have a genetic representation via binary, integer or floating-point parameters, different variation operators can be applied to a population of solutions which, together with selection pressure, evolves the population towards greater average fitness, over a number of generations. Variation operators in GA can range from single/multi point mutation to one/N-point-crossover. For each evolutionary generation of GA, parent selection can also vary from fitness proportionate to direct ranking based on fitness. Child selection also has its diverse methods.

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Particle Swarm Optimization (PSO). Introduced by Kennedy and Eberhart (Kennedy and Eberhart, 1995) in 1995, Particle Swarm Optimization (PSO) is another population based stochastic search and optimization procedure (Kiranyaz et al., 2015). The swarm intelligence paradigm emerged from studies of the collective behaviour and social characteristics of organized, decentralized and complex systems known as swarms. Belonging to this paradigm, PSO was inspired by the behaviour of bird flocks, where the goal is to converge to a global optimum of some multidimensional and possibly non-linear function or system.

Simulated Annealing (SA). Inspired by annealing in metallurgy, Simulated Annealing was first introduced by Metropolis (Metropolis et al., 1953). This method is a Monte Carlo-based technique that generates a sequence of states of the solid. It is the key for achieving the optimal ground state, which is the basis of the annealing as an optimization method (Kiranyaz et al., 2015).

SA along with other EA are sometimes called meta-heuristics, which make few or no assumptions about the problem being optimized and can thus search for a global optimum over a large set of candidate solutions. But as their major difference, SA is not population based and is also not based on "survival of the fittest" philosophy. Suboptimal solutions in SA are sometimes tolerated for the sake of avoiding a local optimum.

Covariance Matrix Adaptation Evolution Strategy (CMA-ES). CMA-ES is currently considered as the 'state of the art' in evolutionary computation which has been adopted in many research labs and industrial environments around the world (Hansen, 2007). Its approach to the evolutionary search space is by estimating a covariance matrix on convex-quadratic function, similar to the inverse Hessian. It is considered to be highly competitive for local optimization (Hansen and Ostermeier, 2001) and also for global optimization (Hansen and Ostermeier, 2001; Hansen, 2009).

2 MAP EXPLORE EXPLOIT (ME2) ALGORITHM

Similar to GA, PSO, SA and CMA-ES, Map Explore & Exploit (ME2) is a meta-heuristic for multi-modal multi-dimension optimization problems. True to its name, the ME2 algorithm has three steps in its process. The first step is where the landscape of the fitness function is randomly **mapped** using a uniform distribution (figure 1(a)). In its second step of **exploration**, an evolutionary strategy is applied to search the local neighbourhood of each of the sample points, to choose their local suboptimal fitness (figure 1(b)). Finally the local suboptimal is **exploited** by applying steepest gradient descent to reach each sample's local optima (figure 1(c)). These steps of ME2 are modular, as each of them independently, can use other suitable algorithms with possibly better performance, as long as the alternative algorithm has the proper aim for the specific search phase (e.g., exploration or exploitation).

Algorithm 1: Map Explore Exploit Algorithm.

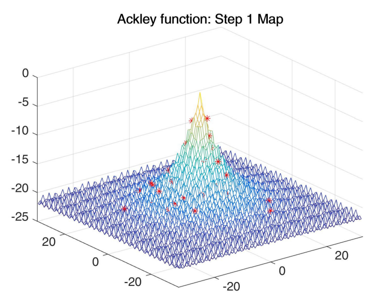
```
 1: procedure ME2($dimension$)
 2:     $noOfSamples \leftarrow 25$
 3:     $noOfNeighbour \leftarrow 25$
 4:     $factor \leftarrow 10$
 5:     $overfactor \leftarrow factor * noOfSamples$
 6:     $samples \leftarrow MAP(dimension, overfactor)$
 7:     $lower \leftarrow$ lower limit of fitness function
 8:     $upper \leftarrow$ upper limit of fitness function
 9:     for $s \leftarrow 0$ to $noOfSamples$ do
10:         $EXPLORE(samples[s])$
11:     end for
12:     for $s \leftarrow 0$ to $noOfSamples$ do
13:         $EXPLOIT(samples[s], s)$
14:     end for
        return Point from $samples$ with the best fitness
15: end procedure
```

2.1 Map

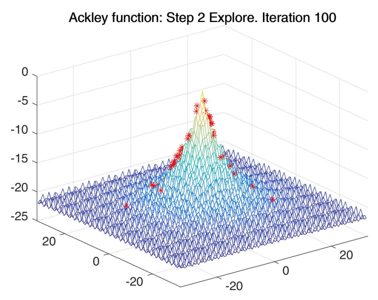
The first step of ME2 begins by generating a set of points spanning the whole search space. Based on sample size, set at the start of the algorithm, an *overFactor* is defined. Hence, uniformly random distributed points are generated between the upper and lower limits of each dimension. The total number of points are equal to the *overFactor* which equals $factor * noOfSamples$, where $factor = 10$. A sample selection is made from this list of over-factors by iterating over 10 points at a time and choosing the one with the best fitness among the ten. The value 10 was selected randomly, but later kept as such as it has an impact in total number of evaluations. In this way the final list of samples is selected. This is the size of the population. Algorithm 2 describes this process in detail.

2.2 Explore

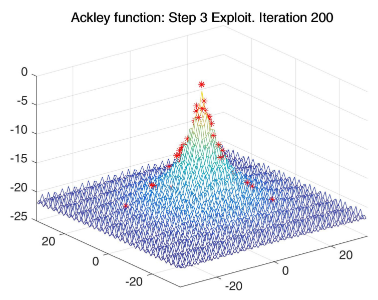
The objective of the exploration phase of ME2 (algorithm 3) is to search the local neighbourhood of each sample point generated from the mapping phase



(a) Step 1 Map



(b) Step 2 Explore. Iteration 100



(c) Step 3 Exploit. Iteration 200

Figure 1: ME2 execution steps. The above figure shows the three phases of ME2 and the state of the sample points at different iterations of the algorithm. The plotted function is Ackley (Ackley, 1987). The plot shows the inverse of the Ackley function so the sample points are visible. Figure 1(a) shows the initial step of **mapping**. Figure 1(b) shows the steps of neighbourhood **exploration** at iteration 100. Figure 1(c) shows the final phase after completion of **exploitation** where points can be observed at the global optimum.

Algorithm 2: Step 1: Map.

```
1:  procedure MAP($dimension, overfactor$)
2:      for $f \leftarrow 0$ to $overfactor$ do
3:          for $d \leftarrow 0$ to $dimension$ do
4:              $completeMap[f][d] \leftarrow$ uniform random between $upper$ and $lower$
5:          end for
6:      end for
7:      $s \leftarrow 0$
8:      for $f \leftarrow 0$ to $overfactor$ do
9:          $samples[s] \leftarrow$ best fitness point between $completeMap[f]$ to $completeMap[f + factor]$
10:         $s \leftarrow s+1$
11:         $f \leftarrow f + factor$
12:     end for
        return $samples$
13: end procedure
```

(sec. 2.1). The algorithm iterates through each sample point, searching its neighbourhood and replacing the initial point with its best possible neighbour. Due to ME2's modular design, any algorithm that has the same objective of local neighbourhood search can be applied during this phase. In our case we used the *DR2* evolutionary strategy.

DR2. In ME2, we applied an evolutionary strategy which uses a derandomized scheme of mutative step-size control (Ostermeier et al., 1994). This adaptation concept *'uses information accumulated from the preceding generations, with an exponential fading of old information, instead of using information from the current generation only'*. Also termed as DR2 evolution strategy, this algorithm creates offspring by mutation, which is parametrized by a global step size δ and local step sizes $\boldsymbol{\delta}_{scal} \in \mathbb{R}^n$ (Bäck et al., 2013).

$$\mathbf{x}' = \mathbf{x} + \delta \cdot \boldsymbol{\delta}_{scal} \otimes \mathbf{z} \text{ where } \mathbf{z} = \mathbf{N(0,I)} \qquad (1)$$

Algorithm 3: Step 2: Explore.

```
1:  procedure EXPLORE(sample)
2:      $d \leftarrow dimension$
3:      $\beta \leftarrow \sqrt{1/d}$
4:      $c \leftarrow \sqrt{1/d}$
5:      $\boldsymbol{\zeta} \leftarrow \{0,...0\}$
6:      $\delta \leftarrow 1$
7:      $\boldsymbol{\delta} \leftarrow \{1,...1\}^T$
8:      $noOfGeneration \leftarrow 0$
9:      while $noOfGeneration < 150$ do
10:         neighFitnsArr[$noOfNeighbors$] $\leftarrow null$
11:         for $n \leftarrow 0$ to $noOfNeighbors - 1$ do
12:             for $i \leftarrow 0$ to $d-1$ do
13:                 $\mathbf{z}_i \leftarrow N(\mathbf{0}, \mathbf{I})$
14:                 $newNeigh \leftarrow sample[i] + \delta \cdot \boldsymbol{\delta} \otimes \mathbf{z}_i$
15:                 if $newNeigh$ within $upper$ and $lower$ then
16:                     $localNeighs[n][i] \leftarrow newNeigh$
17:                 end if
18:             end for
19:             bestLocalNeigh $\leftarrow$ best fitness $localNeighs[n]$
20:             neighFitnsArr[$n$] $\leftarrow fitness$(bestLocalNeigh)
21:         end for
22:         newBestNeigh $\leftarrow$ fitness proportionate selection on neighFitnsArr
23:         if $fitness$(bestOfAllNeigh) $>$ $fitness$(newBestNeigh) then
24:             bestOfAllNeigh $\leftarrow$ newBestNeigh
25:         end if
26:         $\boldsymbol{\zeta} \leftarrow (1-c) \cdot \boldsymbol{\zeta} + c/\delta \cdot$ (bestOfAllNeigh $-$ sample) $\otimes \boldsymbol{\delta}^{-1}$
27:         $\delta \leftarrow \delta \cdot (exp(\frac{\|\boldsymbol{\zeta}\|}{\sqrt{d} \cdot \sqrt{\frac{c}{2-c}}} - 1 + \frac{1}{5d}))^{\beta}$
28:         $\boldsymbol{\delta} \leftarrow \boldsymbol{\delta} \otimes (\frac{|\boldsymbol{\zeta}_d|}{\sqrt{\frac{c}{2c}}} + \frac{7}{20})^{1/d}$
29:         $noOfGeneration \leftarrow noOfGeneration + 1$
30:     end while
        return sample $\leftarrow$ bestOfAllNeigh
31: end procedure
```

DR2 takes into consideration the most successful mutation of the current and all previous generations. The vector $\boldsymbol{\zeta} \in \mathbb{R}^n$ is used for this accumulation, using a factor $c \in (0, 1]$ to control the weight of previous generations relative to the current one.

$$\boldsymbol{\zeta}' = (1-c)\cdot\boldsymbol{\zeta} + c\cdot\mathbf{z}_{sel} \tag{2}$$

The step sizes δ and $\boldsymbol{\delta}_{scal}$ are adapted based on the updated mutation path $\boldsymbol{\zeta}'$.

$$\delta' = \delta\cdot(exp(\frac{\|\boldsymbol{\zeta}'\|}{\sqrt{n}\sqrt{\frac{c}{2-c}}} - 1 + \frac{1}{5n}))^{\beta} \tag{3}$$

$$\boldsymbol{\delta}'_{scal_i} = \boldsymbol{\delta}_{scal_i}\cdot(\frac{|\zeta'_i|}{\sqrt{\frac{c}{2-c}}} + \frac{7}{20})^{\beta_{scal}}\ \forall i \in 1,...,n \tag{4}$$

The parameter c and the exponents β and β_{scal} are expressed as follows,

$$\beta = \sqrt{1/n};\ \beta_{scal} = 1/n;\ c = \sqrt{1/n} \tag{5}$$

The algorithm for explore (algorithm 3) starts by initializing the parameters $c, \beta, \boldsymbol{\zeta}$ and δ. It then iterates over a *noOfGeneration* which in our experiments is set to 150. In each generation, the algorithm initializes a *neighFitnsArr* which stores the fitness values of all the neighbours. This array is later used in fitness proportionate selection of the best neighbour. The *noOfNeighbours* parameter, initially set to 25 in our experiments, is the number of neighbours generated in each generation. Using equation 1, new neighbours are generated and evaluated while the fittest neighbour is stored in the *neighFitnsArr* array. After exiting the inner loop of *noOfNeighbours*, fitness proportionate selection is applied to the set of best local neighbours stored in *neighFitnsArr*. The selected neighbour is compared to the current best neighbour and replaces it if and only if it has greater fitness. Hence, parameters $\boldsymbol{\zeta}, \delta$ and $\boldsymbol{\delta}_{scal}$ are updated using equations 2, 3 and 4. This completes the execution of one generation. This process repeats for 150 generations, at the end of which the fittest neighbour among all the generations takes the place of the original sample point. This point is then used as the starting point for the third stage of optimization: Exploit.

2.3 Exploit

In this final phase, a simple gradient descent algorithm is applied to each of the points, to guide it to its local optimum. The number of generations in this phase is set to 50. For each generation of each sample, the algorithm finds the gradient with the steepest descent, by making a small step forward in one direction and comparing the new point's fitness to the previous one. During those small steps, the algorithm also makes sure the *upper* and *lower* limits of the fitness function are not breached. At the conclusion of this phase, it is hoped that the algorithm would have found an optimal point, within a small local neighbourhood of every starting point, inherited from the exploration phase.

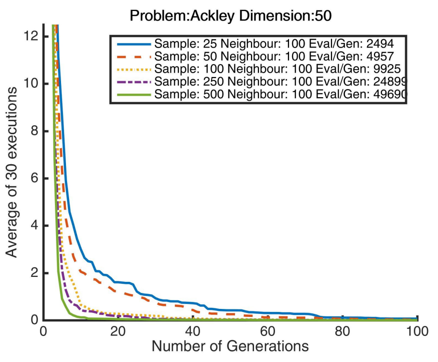
Algorithm 4: Step 3: Exploit.

```
procedure EXPLOIT(sample, $sIndx$)
    $noOfGeneration \leftarrow 0$
    sigma ← 1
    gradien ← $null$
    $delta \leftarrow 0.001$
    while $noOfGeneration < 50$ do
        tSample ← sample
        tSample1 ← sample
        for $i \leftarrow 0$ to $d-1$ do
            $delta \leftarrow delta *$ sigma[$sIndx$][$i$]
            tSample[$i$] ← tSample[$i$] + $delta$
            tSample1[$i$] ← tSample1[$i$] + $delta$
            gradien[$i$] ← $\frac{fitness(\mathbf{tSample1}) - fitness(\mathbf{tSample})}{2\cdot delta}$
            sigma[$sIndx$][$i$] ← 0.5
        end for
        for $i \leftarrow 0$ to $d-1$ do
            tSample[$i$] ← tSample[$i$] + sigma[$sIndx$][$i$] * gradien[$i$]
            if (tSample[$i$] > $upper$) or (tSample[$i$] < $lower$) then
                tSample[$i$] ← sample[$i$]
                sigma[$sIndx$][$i$] ← sigma[$sIndx$][$i$]/2
            end if
        end for
        if $fitness$(tSample) >= $fitness$(sample) then
            tSample ← sample
            for $i \leftarrow 0$ to $d-1$ do
                sigma[$sIndx$][$i$] ← sigma[$sIndx$][$i$]/2
            end for
            $continue$
        end if
        sample ← tSample
        $noOfGeneration \leftarrow noOfGeneration + 1$
    end while
     return sample
end procedure
```

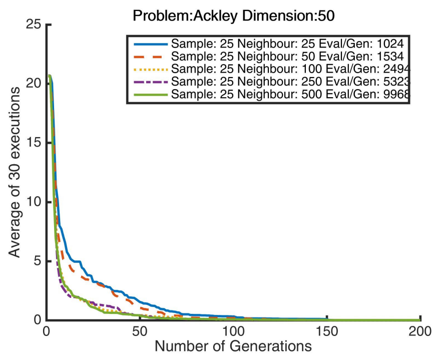
During exploitation, we chose to apply a simple method: gradient descent (Kiranyaz et al., 2015). As previously mentioned, ME2 is modular. So any algorithm capable of finding local optima can potentially replace the gradient descent algorithm we used for this paper. Whether an alternative algorithm will perform better or worse is a matter for empirical testing.

3 COMPARISON

The empirical testing of ME2 was performed in comparison to four other optimization algorithms. Genetic Algorithms (GA), Particle Swarm Optimization (PSO), Simulated Annealing (SA) and Covariance Matrix Adaptation Evolution Strategy (CMA-ES) are benchmark algorithms in the field of optimization.



(a) Varying Sample Parameter



(b) Varying Neighbourhood Parameter

Figure 2: Varying Parameters: In these figures the x-axis represents the number of generations, while the y-axis represents the average fitness of 30 runs. As the sample size is increased in 2(a) number of evaluations keep increasing at a high rate, while improving performance. Figure 2(b) exhibits a situation where the size of the neighbourhood is increased. This improves the performance of the algorithm as well as the number of evaluations.

We use a standard implementation library named ECJ (Evolutionary Computation in Java) (Luke, 1998) to generate the results for the four comparison algorithms (Luke, 2017). ECJ has a complete implementation of GA, PSO and SA along with most of the benchmark test functions. For GA and PSO the algorithm implementation is in its original form. The parameters for these algorithms are also optimized for the test functions in this library. A list of all the parameters for all the algorithms used in our tests is presented in table 2. Source code implementation of ME2 and ECJ extensions done for this work can be found at (Islam, 2019).

The results for CMA-ES were gathered from the implementation by its inventor Nikolaus Hansen, available at: Inria (l'institut national de recherche dédié aux sciences du numérique) (Hansen, 2007). The default parameter for initial mean was updated to a random point within the limits presented in table 1.

4 EXPERIMENTS

The first set of experiments were performed to compare different customizations of ME2, varying two of its most important parameters, *sampleSize* (used for mapping sec. 2.1) and *noOfNeighbours* (used for exploring sec. 2.2). After finding the impact of varying these two parameters, the next step is to compare ME2 with the four other optimization algorithms. All executions of ME2 in our experiments are performed 30 times, with the graphs showing the evolution of average fitness over time.

5 RESULTS AND ANALYSIS

5.1 Parameter Optimization

ME2's two most important parameters are *sampleSize* and *noOfNeighbours*, both of which have a significant impact on its performance. Figure 2 contains the graph comparing these two parameters. By observing figure 2(a) we notice, that increasing the number of samples, increases the number of evaluations significantly. This has a direct impact on the performance of the algorithm, as it results in a reduction in the number of generations necessary for good optimization results. For the experiments in figure 2(a), the other parameters (such as *noOfNeighbours*) remain fixed.

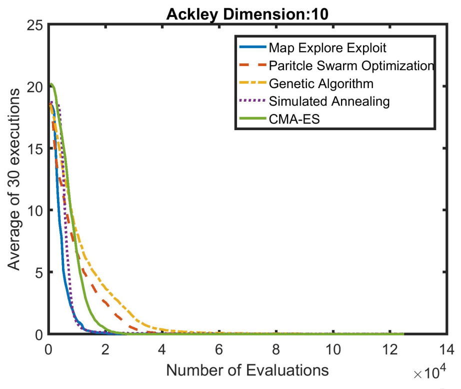
The second parameter that was varied is the *noOfNeighbours* in figure 2(b). In this case, it was observed that as the number of neighbours increase, the performance of the algorithm improves. This empirical relationship was expected, as increasing the number of neighbours, increases the algorithm's ability to perform denser exploration of a local neighbourhood. Increasing the number of neighbours requires an increase in the number of evaluations, impacting computational cost.

5.2 Results of Various Dimensions

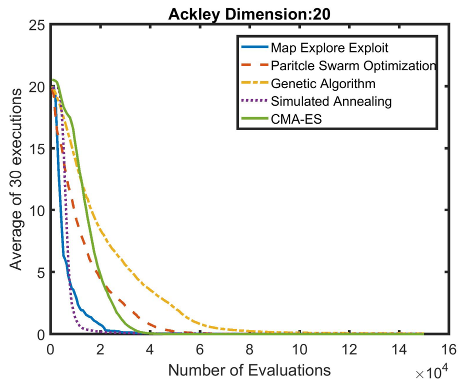
Here we compare the performance of ME2 with other optimization algorithms in the field (GA, PSO, SA and CMA-ES). From the list of benchmark optimization problems, a selection was made of problems that present multi-modal fitness landscapes and can be

Table 1: Optimization Test Functions.

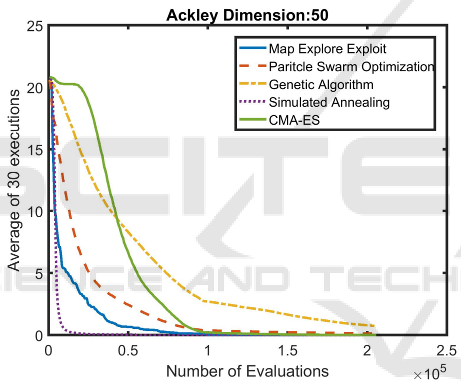
Name/Ref	Function	Global Minima	Limits
(Ackley, 1987)	$-20exp(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{d}x_i^2}) - exp(\frac{1}{d}\sum_{i=1}^{d}cos(2\pi x_i)) + 20 + e$	$f(0,\cdots,0) = 0$	± 32.768
(Rosenbrock, 1960)	$\sum_{i=1}^{d-1}(100(x_i^2 - x_{i+1})^2 + (1 - x_i)^2)$	$f(1,\cdots,1) = 0$	± 2.048
(Schwefel, 1981)	$\sum_{i=1}^{d}(-x_i sin(\sqrt{\lvert x_i \rvert})) + 418.982887 \cdot d$	$f(420.968746,\cdots 420.968746) = 0$	± 512.03
(Rastrigin, 1974)	$10d + \sum_{i=1}^{d}(x_i^2 - 10cos(2\pi x_i))$	$f(0,\cdots,0) = 0$	± 5.12



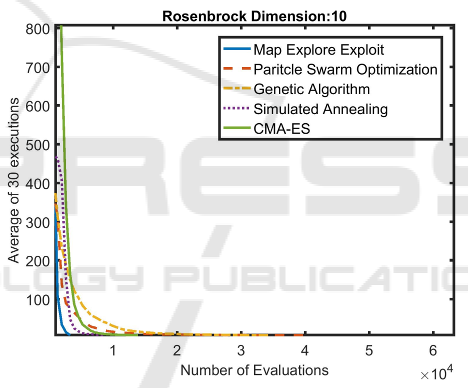
(a) Ackley 10 dimension



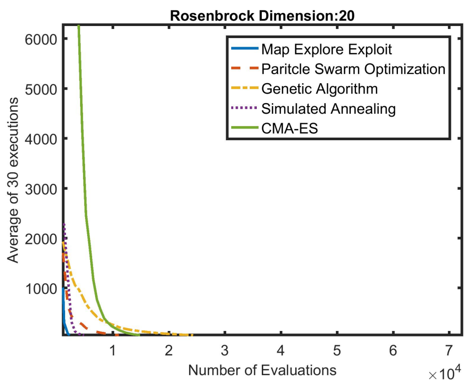
(b) Ackley 20 dimension



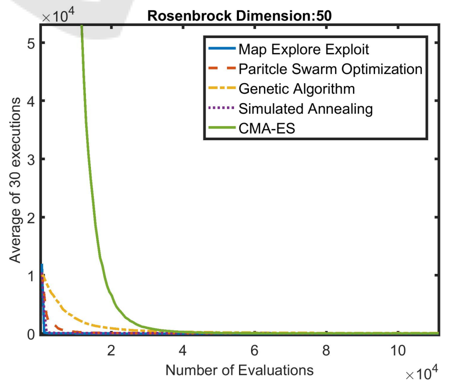
(c) Ackley 50 dimension



(d) Rosenbrock 10 dimension

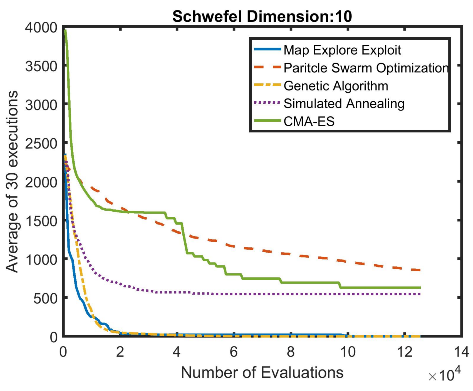


(e) Rosenbrock 20 dimension

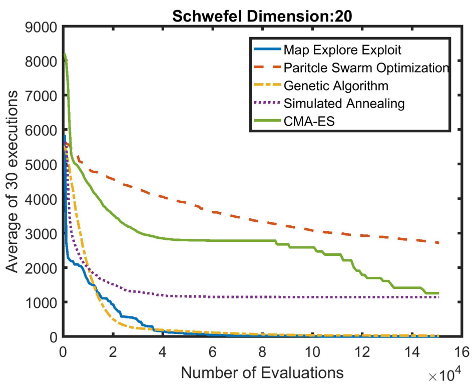


(f) Rosenbrock 50 dimension

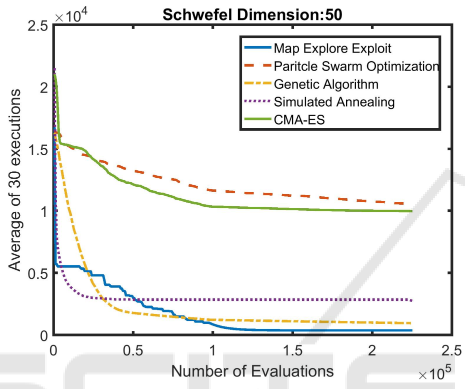
Figure 3: Average fitness comparison between Map Explore Exploit (ME2), Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Simulated Annealing (SA) and Covariance Matrix Adaptation Evolution Strategy (CMA-ES) for test problems: Ackley (Ackley, 1987) and Rosenbrock (Rosenbrock, 1960). Test problem input vector dimensions vary between 10 to 50. The x-axis represents the number of evaluations, while the y-axis represents average fitness over 30 runs. It can be observed that in almost all cases ME2 outperforms GA, PSO, SA and CMA-ES.



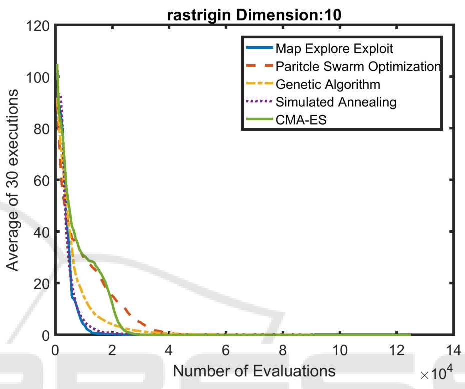
(a) Schwefel 10 dimension



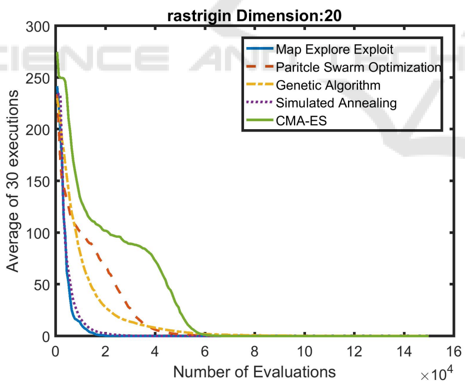
(b) Schwefel 20 dimension



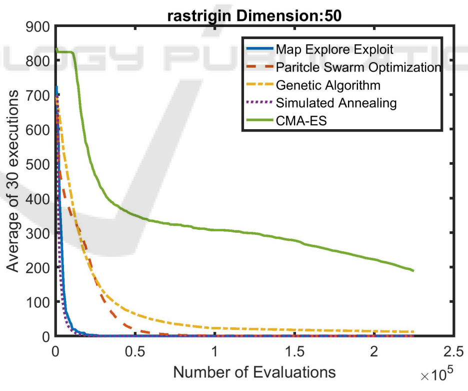
(c) Schwefel 50 dimension



(d) Rastrigin 10 dimension



(e) Rastrigin 20 dimension



(f) Rastrigin 50 dimension

Figure 4: Average fitness comparison between Map Explore Exploit (ME2), Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Simulated Annealing (SA) and Covariance Matrix Adaptation Evolution Strategy (CMA-ES) for test problems: Schwefel (Schwefel, 1981) and Rastrigin (Rastrigin, 1974). Test problem input vector dimensions vary between 10 to 50. The x-axis represents the number of evaluations, while the y-axis represents average fitness over 30 runs. It can be observed that in almost all cases ME2 outperforms GA, PSO, SA and CMA-ES.

scaled up to any number of dimensions. Table 1 contains details of these test functions. Figure 3 contains comparison results for dimensions 10 to 50 of the widely used optimization test functions: Ackley (Ackley, 1987) and Rosenbrock (Rosenbrock, 1960). Figure 4 presents comparison results for the test problems Schwefel (Schwefel, 1981) and Rastrigin (Rastrigin, 1974) for dimensions 10 to 50.

To have a fair comparison between ME2, GA, PSO and CMA-ES, we equalized their number of evaluations per generation. Table 3 contains the number of evaluations used per generation for each of

Table 2: Algorithm Parameters.

Algorithm	Parameters	
Map Explore Exploit	Sample Size	25
	Neighbour Size	25
	No. of Generation: Explore	150
	No. of Generation: Exploit	50
Genetic Algorithm	Tournament Size	2
	Mutation Probability	0.01
	Mutation Type	Gaussian
	Mutation Standard Deviation	1.0
Particle Swarm Optimization	Velocity Coefficient	0.7
	Personal Coefficient	0.4
	Informant Coefficient	0.4
	Global Coefficient	0.0
	Neighbour Size	10
Simulated Annealing	Generations	100000
	Population	1
	Temperature	3000
	Mutation Probability	0.017
	Mutation Standard Dev.	0.89
	Mutation Distribution Index	21
CMA-ES	No. of Generation	200
	Initial Mean	Random
	Standard Deviation	2

Table 3: Number of Evaluations per Generation.

Dimension	Population or No. of Evaluations per Generation		No. of Evaluations
	ME2	GA, PSO, CMA-ES	SA
10	625	625	500
20	750	750	500
50	1029	1050	1000

these algorithms. For ME2, both the sample size and number of neighbours was set to 25 (Table 2). For GA, PSO and CMA-ES, the number of evaluations per generation is equal to the population size. For both of these, each individual is evaluated once per generation. After executing ME2 first, the population size of GA, PSO and of CMA-ES was set to be equal to ME2's number of evaluations per generation.

Having a fair comparison with SA is difficult as SA uses only one individual per generation, and its number of evaluations per generation is also one. To present a fair comparison (figure 3 and 4) the x-axis represents the number of evaluations, rather than the number of generations, from the start of evolution. The y-axis represents the average fitness of 30 runs.

5.3 Analysis

A close analysis of the results in figure 3 and 4, reveals that ME2 returns better performance than GA, PSO and CMA-ES in almost all cases. Also considering Ackley-20d (3(b)), Ackley-50d (3(c)) and Schwefel-50d (4(c)) as exceptions, the performance of ME2 is better or the same as that of SA. Therefore, one can fairly conclude that ME2 is the best overall optimizer, over the specific set of test functions and dimensions, than the set of comparison algorithms.

Algorithm 5: Multi Threaded ME2 Algorithm.

```
1:  procedure ME2_MT($dimension$)
2:      $noOfSamples \leftarrow 25$
3:      $noOfNeighbour \leftarrow 25$
4:      $factor \leftarrow 10$
5:      $overfactor \leftarrow factor * noOfSamples$
6:      $samples \leftarrow MAP(dimension, overfactor)$
7:      $lower \leftarrow$ lower limit of fitness function
8:      $upper \leftarrow$ upper limit of fitness function
9:      for $s \leftarrow 0$ to $noOfSamples$ do
10:         Start new thread, $t$ for $samples[s]$
11:         Join thread $t$ with the main process
12:         execute $EXPLORE(samples[s])$ in thread $t$
13:         execute $EXPLOIT(samples[s], s)$ in thread $t$
14:     end for
        return Point from $samples$ with the best fitness
15: end procedure
```

5.4 Scalability

In addition to modularity and competitive performance, the other core advantage of ME2 is its ability to speed-up its execution, considerably, via parallel processing. The steps of **explore** and **exploit** are performed by ME2 for each point independently of all other points in the population. So given sufficient resources, one can potentially explore around and maximally exploit each point using the resources of one real/virtual processing node, without having to communicate with any of the other nodes. Our implementation of ME2 is done using Java, for which we used Java threads for a multi-threaded implementation. Using a multi core system, we executed both single and multi-threaded versions, and calculated speed-up in terms of execution time. In figure 5 speed-up is defined as f_s/f_t, where f_s is the execution time for the single-threaded version while f_t is the execution time for the multi-threaded version. For the multi-threaded implementation each sample point runs explore and exploit on a single thread. So with a *sampleSize* of 48, there will be 48 parallel threads, running in parallel during exploration and then again during exploitation. For the single threaded implementation, all the operations on all the points are executed in serial fashion. The results show that overall execution time decreased by nearly 7 times as we increased the number of cores from 3 to 24. It is also worth noting that there is an approximately linear increase in the speed of execution as a function of increasing computing nodes. Algorithm 5 describes the multi threaded implementation of ME2.

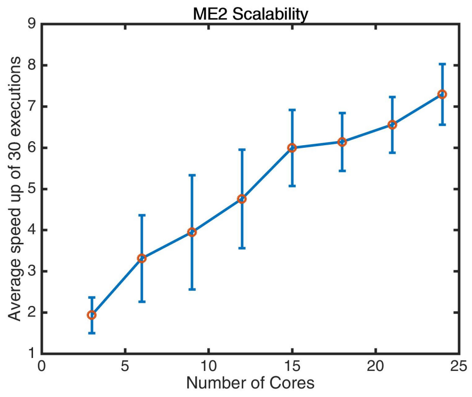


Figure 5: ME2 Scalability using multiple cores. The x-axis of this graph represents the number of cores employed, from 3 to 24. The Y-axis represents the speed-up of execution time between single and multi-threaded ME2 implementations. The multi-threaded implementation of ME2 speeds-up execution by nearly 7 folds when using 24 cores.

6 CONCLUSION

Map Explore & Exploit is a modular and scalable meta-heuristic, suitable for multi-modal multi-dimension optimization, with better or similar performance to other well-known search & optimization algorithms. We present the methodology of ME2 in detail, starting with Map and moving on to Explore and Exploit. We compare ME2 to GA, PSO, SA and CMA-ES. The comparison was carried out using several scalable benchmark functions. ME2's competitive results are shown for dimensions 10, 20 and 50. In addition, we demonstrate the computational scalability of ME2 by comparing a single-threaded version and a multiple-threaded one, running in a multi-core processing environment. The results confirm that ME2, due to the distributed nature of its last two phases, is highly scalable. Multi-threaded ME2's running time decreases in a near-linear fashion, as the number of processing nodes increases. Finally, ME2's tri-modular architecture allows researchers to test other - potentially better - algorithms for each search phase, as long as a proper fitness function is defined for (the conclusion of) each phase. Exploring that potential is our next research objective.

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